

Approximate Fixed Point Theorems For Weak Contractions On Neutrosophic Normed Spaces

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Abstract

In this paper, we define concept of approximate fixed point property of a function and a set in Neutrosophic normed space. Furthermore, we give Neutrosophic version of some class of maps used in fixed point theory and investigate approximate fixed point property of these maps.

Key words: Fixed point, Neutrosophic normed space, Classes of contractions.

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1 Introduction

Fuzzy Set [FS] initiated by Zadeh [21] has influenced profoundly every one of the logical fields since 1965. It is seen that the idea play a vital role to solve many real life problems, but it is not enough to address certain issues. Atanassov [1] introduced Intuitionistic Fuzzy Set [IFS] for such cases. After defining the IFS, it extends the results those are studied over FS. Neutrosophic Set [NS], defined by Smarandache [16], is another variant of the crisp set which is Neutrosophy theory was published in the year 1998 and included in the literature [17]. Park [15] derived Intuitionistic Fuzzy Metric Space [IFMS] as a generalization of Fuzzy Metric Space. Branzei et al [2] further extended these results to multifunctions in Banach spaces. Berinde [3] obtained approximate fixed point theorems for operators satisfying Kannan, Chatterjea and Zamfirescu type of conditions on metric spaces.

After a while, Smarandache introduced the notion of NS, which is the different

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kind of the notation of the classical set theory by adding an intermediate membership function. This set is a formal setting trying to measure the truth, indeterminacy and falsehood. Quite recently, Jeyaraman et al. [10] introduced the notion of Neutrosophic normed space and statistical convergence. Since Neutrosophic Normed Space [NNS] is a natural generalization of IFNS and statistical convergence.

In this paper, we define concept of approximate fixed point property of a function and a set in NNS. Furthermore, we give Neutrosophic version of some class of maps used in fixed point theory and investigate approximate fixed point property of these maps.

2 Preliminaries

Definition 2.1 The 6-tuple $(X, \mu, \nu, \omega, *, \diamond, \otimes)$ is said to be a Neutrosophic Normed Space if X is a vector space, $*$ and $\diamond, \otimes$ be the CTN and CTC, respectively and μ, ν, ω are Normed spaces on $X \times (0, \infty)$ fulfilling the conditions below: For each $x, y \in X$ and for each $s, t > 0, \emptyset \neq 0$,

(NNS-1) $0 \leq \mu(x, t) \leq 1, 0 \leq \nu(x, t) \leq 1, 0 \leq \omega(x, t) \leq 1$, for all $t \in (0, \infty)$;

(NNS-2) $\mu(x, t) + \nu(x, t) + \omega(x, t) \leq 3$;

(NNS-3) $\mu(x, t) > 0$;

(NNS-4) $\mu(x, t) = 1$ if and only if $x = 0$;

(NNS-5) $\mu(\emptyset x, t) = \mu(x, \frac{1}{|\emptyset|})$, for each $\emptyset \neq 0$;

(NNS-6) $\mu(x, t) * \mu(y, s) \leq \mu(x + y, t + s)$;

(NNS-7) $\mu(x, .) : (0, \infty) \rightarrow [0, 1]$ is continuous and increasing;

(NNS-8) $\lim_{t \rightarrow \infty} \mu(x, t) = 1$ and $\lim_{t \rightarrow 0} \mu(x, t) = 0$;

(NNS-9) $\nu(x, t) < 1$;

(NNS-10) $\nu(x, t) = 0$ if and only if $\nu = 0$;

(NNS-11) $\nu(\emptyset x, t) = \nu(x, \frac{t}{|\emptyset|})$, for each $\emptyset \neq 0$;

(NNS-12) $\nu(x, t) \diamond \nu(y, s) \geq \nu(x + y, t + s)$;

(NNS-13) $\nu(x, .) : (0, \infty) \rightarrow [0, 1]$ is continuous and increasing;

(NNS-14) $\lim_{t \rightarrow \infty} \nu(x, t) = 0$ and $\lim_{t \rightarrow 0} \nu(x, t) = 1$;

(NNS-15) $\omega(x, t) < 1$;

(NNS-16) $\omega(x, t) = 0$ if and only if $\omega = 0$;

(NNS-17) $\omega(\emptyset x, t) = \omega(x, \frac{t}{|\emptyset|})$, for each $\emptyset \neq 0$;

(NNS-18) $\omega(x, t) \otimes \omega(y, s) \geq \omega(x + y, t + s)$;

(NNS-19) $\omega(x, .) : (0, \infty) \rightarrow [0, 1]$ is continuous and increasing;



(NNS-20) $\lim_{t \rightarrow \infty} \omega(x, t) = 0$ and $\lim_{t \rightarrow 0} \omega(x, t) = 1$;
 Then (μ, ν, ω) is called Neutrosophic Norm [NN].

Lemma 2.2 Let (μ, ν, ω) be NN on X . The following hold:

- (i) $\mu(x, \cdot), \nu(x, \cdot)$ and $\omega(x, \cdot)$ are non-decreasing, non-increasing and non-increasing for all $x \in X$, respectively.
- (ii) $\mu(x - y, t) = \mu(y - x, t), \nu(x - y, t) = \nu(y - x, t)$ and $\omega(x - y, t) = \omega(y - x, t)$, for any $t > 0$.

Definition 2.3 A sequence (x_k) in $(X, \mu, \nu, \omega, *, \diamond, \otimes)$ converges to x if and only if $\mu(x_k - x, t) \rightarrow 1, \nu(x_k - x, t) \rightarrow 0$ and $\omega(x_k - x, t) \rightarrow 0$ as $k \rightarrow \infty$, for each $t > 0$. We denote the convergence of (x_k) to x by $x_k \xrightarrow{(\mu, \nu, \omega)} x$.

Definition 2.4 Let $(X, \mu, \nu, \omega, *, \diamond, \otimes)$ be a NNS. $(X, \mu, \nu, \omega, *, \diamond, \otimes)$ is said to be complete if every Cauchy sequence in $(X, \mu, \nu, \omega, *, \diamond, \otimes)$ is convergent.

