Abstract

Let \( V(G) \), be the vertex set of graph \( G \), we define \( V_d = \{ v \in V(G) : S_G(u) = d \} \) in which \( S_G(u) = \sum_{v \in N_G(u)} d_G(v) \) and \( N_G(u) = \{ v \in V(G) : uv \in E(G) \} \). In this paper we express the explicit formula for Sanskruti index \( S(G) \), fifth \( M_1 \) and \( M_2 \) Zagreb indices, fifth \( M_1 \) and \( M_2 \) multiplicative Zagreb indices for Dutch windmill graph, Kulli cyclic windmill graph, Kulli path windmill graph and French windmill graph.

Key words: Topological index, Dutch windmill graph, Kulli cyclic windmill graph, Kulli path windmill graph, French windmill graph.

AMS classification: 05C09, 05C92, 05C99

1. Introduction

Let \( G \) be a simple graph having vertex set \( V(G) \) and edge set \( E(G) \). Let \( p \) the order of a graph i.e. \( |V(G)| = p \) and the size of a graph be \( q \) i.e. \( |E(G)| = q \). Generally, the degree of a vertex \( v \) is denoted by \( d_G(v) \) which is the number of edges incident to the vertex \( v \). Many topological indices are defined based on the degree of the vertex; the topological index is a numerical value associated with the graph. Topological indices have many applications in the various disciplines like chemistry, biology, etc. Some of the standard topological indices are listed below.

The Sanskruti index was introduced by S. Hosamani [4], which is defined as follow

\[
S(G) = \sum_{uv \in E(G)} \left( \frac{S_G(u)S_G(v)}{S_G(u) + S_G(v) - 2} \right)^3
\]
The fifth $M_1$ and $M_2$ Zagreb indices introduced by Graovac [1] and defined as

$$M_1 G_5(G) = \sum_{uv \in E(G)} (S_G(u) + S_G(v))$$

$$M_2 G_5(G) = \sum_{uv \in E(G)} (S_G(u) \cdot S_G(v))$$

The fifth $M_1$ and $M_2$ multiplicative Zagreb indices proposed by Kulli [6] defined as

$$M_1 G_5 \prod(G) = \prod_{uv \in E(G)} (S_G(u) + S_G(v))$$

$$M_2 G_5 \prod(G) = \prod_{uv \in E(G)} (S_G(u) \cdot S_G(v))$$

2. Dutch Windmill Graph

For $m \geq 2, n \geq 5$, the graph constructed by taking $m$ copies of the cycle $C_n$ with a vertex in common is called Dutch windmill graph, and is generally denoted by $D_n^m$, which is shown in Figure 1.

![Figure 1: Dutch windmill graph $D_n^m$](image)

For $m \geq 2, n \geq 5$, let $G = D_n^m$ be a Dutch windmill graph then $G$ has $1 + m(n - 1)$ vertices and $mn$ edges, moreover, $G$ has 3 types of edge partitions.
(S_u, S_v) where \( u \in E(G) \) where \( u \in E(G) \)

\[ \begin{array}{|c|c|c|c|}
\hline
&S_u, S_v & (4m, 2(m + 1)) & (2(m + 1), 4) & (4, 4) \\
\hline
No. of edges & 2m & 2m & mn - 4m \\
\hline
\end{array} \]

**Theorem 2.1** For the Dutch windmill graph \( G = D_m^n \), Sanskruti index is

\[ S(G) = \frac{128m}{27(m+2)^3}(m^6 + 9m^5 + 33m^4 + 90m^3 + 143m^2 + 117m + 4n + 19). \]