Definition 2.5 Let X and Y be two NNSs. $f : X \rightarrow Y$ is continuous at $x_0 \in X$ if $(f(x_k))$ in Y converges to $f(x_0)$ for any (x_k) in X converging to x_0 . If $f : X \rightarrow Y$ is continuous at each element of X , then $f : X \rightarrow Y$ is said to be continuous on X .

Definition 2.6 Let $(X, \mu, \nu, \omega, *, \diamond, \otimes)$ be a NNS. $A \subset X$ is dense in $(X, \mu, \nu, \omega, *, \diamond, \otimes)$ if there exists a sequence (x_k) in A such that $x_k \xrightarrow{(\mu, \nu, \omega)} x$ for all $x \in X$.

Definition 2.7 Let $(X, \mu, \nu, \omega, *, \diamond, \otimes)$ be a NNS. We call the mapping $f : X \rightarrow X$ Neutrosophic contraction map, if there exists $a \in (0, 1)$ such that $\mu(f(x), f(y), at) \geq \mu(x, y, t), \nu(f(x), f(y), at) \leq \nu(x, y, t)$ and $(f(x), f(y), at) \leq \omega(x, y, t)$, for all $x, y \in X$ and $t > 0$.

Definition 2.8 Let $(X, \mu, \nu, \omega, *, \diamond, \otimes)$ be a NNS. We call the mapping $f : X \rightarrow X$ Neutrosophic nonexpansive, if $\mu(f(x), f(y), t) \geq \mu(x, y, t), \nu(f(x), f(y), t) \leq \nu(x, y, t)$ and $\omega(f(x), f(y), t) \leq \omega(x, y, t)$, for all $x, y \in X$ and $t > 0$.

Lemma 2.9 Let $(X, \mu, \nu, \omega, *, \diamond, \otimes)$ be a NNS.



(i) If $x_k \xrightarrow{(\mu, \nu, \omega)} x$ and $y_k \xrightarrow{(\mu, \nu, \omega)} y$,

$$\begin{aligned}\mu(x, y, t) &\leq \liminf_{k \rightarrow \infty} \mu(x_k, y_k, t) \\ \nu(x, y, t) &\geq \limsup_{k \rightarrow \infty} \nu(x_k, y_k, t) \text{ and} \\ \omega(x, y, t) &\geq \limsup_{k \rightarrow \infty} \omega(x_k, y_k, t), \text{ for all } t > 0.\end{aligned}$$

(ii) If $x_k \xrightarrow{(\mu, \nu, \omega)} x$ and $y_k \xrightarrow{(\mu, \nu, \omega)} y$,

$$\begin{aligned}\mu(x, y, t) &\geq \limsup_{k \rightarrow \infty} \mu(x_k, y_k, t), \\ \nu(x, y, t) &\leq \liminf_{k \rightarrow \infty} \nu(x_k, y_k, t) \text{ and} \\ \omega(x, y, t) &\leq \liminf_{k \rightarrow \infty} \omega(x_k, y_k, t), \text{ for all } t > 0.\end{aligned}$$

3 Main Results

Firstly, we define approximate fixed property, diameter of a set in NNSs and give examples.

Definition 3.1 Let $(X, \mu, \nu, \omega, *, \diamond, \otimes)$ be a NNS and $f : X \rightarrow X$ be a function. Given $\epsilon > 0$. It is said that $x_0 \in X$ is a neutrosophic ϵ - fixed point or approximate fixed point of f if $\mu(f(x_0) - x_0, t) > 1 - \epsilon$, $\nu(f(x_0) - x_0, t) < \epsilon$ and $\omega(f(x_0) - x_0, t) < \epsilon$, for all $t > 0$. We denote the set of neutrosophic ϵ - fixed points of f with $F_\epsilon^{(\mu, \nu, \omega)}(f)$.

Definition 3.2 It is said that f has the Neutrosophic Approximate Fixed Point Property [NAFPP] if $F_\epsilon^{(\mu, \nu, \omega)}(f)$ is not empty for every $\epsilon > 0$.

Example 3.3 Consider $f(x) = x^2$, defined on $(0, 1)$. $((0, 1), \mu, \nu, \omega, *, \diamond, \otimes)$ is an NNS with $\mu(x, t) = \frac{t}{t + |x|}$, $\nu(x, t) = \frac{|x|}{t + |x|}$ and $\omega(x, t) = \frac{|x|}{t}$, where $|\cdot|$ is usual norm on $(0, 1)$, $a * b = a.b$ and $a \diamond b = \min\{a + b, 1\}$, for all $a, b \in [a, b]$. As known, f has not any fixed point on $(0, 1)$. We investigate neutrosophic approximate fixed point of f . For every $\epsilon > 0$ and $t > 0$, there exists $x \in (0, 1)$ such that x satisfies

$$\begin{aligned}\mu(f(x) - x, t) &= \frac{t}{t + |f(x) - x|} > 1 - \epsilon, \\ \nu(f(x) - x, t) &= \frac{|f(x) - x|}{t + |f(x) - x|} < \epsilon \text{ and}\end{aligned}$$

$$\omega(f(x) - x, t) = \frac{|f(x) - x|}{t} < \epsilon, \text{ that is } |x^2 - x| < \frac{\epsilon t}{1 - \epsilon}.$$

So, f has the NAFPP, since $F_\epsilon^{(\mu, \nu, \omega)}(f)$ is not empty for every $\epsilon > 0$.

Definition 3.4 Let K be nonempty subset of $(X, \mu, \nu, \omega, *, \diamond, \otimes)$. We say that $(\delta_\mu(K), \delta_\nu(K), \delta_\omega(K))$ is Neutrosophic Diameter [ND] of K with respect to t , where $\delta_\mu(K) = \inf\{\mu(x - y, t) : x, y \in K\}$, $\delta_\nu(K) = \sup\{\nu(x - y, t) : x, y \in K\}$ and $\delta_\omega(K) = \sup\{\omega(x - y, t) : x, y \in K\}$, for $t > 0$.

Theorem 3.5 Let X be a NNS and $f : X \rightarrow X$ be a function. We suppose that

- (i) $F_\epsilon^{(\mu, \nu, \omega)}(f) \neq \emptyset$,
- (ii) There exists $\mu(\epsilon_1), \nu(\epsilon_2)$ and $\omega(\epsilon_3)$ such that

$$\begin{aligned} \mu(x - y, t) \geq \epsilon_1 * \mu(f(x) - f(y), t_1) &\Rightarrow \mu(x - y, t) \geq \vartheta(\epsilon_1), \\ \nu(x - y, t) \leq \epsilon_2 \diamond \nu(f(x) - f(y), t_2) &\Rightarrow \nu(x - y, t) \leq \vartheta(\epsilon_2) \text{ and} \\ \omega(x - y, t) \leq \epsilon_3 \diamond \omega(f(x) - f(y), t_3) &\Rightarrow \omega(x - y, t) \leq \vartheta(\epsilon_3), \end{aligned}$$

for $x, y \in F_\epsilon^{(\mu, \nu, \omega)}(f)$ and $\epsilon_1, \epsilon_2, \epsilon_3 \in (0, 1)$.

Then $\left(\delta_\mu\left(F_\epsilon^{(\mu, \nu, \omega)}(f)\right), \delta_\nu\left(F_\epsilon^{(\mu, \nu, \omega)}(f)\right), \delta_\omega\left(F_\epsilon^{(\mu, \nu, \omega)}(f)\right)\right) = (\vartheta(1 - \epsilon), \vartheta(\epsilon), \vartheta(\epsilon))$.

Proof: Let $\epsilon > 0$ and $x, y \in F_\epsilon^{(\mu, \nu, \omega)}(f)$. Then

$$\mu(f(x) - x, t) > 1 - \epsilon, \nu(f(x) - x, t) < \epsilon \text{ and } \omega(f(x) - x, t) < \epsilon,$$

$$\mu(f(y) - y, t) > 1 - \epsilon, \nu(f(y) - y, t) < \epsilon \text{ and } \omega(f(y) - y, t) < \epsilon.$$

It can be written

$$\begin{aligned} \mu(x - y, t) &\geq \mu\left(f(x) - x, \frac{t}{3}\right) * \mu\left(f(x) - f(y), \frac{t}{3}\right) * \mu\left(f(y) - y, \frac{t}{3}\right) \\ &\geq (1 - \epsilon) * (1 - \epsilon) * \mu\left(f(x) - f(y), \frac{t}{3}\right) \\ &= (1 - \epsilon) * \mu\left(f(x) - f(y), \frac{t}{3}\right), \\ \nu(x - y, t) &\leq \nu\left(f(x) - x, \frac{t}{3}\right) \diamond \nu\left(f(x) - f(y), \frac{t}{3}\right) \diamond \nu\left(f(y) - y, \frac{t}{3}\right) \end{aligned}$$



$$\begin{aligned} &\leq \epsilon \diamond \epsilon \diamond \nu \left(f(x) - f(y), \frac{t}{3} \right) = \epsilon \diamond \nu \left(f(x) - f(y), \frac{t}{3} \right) \text{ and} \\ \omega(x - y, t) &\leq \omega \left(f(x) - x, \frac{t}{3} \right) \otimes \omega \left(f(x) - f(y), \frac{t}{3} \right) \otimes \omega \left(f(y) - y, \frac{t}{3} \right) \\ &\leq \epsilon \otimes \epsilon \otimes \omega \left(f(x) - f(y), \frac{t}{3} \right) = \epsilon \otimes \omega \left(f(x) - f(y), \frac{t}{3} \right). \end{aligned}$$

By (ii), for every $x, y \in F_{\epsilon}^{(\mu, \nu, \omega)}(f)$, we get

$$\mu(x - y, t) \geq \vartheta(1 - \epsilon), \nu(x - y, t) \leq \vartheta(\epsilon) \text{ and } \omega(x - y, t) \leq \vartheta(\epsilon).$$

Hence, $\left(\delta_{\mu} \left(F_{\epsilon}^{(\mu, \nu, \omega)}(f) \right), \delta_{\nu} \left(F_{\epsilon}^{(\mu, \nu, \omega)}(f) \right), \delta_{\omega} \left(F_{\epsilon}^{(\mu, \nu, \omega)}(f) \right) \right) = (\vartheta(1 - \epsilon), \vartheta(\epsilon), \vartheta(\epsilon)).$

Now, we introduce neutrosophic asymptotic regularity to investigate NAFPP of some operators.

Definition 3.6 Let $(X, \mu, \nu, \omega, *, \diamond, \otimes)$ be a NNS and $f : X \rightarrow X$ be a function. It is said that f is Neutrosophic Asymptotic Regular [NAR] if

$$\begin{aligned} \lim_{k \rightarrow \infty} \mu(f^{k+1}(x) - f^k(x), t) &= 1, \\ \lim_{k \rightarrow \infty} \nu(f^{k+1}(x) - f^k(x), t) &= 0 \text{ and} \\ \lim_{k \rightarrow \infty} \omega(f^{k+1}(x) - f^k(x), t) &= 0, \end{aligned}$$

for every $x \in X$ and $t > 0$.

Theorem 3.7 Let $(X, \mu, \nu, \omega, *, \diamond, \otimes)$ be a NNS and $f : X \rightarrow X$ be a function. If f has NAR, then f has NAFPP.

Proof: Let x_0 be arbitrary element of X .