Proof: By the definition of Sanskruti index and the information from Table 1, we have

\[
S(G) = 2m \left( \frac{4m \cdot 2(m + 1)}{4m \cdot 2(m + 2) - 2} \right)^3 + 2m \left( \frac{2(m + 1) \cdot 4}{2(m + 2) + 4 - 2} \right)^3 + (mn - 4m) \left( \frac{16}{6} \right)^3 \\
= 2m \left( \frac{8m^2 + 8m}{6m} \right)^3 + 2m \left( \frac{8m + m}{2m + 4} \right)^3 + (mn - 4m) \left( \frac{8}{3} \right)^3 \\
= \frac{128m}{27} (m + 1)^3 + 128m \left( \frac{m + 1}{m + 2} \right)^3 + (mn - 4m) \left( \frac{8}{3} \right)^3 \\
= \frac{128m}{27} \left( (m + 1)^3 + 27 \left( \frac{m + 1}{m + 2} \right)^3 + 4(n - 4) \right) \\
= \frac{128m}{27} \left( (m + 1)^3(m + 2)^3 + 27(m + 1)^3 + (4n - 16)(m + 2)^3 \right)
\]

**Theorem 2.2** For the Dutch windmill graph \( G = D_m^n \), the fifth \( M_1 \) and \( M_2 \) Zagreb indices are

\[
M_1 G_5(G) = 16m^2 - 16m + 8mn \\
M_2 G_5(G) = 16m^3 + 32m^2 + 16mn - 48m
\]

Proof: By the definition of the fifth \( M_1 \) and \( M_2 \) Zagreb indices and the information from Table 1, we have

Case:1

\[
M_1 G_5(G) = 2m[4m + 2(m + 1)] + 2m[2(m + 1) + 4] + (mn - 4m)[4 + 4] \\
= 2m[6m + 2] + 2m[2m + 6] + 8mn - 32m \\
= 16m^2 - 16m + 8mn
\]
$M_2 G_5(G) = 2m[4m \cdot 2(m + 1)] + 2m[2(m + 1) \cdot 4] + (mn - 4m)[4 \cdot 4]$

$= 2m[8m^2 + 8m] + 2m[8m + 8] + 16mn - 64m$

$= 16m^3 + 32m^2 + 16mn - 48m$

(1)

**Theorem 2.3** For the Dutch windmill graph $G = D_n^m$, fifth $M_1$ and $M_2$ multiplicative Zagreb indices are

$M_1 G_5 \prod(G) = (12m^2 + 40m + 12)^{2m} \cdot 8^{(mn - 4m)}.$

$M_2 G_5 \prod(G) = (64m^3 + 128m^2 + 64m)^{2m} \cdot 16^{(mn - 4m)}.$

Proof: By the definition of the fifth $M_1$ and $M_2$ multiplicative Zagreb indices and from Table 1, we have

Case: 1

$M_1 G_5 \prod(G) = (4m + 2(m + 1))^{2m} \cdot (2(m + 1) + 4)^{2m} \cdot (4 + 4)^{(mn - 4m)}$

$= (6m + 2)^2 m \cdot (2m + 6)^{2m} \cdot 8^{(mn - 4m)}$

$= ((6m + 2)(2m + 6))^{2m} \cdot 8^{(mn - 4m)}$

$= (12m^2 + 40m + 12)^{2m} \cdot 8^{(mn - 4m)}$

Case: 2

$M_2 G_5 \prod(G) = (4m \cdot 2(m + 1))^{2m} \cdot (2(m + 1) \cdot 4)^{2m} \cdot (4 \cdot 4)^{(mn - 4m)}$

$= (8m^2 + 8m)^{2m} \cdot (8m + 8)^{2m} \cdot 16^{(mn - 4m)}$

$= ((8m^2 + 8m)(8m + 8))^{2m} \cdot 16^{(mn - 4m)}$

$= (64m^3 + 128m^2 + 64m)^{2m} \cdot 16^{(mn - 4m)}$

**3. Kulli Cyclic Windmill Graph**

For $n \geq 3$ with a vertex $K_1$ in common, the graph constructed by taking $m$ copies of the cycle $K_1 + C_n$ is called Dutch windmill graph, and is generally denoted by $C_{n+1}^m$, which is shown in Figure 2.