Since f is NAR,

$$\begin{aligned} \lim_{k \rightarrow \infty} \mu(f^{k+1}(x_0) - f^k(x_0), t) &= 1, \\ \lim_{k \rightarrow \infty} \nu(f^{k+1}(x_0) - f^k(x_0), t) &= 0 \text{ and} \\ \lim_{k \rightarrow \infty} \omega(f^{k+1}(x_0) - f^k(x_0), t) &= 0. \end{aligned}$$

In this case, for every $\epsilon > 0$ there exists $k_0(\epsilon, t) \in \mathbb{N}$ such that

$$\mu(f^{k+1}(x_0) - f^k(x_0), t) > 1 - \epsilon,$$

$$\begin{aligned}\nu(f^{k+1}(x_0) - f^k(x_0), t) &< \epsilon \text{ and} \\ \omega(f^{k+1}(x_0) - f^k(x_0), t) &< \epsilon, \text{ for every } k > k_0(\epsilon, t).\end{aligned}$$

If we, denote $f^k(x_0)$ by y_0 , we have

$$\begin{aligned}\mu(f^{k+1}(x_0) - f^k(x_0), t) &= \mu(f(f^k(x_0)) - f^k(x_0), t) = \mu(f(y_0) - y_0, t) > 1 - \epsilon, \\ \nu(f^{k+1}(x_0) - f^k(x_0), t) &= \nu(f(f^k(x_0)) - f^k(x_0), t) = \nu(f(y_0) - y_0, t) < \epsilon \quad \text{and} \\ \omega(f^{k+1}(x_0) - f^k(x_0), t) &= \omega(f(f^k(x_0)) - f^k(x_0), t) = \omega(f(y_0) - y_0, t) < \epsilon.\end{aligned}$$

This shows that y_0 is neutrosophic approximate fixed point of f .

Theorem 3.8 Let $(X, \mu, \nu, \omega, *, \diamond, \otimes)$ be a NNS and $f : X \rightarrow X$ be a neutrosophic contraction. Then $F_\epsilon^{(\mu, \nu, \omega)}(f) \neq \emptyset$ for every $\epsilon \in (0, 1)$.

Proof: Let $x \in X$ and $\epsilon \in (0, 1), t > 0$.

$$\begin{aligned}\mu(f^k(x) - f^{k+1}(x), t) &= \mu(f(f^{k-1}(x)) - f(f^k(x)), t) \geq \mu\left(f^{k-1}(x) - f^k(x), \frac{t}{a}\right) \\ &\geq \mu\left(f^{k-2}(x) - f^{k-1}(x), \frac{t}{a^2}\right) \geq \cdots \geq \mu\left(x - f(x), \frac{t}{a^k}\right), \\ \nu(f^k(x) - f^{k+1}(x), t) &= \nu(f(f^{k-1}(x)) - f(f^k(x)), t) \leq \nu\left(f^{k-1}(x) - f^k(x), \frac{t}{a}\right) \\ &\leq \nu\left(f^{k-2}(x) - f^{k-1}(x), \frac{t}{a^2}\right) \leq \cdots \leq \nu\left(x - f(x), \frac{t}{a^k}\right) \text{ and} \\ \omega(f^k(x) - f^{k+1}(x), t) &= \omega(f(f^{k-1}(x)) - f(f^k(x)), t) \leq \omega\left(f^{k-1}(x) - f^k(x), \frac{t}{a}\right) \\ &\leq \omega\left(f^{k-2}(x) - f^{k-1}(x), \frac{t}{a^2}\right) \leq \cdots \leq \omega\left(x - f(x), \frac{t}{a^k}\right).\end{aligned}$$

For $a \in (0, 1), k \rightarrow \infty \Rightarrow \frac{t}{a^k} \rightarrow 0$, by properties (NNS-8), (NNS-14), (NNS-20) of NN

$$\begin{aligned}\mu(f^k(x) - f^{k+1}(x), t) &\rightarrow 1, \\ \nu(f^k(x) - f^{k+1}(x), t) &\rightarrow 0 \text{ and} \\ \omega(f^k(x) - f^{k+1}(x), t) &\rightarrow 0.\end{aligned}$$

By Theorem (3.7) it follows that $F_\epsilon^{(\mu, \nu, \omega)}(f) \neq \emptyset$ for every $\epsilon \in (0, 1)$.

Example 3.9 The open interval $(0, 1)$ is a NNS with NN, t -norm and t -conorm given in Example (3.3). Consider $f : (0, 1) \rightarrow (0, 1)$ given by $f(x) = \frac{1}{2}x$. This map has not any fixed point in $(0, 1)$. Furthermore, f is a neutrosophic contraction map.

$$\begin{aligned}\mu\left(f(x) - f(y), \frac{t}{2}\right) &= \frac{\frac{t}{2}}{\frac{t}{2} + \left|\frac{x}{2} - \frac{y}{2}\right|} = \mu(x - y, t), \\ \nu\left(f(x) - f(y), \frac{t}{2}\right) &= \frac{\left|\frac{x}{2} - \frac{y}{2}\right|}{\frac{t}{2} + \left|\frac{x}{2} - \frac{y}{2}\right|} = \nu(x - y, t) \text{ and} \\ \omega\left(f(x) - f(y), \frac{t}{2}\right) &= \frac{\left|\frac{x}{2} - \frac{y}{2}\right|}{\frac{t}{2}} = \omega(x - y, t),\end{aligned}$$

for every $x, y \in (0, 1)$ and $t > 0$.

We write $\frac{1}{2}x < \frac{\epsilon t}{1-\epsilon}$ from

$$\begin{aligned}\mu(x - f(x), t) &= \mu\left(x - \frac{x}{2}, t\right) = \mu\left(\frac{x}{2}, t\right) = \frac{t}{t + \frac{x}{2}} > 1 - \epsilon \\ \nu(x - f(x), t) &= \nu\left(x - \frac{x}{2}, t\right) = \nu\left(\frac{x}{2}, t\right) = \frac{\frac{x}{2}}{t + \frac{x}{2}} < \epsilon \text{ and} \\ \omega(x - f(x), t) &= \omega\left(x - \frac{x}{2}, t\right) = \omega\left(\frac{x}{2}, t\right) = \frac{\frac{x}{2}}{t} < \epsilon.\end{aligned}$$

For every $\epsilon \in (0, 1)$ and $t > 0$ there exists $x \in (0, 1)$ such that $\frac{1}{2}x < \frac{\epsilon t}{1-\epsilon}$. So, f has NAFPP.

Definition 3.10 Let X be a NNS. If there exists $a \in (0, \frac{1}{2})$ such that

$$\begin{aligned}\mu(f(x) - f(y), at) &\geq \mu(x - f(x), t) * \mu(y - f(y), t), \\ \nu(f(x) - f(y), at) &\leq \nu(x - f(x), t) \diamond \nu(y - f(y), t) \text{ and} \\ \omega(f(x) - f(y), at) &\leq \omega(x - f(x), t) \diamond \omega(y - f(y), t),\end{aligned}$$

for every $x, y \in X$ and $t > 0$, then $f : X \rightarrow X$ is called neutrosophic Kannan operator.

Theorem 3.11 Let $(X, \mu, \nu, \omega, *, \diamond, \otimes)$ be a NNS having partial order relation denoted by $\preceq$, where $a * b = \min\{a, b\}$ and $a \diamond b = \max\{a, b\}$ and $f : X \rightarrow X$



be a neutrosophic Kannan operator satisfying $x \preceq f(x)$ for every $x \in X$. Assume that $\preceq \subset X \times X$ holds one of the following conditions:

- (i) $\preceq$ is subvector space,
- (ii) X is a totally ordered space.

If $\mu(., t)$ is non-decreasing, $\nu(., t)$ is non-increasing and $\omega(., t)$ is non-increasing, for all $t \in (0, \infty)$, $x \succeq \theta$ (θ is unit element in vector space X), then $F_\epsilon^{(\mu, \nu, \omega)}(f) \neq \emptyset$ for every $\epsilon \in (0, 1)$.

Proof: Let $x \in X$ and $\epsilon \in (0, 1)$, $t > 0$. We can write from $x \preceq f(x)$ for every $x \in X$, $x \preceq f(x) \preceq f^2(x) \preceq f^3(x) \preceq \dots \preceq f^k(x) \preceq \dots$

Considering assumptions, we have

$$\begin{aligned}
 \mu(f^{k+1}(x) - f^k(x), t) &= \mu(f(f^k(x)) - f(f^{k-1}(x)), t) \\
 &\geq \mu\left(f^k(x) - f^{k+1}(x), \frac{t}{a}\right) * \mu\left(f^{k-1}(x) - f^k(x), \frac{t}{a}\right) \\
 &\geq \mu\left(f^k(x) - f^{k-1}(x), \frac{t}{2a}\right) * \mu\left(f^{k-1}(x) - f^{k+1}(x), \frac{t}{2a}\right) * \mu\left(f^{k-1}(x) - f^k(x), \frac{t}{a}\right) \\
 &\geq \mu\left(f^k(x) - f^{k-1}(x), \frac{t}{2a}\right) * \mu\left(f^{k-1}(x) - f^{k+1}(x), \frac{t}{2a}\right) * \mu\left(f^{k-1}(x) - f^k(x), \frac{t}{2a}\right) \\
 &= \mu\left(f^k(x) - f^{k-1}(x), \frac{t}{2a}\right) * \mu\left(f^{k-1}(x) - f^{k+1}(x), \frac{t}{2a}\right) \\
 &= \min \left\{ \mu\left(f^k(x) - f^{k-1}(x), \frac{t}{2a}\right), \mu\left(f^{k+1}(x) - f^{k-1}(x), \frac{t}{2a}\right) \right\} \\
 &= \mu\left(f^k(x) - f^{k-1}(x), \frac{t}{2a}\right) \\
 &\geq \mu\left(f^{k-1}(x) - f^k(x), \frac{t}{2a^2}\right) * \mu\left(f^{k-2}(x) - f^{k-1}(x), \frac{t}{2a^2}\right) \\
 &\geq \mu\left(f^{k-1}(x) - f^{k-2}(x), \frac{t}{4a^2}\right) * \mu\left(f^{k-2}(x) - f^k(x), \frac{t}{4a^2}\right) \\
 &\quad * \mu\left(f^{k-2}(x) - f^{k-1}(x), \frac{t}{2a^2}\right) \\
 &\geq \mu\left(f^{k-1}(x) - f^{k-2}(x), \frac{t}{4a^2}\right) * \mu\left(f^{k-2}(x) - f^k(x), \frac{t}{4a^2}\right) \\
 &\quad * \mu\left(f^{k-2}(x) - f^{k-1}(x), \frac{t}{4a^2}\right)
 \end{aligned}$$