For $m \geq 2, n \geq 5$, let $G = C_{n+1}^m$ be a Kulli cycle windmill graph with $mn + 1$
Figure 2: Kulli Cyclic windmill graph $C_{(n+1)}^m$

vertices and $2mn$ edge then G has two type of edge partition as given in Table 2.

<table>
<thead>
<tr>
<th>$(S_u, S_v)$ where $uv \in E(G)$</th>
<th>$(3mn, mn + 6)$</th>
<th>$(nm + 6, nm + 6)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of edges</td>
<td>$nm$</td>
<td>$nm$</td>
</tr>
</tbody>
</table>

**Theorem 3.1** For the Kulli cyclic windmill graph $G = C_{n+1}^m$, Sanskruti index is

\[
S(G) = \frac{nm(nm+6)^3}{8} \left( \left( \frac{3nm}{2nm+2} \right)^3 + \left( \frac{nm+6}{nm+5} \right)^3 \right).
\]

Proof: By the definition of Sanskruti index and the information from Table 2, we have

\[
S(G) = nm\left( \frac{3nm \cdot (nm + 6)}{3nm + nm + 6 - 2} \right)^3 + nm\left( \frac{(nm+6) \cdot (nm+6)}{nm+6+nm+6-2} \right)^3
\]

\[
= nm(nm+6)^3\left( \frac{3nm}{4nm+4} \right)^3 + \left( \frac{nm+6}{2nm+10} \right)^3
\]

\[
= \frac{nm(nm+6)^3}{8} \left( \left( \frac{3nm}{2nm+2} \right)^3 + \left( \frac{nm+6}{nm+5} \right)^3 \right)
\]

**Theorem 3.2** For the Kulli cyclic windmill graph $G = C_{(n+1)}^m$, fifth $M_1$ and $M_2$ Zagreb indices are
Proof: By the definition of the fifth $M_1$ and $M_2$ Zagreb indices and from Table 2, we have
Case: 1

\[ M_1 G_5(G) = nm(3nm + nm + 6) + nm(nm + 6 + nm + 6) \]
\[ = nm(4nm + 6) + nm(2nm + 12) \]
\[ = 4n^2m^2 + 6nm + 2n^2m^2 + 12nm \]
\[ = 6n^2m^2 + 18nm \]

Case: 2

\[ M_2 G_5(G) = nm[3nm \cdot (nm + 6)] + nm[(nm + 6) \cdot (nm + 6)] \]
\[ = nm(3n^2m^2 + 18nm) + nm(n^2m^2 + 12nm + 36) \]
\[ = 3n^3m^3 + 18n^2m^2 + n^3m^3 + 12n^2m^2 + 36nm \]
\[ = 4n^3m^3 + 30n^2m^2 + 36nm \]

**Theorem 3.3** For the Kulli cyclic windmill graph $G = C_{(n+1)}^m$, fifth $M_1$ and $M_2$ multiplicative Zagreb indices are

\[ M_1 G_5 \prod (G) = (8n^2m^2 + 84nm + 72)^{nm} \]
\[ M_2 G_5 \prod (G) = (3n^4m^4 + 54n^3m^3 + 324n^2m^2 + 648nm)^{nm} \]

Proof: By the definition of the fifth $M_1$ and $M_2$ multiplicative Zagreb indices and from Table 2, we have
Case: 1

\[ M_1 G_5 \prod (G) = (3nm + nm + 6)^{nm} \cdot (nm + 6 + nm + 6)^{nm} \]
\[ = (4nm + 6)^{nm} \cdot (2nm + 12)^{nm} \]
\[ = ((4nm + 6) \cdot (2nm + 12))^{nm} \]
\[ = [8n^2m^2 + 84nm + 72]^{nm} \]
Case: 2

\[ M_2 G_5 \prod (G) = (3nm \cdot (nm + 6))^{nm} \cdot ((nm + 6) \cdot (nm + 6))^{nm} \]
\[ = ((3n^2m^2 + 18nm) \cdot (nm + 6)^2)^{nm} \]
\[ = ((3n^2m^2 + 18nm) \cdot (n^2m^2 + 12nm + 36))^{nm} \]
\[ = (3n^4m^4 + 54nm^3m^3 + 324nm^2m^2 + 648nm)^{nm} \]