$$\begin{aligned}
&= \mu \left(f^{k-1}(x) - f^{k-2}(x), \frac{t}{4a^2} \right) * \mu \left(f^{k-2}(x) - f^k(x), \frac{t}{4a^2} \right) \\
&= \min \left\{ \mu \left(f^{k-1}(x) - f^{k-2}(x), \frac{t}{4a^2} \right), \mu \left(f^k(x) - f^{k-2}(x), \frac{t}{4a^2} \right) \right\} \\
&= \mu \left(f^{k-1}(x) - f^{k-2}(x), \frac{t}{4a^2} \right) \geq \dots \geq \mu \left(f^{k-(k-2)}(x) - f^{k-(k-1)}(x), \frac{t}{2^{k-1}a^{k-1}} \right) \\
&= \mu \left(f^2(x) - f(x), \frac{t}{2^{k-1}a^{k-1}} \right) \\
&\geq \mu \left(f^2(x) - f(x), \frac{t}{2^{k-1}a^k} \right) * \mu \left(x - f(x), \frac{t}{2^{k-1}a^k} \right) \\
&\geq \mu \left(f^2(x) - x, \frac{t}{2^k a^k} \right) * \mu \left(x - f(x), \frac{t}{2^k a^k} \right) * \mu \left(x - f(x), \frac{t}{2^{k-1}a^k} \right) \\
&\geq \mu \left(f^2(x) - x, \frac{t}{2^k a^k} \right) * \mu \left(x - f(x), \frac{t}{2^k a^k} \right) * \mu \left(x - f(x), \frac{t}{2^k a^k} \right) \\
&= \mu \left(f^2(x) - x, \frac{t}{2^k a^k} \right) * \mu \left(x - f(x), \frac{t}{2^k a^k} \right) \\
&= \min \left\{ \mu \left(x - f^2(x), \frac{t}{2^k a^k} \right), \mu \left(x - f(x), \frac{t}{2^k a^k} \right) \right\} = \mu \left(x - f(x), \frac{t}{2^k a^k} \right) \text{ and} \\
&\nu(f^{k+1}(x) - f^k(x), t) = \nu(f(f^k(x)) - f(f^{k-1}(x)), t) \\
&\leq \nu \left(f^k(x) - f^{k+1}(x), \frac{t}{a} \right) \diamond \nu \left(f^{k-1}(x) - f^k(x), \frac{t}{a} \right) \\
&\leq \nu \left(f^k(x) - f^{k-1}(x), \frac{t}{2a} \right) \diamond \nu \left(f^{k-1}(x) - f^{k+1}(x), \frac{t}{2a} \right) \diamond \nu \left(f^{k-1}(x) - f^k(x), \frac{t}{a} \right) \\
&\leq \nu \left(f^k(x) - f^{k-1}(x), \frac{t}{2a} \right) \diamond \nu \left(f^{k-1}(x) - f^{k+1}(x), \frac{t}{2a} \right) \diamond \nu \left(f^{k-1}(x) - f^k(x), \frac{t}{2a} \right) \\
&= \nu \left(f^k(x) - f^{k-1}(x), \frac{t}{2a} \right) \diamond \nu \left(f^{k-1}(x) - f^{k+1}(x), \frac{t}{2a} \right) \\
&= \max \left\{ \nu \left(f^k(x) - f^{k-1}(x), \frac{t}{2a} \right), \nu \left(f^{k+1}(x) - f^{k-1}(x), \frac{t}{2a} \right) \right\} \\
&= \nu \left(f^k(x) - f^{k-1}(x), \frac{t}{2a} \right) \\
&\leq \nu \left(f^{k-1}(x) - f^k(x), \frac{t}{2a^2} \right) \diamond \nu \left(f^{k-2}(x) - f^{k-1}(x), \frac{t}{2a^2} \right) \\
&\leq \nu \left(f^{k-1}(x) - f^{k-2}(x), \frac{t}{4a^2} \right) \diamond \nu \left(f^{k-2}(x) - f^k(x), \frac{t}{4a^2} \right)
\end{aligned}$$

$$\begin{aligned}
& \diamond \nu \left(f^{k-2}(x) - f^{k-1}(x), \frac{t}{2a^2} \right) \\
& \leq \nu \left(f^{k-1}(x) - f^{k-2}(x), \frac{t}{4a^2} \right) \diamond \nu \left(f^{k-2}(x) - f^k(x), \frac{t}{4a^2} \right) \\
& \quad \diamond \nu \left(f^{k-2}(x) - f^{k-1}(x), \frac{t}{4a^2} \right) \\
& = \nu \left(f^{k-1}(x) - f^{k-2}(x), \frac{t}{4a^2} \right) \diamond \nu \left(f^{k-2}(x) - f^k(x), \frac{t}{4a^2} \right) \\
& = \max \left\{ \nu \left(f^{k-1}(x) - f^{k-2}(x), \frac{t}{4a^2} \right), \nu \left(f^k(x) - f^{k-2}(x), \frac{t}{4a^2} \right) \right\} \\
& = \nu \left(f^{k-1}(x) - f^{k-2}(x), \frac{t}{4a^2} \right) \leq \dots \leq \nu \left(f^{k-(k-2)}(x) - f^{k-(k-1)}(x), \frac{t}{2^{k-1}a^{k-1}} \right) \\
& = \nu \left(f^2(x) - f(x), \frac{t}{2^{k-1}a^{k-1}} \right) \\
& \leq \nu \left(f^2(x) - f(x), \frac{t}{2^{k-1}a^k} \right) \diamond \nu \left(x - f(x), \frac{t}{2^{k-1}a^k} \right) \\
& \leq \nu \left(f^2(x) - x, \frac{t}{2^ka^k} \right) \diamond \nu \left(x - f(x), \frac{t}{2^ka^k} \right) \diamond \nu \left(x - f(x), \frac{t}{2^{k-1}a^k} \right) \\
& \leq \nu \left(f^2(x) - x, \frac{t}{2^ka^k} \right) \diamond \nu \left(x - f(x), \frac{t}{2^ka^k} \right) \diamond \nu \left(x - f(x), \frac{t}{2^ka^k} \right) \\
& = \nu \left(f^2(x) - x, \frac{t}{2^ka^k} \right) \diamond \nu \left(x - f(x), \frac{t}{2^ka^k} \right) \\
& = \max \left\{ \nu \left(x - f^2(x), \frac{t}{2^ka^k} \right), \nu \left(x - f(x), \frac{t}{2^ka^k} \right) \right\} \\
& = \nu \left(x - f(x), \frac{t}{2^ka^k} \right) \text{ and} \\
& \omega \left(f^{k+1}(x) - f^k(x), t \right) = \omega \left(f \left(f^k(x) \right) - f \left(f^{k-1}(x) \right), t \right) \\
& \leq \omega \left(f^k(x) - f^{k+1}(x), \frac{t}{a} \right) \otimes \omega \left(f^{k-1}(x) - f^k(x), \frac{t}{a} \right) \\
& \leq \omega \left(f^k(x) - f^{k-1}(x), \frac{t}{2a} \right) \otimes \omega \left(f^{k-1}(x) - f^{k+1}(x), \frac{t}{2a} \right) \\
& \quad \otimes \omega \left(f^{k-1}(x) - f^k(x), \frac{t}{a} \right) \\
& \leq \omega \left(f^k(x) - f^{k-1}(x), \frac{t}{2a} \right) \otimes \omega \left(f^{k-1}(x) - f^{k+1}(x), \frac{t}{2a} \right)
\end{aligned}$$

$$\begin{aligned}
& \otimes \omega \left(f^{k-1}(x) - f^k(x), \frac{t}{2a} \right) \\
&= \omega \left(f^k(x) - f^{k-1}(x), \frac{t}{2a} \right) \otimes \omega \left(f^{k-1}(x) - f^{k+1}(x), \frac{t}{2a} \right) \\
&= \max \left\{ \omega \left(f^k(x) - f^{k-1}(x), \frac{t}{2a} \right), \omega \left(f^{k+1}(x) - f^{k-1}(x), \frac{t}{2a} \right) \right\} \\
&= \omega \left(f^k(x) - f^{k-1}(x), \frac{t}{2a} \right) \\
&\leq \omega \left(f^{k-1}(x) - f^k(x), \frac{t}{2a^2} \right) \otimes \omega \left(f^{k-2}(x) - f^{k-1}(x), \frac{t}{2a^2} \right) \\
&\leq \omega \left(f^{k-1}(x) - f^{k-2}(x), \frac{t}{4a^2} \right) \otimes \omega \left(f^{k-2}(x) - f^k(x), \frac{t}{4a^2} \right) \\
&\quad \otimes \omega \left(f^{k-2}(x) - f^{k-1}(x), \frac{t}{2a^2} \right) \\
&\leq \omega \left(f^{k-1}(x) - f^{k-2}(x), \frac{t}{4a^2} \right) \otimes \omega \left(f^{k-2}(x) - f^k(x), \frac{t}{4a^2} \right) \\
&\quad \otimes \omega \left(f^{k-2}(x) - f^{k-1}(x), \frac{t}{4a^2} \right) \\
&= \omega \left(f^{k-1}(x) - f^{k-2}(x), \frac{t}{4a^2} \right) \otimes \omega \left(f^{k-2}(x) - f^k(x), \frac{t}{4a^2} \right) \\
&= \max \left\{ \omega \left(f^{k-1}(x) - f^{k-2}(x), \frac{t}{4a^2} \right), \omega \left(f^k(x) - f^{k-2}(x), \frac{t}{4a^2} \right) \right\} \\
&= \omega \left(f^{k-1}(x) - f^{k-2}(x), \frac{t}{4a^2} \right) \leq \dots \leq \omega \left(f^{k-(k-2)}(x) - f^{k-(k-1)}(x), \frac{t}{2^{k-1}a^{k-1}} \right) \\
&= \omega \left(f^2(x) - f(x), \frac{t}{2^{k-1}a^{k-1}} \right) \\
&\leq \omega \left(f^2(x) - f(x), \frac{t}{2^{k-1}a^k} \right) \otimes \omega \left(x - f(x), \frac{t}{2^{k-1}a^k} \right) \\
&\leq \omega \left(f^2(x) - x, \frac{t}{2^k a^k} \right) \otimes \omega \left(x - f(x), \frac{t}{2^k a^k} \right) \otimes \omega \left(x - f(x), \frac{t}{2^{k-1}a^k} \right) \\
&\leq \omega \left(f^2(x) - x, \frac{t}{2^k a^k} \right) \otimes \omega \left(x - f(x), \frac{t}{2^k a^k} \right) \otimes \omega \left(x - f(x), \frac{t}{2^k a^k} \right) \\
&= \omega \left(f^2(x) - x, \frac{t}{2^k a^k} \right) \otimes \omega \left(x - f(x), \frac{t}{2^k a^k} \right) \\
&= \max \left\{ \omega \left(x - f^2(x), \frac{t}{2^k a^k} \right), \omega \left(x - f(x), \frac{t}{2^k a^k} \right) \right\}
\end{aligned}$$

$$= \omega \left(x - f(x), \frac{t}{2^k a^k} \right).$$

Now, if we take limit for $k \rightarrow \infty$, $\frac{t}{(2a)^k}$ tends to infinity for $a \in (0, \frac{1}{2})$.
 Using properties (NNS-8), (NNS-14), (NNS-20) of NN,

$$\begin{aligned} \lim_{k \rightarrow \infty} \mu(f^k(x) - f^{k+1}(x), t) &\geq \lim_{k \rightarrow \infty} \mu(x - f(x), \frac{t}{(2a)^k}) = 1, \\ \lim_{k \rightarrow \infty} \nu(f^k(x) - f^{k+1}(x), t) &\leq \lim_{k \rightarrow \infty} \nu(x - f(x), \frac{t}{(2a)^k}) = 0 \text{ and} \\ \lim_{k \rightarrow \infty} \omega(f^k(x) - f^{k+1}(x), t) &\leq \lim_{k \rightarrow \infty} \omega(x - f(x), \frac{t}{(2a)^k}) = 0. \end{aligned}$$

This means that neutrosophic Kannan operator is NAR. That is, by Theorem (3.7) neutrosophic Kannan operator has NAFPP.

Corollary 3.12 In the Theorem (3.11), if $x \succeq f(x)$ for neutrosophic Kannan operator f and $\mu(., t)$ is non-increasing, $\nu(., t)$ is non-decreasing and $\omega(., t)$ is non-decreasing for every $t \in (0, \infty)$, $x \succeq \theta$, f has still NAFPP.

Definition 3.13 Let X be an NNS. If there exists $\alpha \in (0, \frac{1}{2})$ such that

$$\begin{aligned} \mu(f(x) - f(y), at) &\geq \mu(x - f(y), t) * \mu(y - f(x), t), \\ \nu(f(x) - f(y), at) &\leq \nu(x - f(y), t) \diamond \nu(y - f(x), t) \text{ and} \\ \omega(f(x) - f(y), at) &\leq \omega(x - f(y), t) \otimes \omega(y - f(x), t), \end{aligned}$$

for every $x, y \in X$ and $t > 0$, then $f : X \rightarrow X$ is called neutrosophic Chatterjea operator.