4. Kulli Path Windmill Graph:

Kulli path windmill graph is the graph contains \( m \) sets of the graph \( K_1 + P_n \) with a common vertex and it is denoted by \( P^m_{(n+1)} \) and it is shown in Figure 3. Let

\[ G = P^m_{(n+1)}, \text{ for } m \geq 2, n \geq 6 \] be a Kulli path windmill graph then \( G \) has \( mn + 1 \) vertices, \( 2nm - m \) edges and also \( G \) has six types of edge partition given in Table 3.

**Table 3.**

<table>
<thead>
<tr>
<th>(( S_u, S_v )) where ( uv \in E(G) )</th>
<th>(3 + nm, 3nm − 2m)</th>
<th>(5 + nm, 3 + nm)</th>
<th>(6 + nm, 5 + nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of edges</td>
<td>2m</td>
<td>2m</td>
<td>2m</td>
</tr>
<tr>
<td>(( S_u, S_v )) where ( uv \in E(G) )</td>
<td>(6 + nm, 6 + nm)</td>
<td>(5 + nm, 3nm − 2m)</td>
<td>(6 + nm, 3nm − 2m)</td>
</tr>
<tr>
<td>No. of edges</td>
<td>nm − 5m</td>
<td>2m</td>
<td>nm − 4m</td>
</tr>
</tbody>
</table>
Theorem 4.1 For the Kulli path windmill graph $G = P_{n+1}^m$, Sanskruti index is

$$S(G) = 2m\left(\frac{3n^2m^2+9nm-2nm^2-6m}{4nm-2m+1}\right)^3 + 2m\left(\frac{n^2m^2+8nm+15}{2nm+6}\right)^3 + 2m\left(\frac{n^2m^2+11nm+30}{2nm+9}\right)^3 +$$

$$(nm - 5m)\left(\frac{n^2m^2+12nm+36}{nm+10}\right)^3 + 2m\left(\frac{3n^2m^2+15nm-2nm^2-10m}{4nm-2m+3}\right)^3 + (nm - 4m)\left(\frac{3n^2m^2+18nm-2nm^2-12m}{5nm-2m+4}\right)^3.$$

Proof: By the definition of Sanskruti index and the information from Table 3, we have

$$S(G) = 2m\left(\frac{(3 + nm)(3nm - 2m)}{3 + nm + 3nm - 2m - 2}\right)^3 + 2m\left(\frac{(5 + nm)(3 + nm)}{5 + nm + 3 + nm - 2}\right)^3 +$$

$$+ 2m\left(\frac{(6 + nm)(5 + nm)}{6 + nm + 5 + nm - 2}\right)^3 + (nm - 5m)\left(\frac{(6 + nm)(6 + nm)}{6 + nm + 6 + nm - 2}\right)^3 +$$

$$+ 2m\left(\frac{(5 + nm)(3nm - 2m)}{5 + nm + 3nm - 2m - 2}\right)^3 + (nm - 4m)\left(\frac{(6 + nm)(3nm - 2m)}{6 + nm + 3nm - 2m - 2}\right)^3 +$$

$$= 2m\left(\frac{3n^2m^2 + 9nm - 2nm^2 - 6m}{4nm - 2m + 1}\right)^3 + 2m\left(\frac{n^2m^2 + 8nm + 15}{2nm + 6}\right)^3 +$$

$$+ 2m\left(\frac{n^2m^2 + 11nm + 30}{2nm + 9}\right)^3 + (nm - 5m)\left(\frac{n^2m^2 + 12nm + 36}{nm + 10}\right)^3 +$$

$$+ 2m\left(\frac{3n^2m^2 + 15nm - 2nm^2 - 10m}{4nm - 2m + 3}\right)^3 + (nm - 4m)\left(\frac{3n^2m^2 + 18nm - 2nm^2 - 12m}{5nm - 2m + 4}\right)^3.$$

Theorem 4.2 For the Kulli path windmill graph $G = P_{n+1}^m$ the fifth $M_1$ and $M_2$ Zagreb indices are

$$M_1G_5(G) = 6n^2m^2 + 18nm - 4nm^2 - 30m$$

$$M_2G_5(G) = 4n^3m^3 - 3n^2m^3 - 58nm^2 + 30n^2m^2 + 16m^2 + 36nm - 90m$$

Proof: By the definition of the fifth $M_1$ and $M_2$ Zagreb indices and from Table 3, we have
Case:1

\[ M_1 G_5(G) = 2m(3 + nm + 3nm - 2m) + 2m(5 + nm + 3 + nm) + 2m(6 + nm + 5 + nm) \\
+ (nm - 5m)(6 + nm + 6 + nm) + 2m(5 + nm + 3nm - 2m) \\
+ (nm - 4m)(6 + nm + 3nm - 2) \]

\[ = 2m(4nm - 2m + 3) + 2m(2nm + 8) + 2m(2nm + 11) + (nm - 5m)(2nm + 12) \\
+ 2m(4nm - 2m + 5) + (nm - 4m)(4nm - 2m + 6) \]

\[ = 8nm^2 - 4m^2 + 6m + 4nm^2 + 16m + 4n^2m^2 + 22m + 2n^2m^2 + 12nm - 10nm^2 \\
- 60m + 8n^2m - 4m^2 + 10m + 4n^2m^2 - 2nm^2 + 6nm - 16nm^2 + 8m^2 - 24m \]

\[ = 6n^2m^2 + 18nm - 4nm^2 - 30m \]

case:2

\[ M_2 G_5(G) = 2m((3 + nm) \cdot (3nm - 2m)) + 2m((5 + nm) \cdot (3 + nm)) + 2m((6 + nm) \cdot (5 + nm) \\
+ (nm - 5m)(6 + nm) \cdot (6 + nm) + 2m((5 + nm)(3nm - 2m)) \\
+ (nm - 4m)(6 + nm) \cdot (3nm - 2m) \]

\[ = 2m(3n^2m^2 - 2nm^2 + 9nm - 6m) + 2m(n^2m^2 + 8nm + 15) + 2m(n^2m^2 + 11nm + 3) \\
+ (nm - 5m)(n^2m^2 + 12nm + 36) + 2m(3n^2m^2 - 2nm^2 + 15nm - 10m) \\
+ (nm - 4m)(3n^2m^2 - 2nm^2 + 18nm - 12m) \]

\[ = 6n^2m^3 - 4nm^3 + 18nm^2 - 12m^2 + 2n^2m^3 + 16nm^2 + 30m + 2n^2m^3 + 22nm^2 \\
+ 60m + n^3m^3 + 12n^2m^2 + 36nm - 5n^2m^3 - 60nm^2 - 180m + 6n^2m^3 - 4nm^3 \\
+ 30n^2m^2 - 20m^2 + 3n^3m^3 - 2n^2m^3 + 18n^2m^2 - 12nm^2 - 12n^2m^3 \\
+ 8nm^3 - 72nm^2 + 48m^2 \]

\[ = 4n^3m^3 - 3n^2m^3 - 58nm^2 + 30n^2m^2 + 16m^2 + 36nm - 90m \]

**Theorem 4.3** For the Kulli path windmill graph \( G = P_{n+1}^m \), fifth \( M_1 \) and \( M_2 \) multiplicative Zagreb indices are

\[ M_1 G_5 \prod(G) = (4nm - 2m + 3)^{2m} \cdot (2nm + 8)^{2m} \cdot (2nm + 11)^{2m} \cdot (2nm + 12)^{(nm - 5m)} \cdot \\
(4nm - 2m + 5)^{2m} \cdot (2nm - 2m + 6)^{(nm - 4m)} \]

\[ M_2 G_5 \prod(G) = (3n^2m^2 - 2nm^2 + 9nm - 6m)^{2m} \cdot (n^2m^2 + 8nm + 15)^{2m} \cdot (n^2m^2 + 11nm + 30)^{2m} \cdot (n^2m^2 + 12nm + 36)^{(nm - 5m)} \cdot (3n^2m^2 - 2nm^2 + 15nm - 10m)^{2m} \cdot (3n^2m^2 -
Proof: By the definition of the fifth $M_1$ and $M_2$ multiplicative Zagreb indices and from Table 3. we have

Case: 1

\[ M_1G_5 \prod(G) = (3 + nm + 3nm - 2m)^{2m} \cdot (5 + nm + 3 + nm)^{2m} \cdot (6 + nm + 5 + nm)^{2m} \]
\[ \cdot (6 + nm + 6 + nm)^{(nm-5m)} \cdot (5 + nm + 3nm - 2m)^{2m} \]
\[ \cdot (6 + nm + 3nm - 2m)^{(nm-4m)} \]
\[ = (4nm - 2m + 3)^{2m} \cdot (2nm + 8)^{2m} \cdot (2nm + 11)^{2m} \]
\[ \cdot (2nm + 12)^{(nm-5m)} \cdot (4nm - 2m + 5)^{2m} \cdot (2nm - 2m + 6)^{(nm-4m)} \]

Case: 2

\[ M_2G_5 \prod(G) = ((3 + nm) \cdot (3nm - 2m))^{2m} \cdot ((5 + nm) \cdot (3 + nm))^{2m} \cdot ((6 + nm) \cdot (5 + nm))^{2m} \]
\[ \cdot ((6 + nm) \cdot (6 + nm))^{(nm-5m)} \cdot (5 + nm) \cdot (3nm - 2m)^{2m} \]
\[ \cdot ((6 + nm) \cdot (3nm - 2m))^{(nm-4m)} \]
\[ = (3n^2m^2 - 2nm^2 + 9nm - 6m)^{2m} \cdot (n^2m^2 + 8nm + 15)^{2m} \cdot (n^2m^2 + 11nm + 30)^{2m} \]
\[ \cdot (n^2m^2 + 12nm + 36)^{(nm-5m)} \cdot (3n^2m^2 - 2nm^2 + 15nm - 10m)^{2m} \]
\[ \cdot (3n^2m^2 - 2nm^2 + 18nm - 12m)^{(nm-4m)} \]

5. French Windmill Graph:

For $m \geq 2, n \geq 5$ with a common vertex, the graph constructed by taking $m$ copies of complete graph $K_n$ is called French Windmill Graph, and is generally denoted by $F^m_n$, which is shown in Figure 4. Let $G = F^m_n$ for $m \geq 2, n \geq 2$ be a French Windmill Graph then $G$ has $1 + m(n - 1)$ vertices, $mn/2(n - 1)$ edges and also $G$ has two types of edge partition given in Table 4.
(\text{No. of edges}) & m(n-1) & \frac{1}{2}m(n-1)(n-2) \\
\hline
\end{tabular}

**Theorem 5.1** For the French Windmill graph $G = F^m_n$ the Sanskruti index is

$$S(G) = m(n-1)^7(m+n-2)^3 \left( \frac{m^3(n-1)^3}{(m(n-1)^2 + (n-1)(m+n-2)-2)^3} + \frac{(n-2)(m+n-2)^3}{16((n-1)(m+n-2)-1)^3} \right)$$

Proof: By definition of Sanskruti index and from Table 4, we get

$$S(G) = m(n-1)^7(m+n-2)^3 \left( \frac{m(n-1)^2 \cdot (n-1)(m+n-2) \cdot (n-1)(m+n-2)}{(m(n-1)^2 + (n-1)(m+n-2)-2)} \right)^3$$

$$+ \frac{1}{2} m(n-1)(n-2) \left( \frac{(n-1)(m+n-2) \cdot (n-1)(m+n-2)}{(n-1)(m+n-2) + (n-1)(m+n-2)-2} \right)^3$$

$$= m(n-1)^7(m+n-2)^3 \left( \frac{m^3(n-1)^3}{(m(n-1)^2 + (n-1)(m+n-2)-2)^3} \right)$$

$$+ \left( \frac{(n-2)(m+n-2)^3}{16((n-1)(m+n-2)-1)^3} \right).$$

**Theorem 5.2** For the French Windmill graph $G = F^m_n$ the fifth $M_1$ and $M_2$ Zagreb indices are

$$M_1G_5(G) = m(n-1)^2(n^2 + 2nm - 2m - 3n + 2)$$

$$M_2G_5(G) = \frac{1}{2}m(n-1)^3(m + n - 2)(n^2 + 3nm - 4m - 4n + 4)$$
Proof: By the definition of the fifth $M_1$ and $M_2$ Zagreb indices and from Table 4, we have

Case:1

$$M_1G_5(G) = m(n - 1)(m(n - 1)^2 + (n - 1)(m + n - 2))$$
$$+ {1 \over 2}m(n - 1)(m - 2)((n - 1)(m + n - 2) + (n - 1)(m + n - 2))$$
$$= m(n - 1)^2(m - 1) + (m + n - 2)(m + n - 2)$$
$$= m(n - 1)^2(n^2 + 2mn - 2m - 3n + 2)$$

Case:2

$$M_2G_5(G) = m(n - 1)(m(n - 1)^2 \cdot (n - 1)(m + n - 2))$$
$$+ {1 \over 2}m(n - 1)(m - 2)((n - 1)(m + n - 2) \cdot (n - 1)(m + n - 2))$$
$$= m^2(n - 1)^2(m + n - 2) + {1 \over 2}m(n - 1)^3(n - 2)(m + n - 2)^2$$
$$= {1 \over 2}m(n - 1)^3(m + n - 2)(2m(n - 1) + (n - 2)(m + n - 2))$$
$$= {1 \over 2}m(n - 1)^3(m + n - 2)(n^2 + 3mn - 4m - 4n + 4)$$

**Theorem 5.3** For the French windmill graph $G = F^m_n$, fifth $M_1$ and $M_2$ multiplicative Zagreb indices are

$$M_1G_5 \prod(G) = (m(n - 1)^2 + (n - 1)(m + n - 2))^{m(n - 1)}. (2(n - 1)(m + n - 2))^{(1/2)m(n - 1)(n - 2)}$$

$$M_2G_5 \prod(G) = (m(n - 1)^3(m + n - 2))^{m(n - 1)}$$

Proof: By the definition of the fifth $M_1$ and $M_2$ multiplicative Zagreb indices and from Table 4, we have

Case:1

$$M_1G_5 \prod(G) = (m(n - 1)^2 + (n - 1)(m + n - 2))^{m(n - 1)}. ((n - 1)(m + n - 2))^{(1/2)m(n - 1)(n - 2)}$$

$$+ (n - 1)(m + n - 2))^{(1/2)m(n - 1)(n - 2)}$$

$$= (m(n - 1)^2 + (n - 1)(m + n - 2))^{m(n - 1)}. (2(n - 1)(m + n - 2))^{(1/2)m(n - 1)(n - 2)}$$
Case: 2

\[ M_2 G_5 \prod (G) = (m(n-1)^2(n-1)(m+n-2))^{m(n-1)} \cdot ((n-1)(m+n-2)^{(1/2)(m(n-1)(n-2))}}
\]
\[ = (m(n-1)^3(m+n-2)^{m(n-1)}((n-1)(m+n-2)^{m(n-1)(n-2)})^3 \]

6 Conclusion

For the given graph \( G \) with number of vertex or edges, above results gives explicit formula for Sanskruti index, fifth \( M_1 \) and \( M_2 \) Zagreb indices, fifth \( M_1 \) and \( M_2 \) multiplicative Zagreb indices of Dutch windmill graph, Kulli cyclic windmill graph, Path windmill graph and French windmill graph. These developments achieve remarkable advances in chemical sciences in which compounds undergo a similar structure of the considered graph.

References