Theorem 3.14 Let $(X, \mu, \nu, \omega, *, \diamond, \otimes)$ be a NNS having partial order relation denoted by $\preceq$, where $a * b = \min\{a, b\}$ and $a \diamond b = \max\{a, b\}$, and $f : X \rightarrow X$ be a neutrosophic Chatterjea operator satisfying $x \preceq f(x)$ for every $x \in X$. Assume that $\preceq \subset X \times X$ holds one of the following conditions:

- (i) $\preceq$ is subvector space
- (ii) X is a totally ordered space.

If $\mu(., t)$ is non-decreasing, $\nu(., t)$ is non-increasing and $\omega(., t)$ is non-increasing, for

all $t \in (0, \infty)$, $x \succeq \theta$ (θ is unit element in vector space X), then $F_{\epsilon}^{(\mu, \nu, \omega)}(f) \neq \emptyset$ for every $\epsilon \in (0, 1)$.

Proof: By taking into consideration assumption of theorem, we get

$$\begin{aligned}
 \mu(f^{k+1}(x) - f^k(x), t) &\geq \mu(f^k(x) - f^k(x), \frac{t}{a}) * \mu\left(f^{k-1}(x) - f^{k+1}(x), \frac{t}{a}\right) \\
 &= 1 * \mu\left(f^{k-1}(x) - f^{k+1}(x), \frac{t}{a}\right) = \mu\left(f^{k-1}(x) - f^{k+1}(x), \frac{t}{a}\right) \\
 &\geq \mu\left(f^{k-1}(x) - f^k(x), \frac{t}{2a}\right) * \mu\left(f^k(x) - f^{k+1}(x), \frac{t}{2a}\right) \\
 &= \min\left\{\mu\left(f^{k-1}(x) - f^k(x), \frac{t}{2a}\right), \mu\left(f^{k+1}(x) - f^k(x), \frac{t}{2a}\right)\right\} \\
 &= \mu\left(f^{k-1}(x) - f^k(x), \frac{t}{2a}\right) \\
 &\geq \mu\left(f^{k-2}(x) - f^k(x), \frac{t}{2a^2}\right) * \mu\left(f^{k-1}(x) - f^{k-1}(x), \frac{t}{2a^2}\right) \\
 &= \mu\left(f^{k-2}(x) - f^k(x), \frac{t}{2a^2}\right) * 1 = \mu\left(f^{k-2}(x) - f^k(x), \frac{t}{2a^2}\right) \\
 &\geq \mu\left(f^{k-2}(x) - f^{k-1}(x), \frac{t}{4a^2}\right) * \mu\left(f^{k-1}(x) - f^k(x), \frac{t}{4a^2}\right) \\
 &= \min\left\{\mu\left(f^{k-2}(x) - f^{k-1}(x), \frac{t}{4a^2}\right), \mu\left(f^k(x) - f^{k-1}(x), \frac{t}{4a^2}\right)\right\} \\
 &= \mu\left(f^{k-2}(x) - f^{k-1}(x), \frac{t}{4a^2}\right) \geq \dots \geq \mu\left(f^{k-(k-2)}(x) - f^{k-(k-3)}(x), \frac{t}{2^{k-2}a^{k-2}}\right) \\
 &= \mu\left(f^2(x) - f^3(x), \frac{t}{2^{k-2}a^{k-2}}\right) \\
 &\geq \mu\left(f(x) - f^3(x), \frac{t}{2^{k-2}a^{k-1}}\right) * \mu\left(f^2(x) - f^3(x), \frac{t}{2^{k-2}a^{k-1}}\right) \\
 &= \mu\left(f(x) - f^3(x), \frac{t}{2^{k-2}a^{k-1}}\right) * 1 = \mu\left(f(x) - f^3(x), \frac{t}{2^{k-2}a^{k-1}}\right) \\
 &\geq \mu\left(f(x) - f^2(x), \frac{t}{2^{k-1}a^{k-1}}\right) * \mu\left(f^2(x) - f^3(x), \frac{t}{2^{k-1}a^{k-1}}\right) \\
 &= \min\left\{\mu\left(f(x) - f^2(x), \frac{t}{2^{k-1}a^{k-1}}\right), \mu\left(f^3(x) - f^2(x), \frac{t}{2^{k-1}a^{k-1}}\right)\right\} \\
 &= \mu\left(f(x) - f^2(x), \frac{t}{2^{k-1}a^{k-1}}\right)
 \end{aligned}$$

$$\begin{aligned}
&\geq \mu \left(x - f^2(x), \frac{t}{2^{k-1}a^k} \right) * \mu \left(f(x) - f(x), \frac{t}{2^{k-1}a^k} \right) \\
&= \mu \left(x - f^2(x), \frac{t}{2^{k-1}a^k} \right) * 1 = \mu \left(x - f^2(x), \frac{t}{2^{k-1}a^k} \right) \\
&\geq \mu \left(x - f(x), \frac{t}{2^ka^k} \right) * \mu \left(f(x) - f^2(x), \frac{t}{2^ka^k} \right) \\
&= \min \left\{ \mu \left(x - f(x), \frac{t}{2^ka^k} \right), \mu \left(f^2(x) - f(x), \frac{t}{2^ka^k} \right) \right\} \\
&= \mu \left(x - f(x), \frac{t}{2^ka^k} \right), \\
\nu(f^{k+1}(x) - f^k(x), t) &\leq \nu \left(f^k(x) - f^k(x), \frac{t}{a} \right) \Diamond \nu \left(f^{k-1}(x) - f^{k+1}(x), \frac{t}{a} \right) \\
&= 0 \Diamond \nu \left(f^{k-1}(x) - f^{k+1}(x), \frac{t}{a} \right) = \nu \left(f^{k-1}(x) - f^{k+1}(x), \frac{t}{a} \right) \\
&\leq \nu \left(f^{k-1}(x) - f^k(x), \frac{t}{2a} \right) \Diamond \nu \left(f^k(x) - f^{k+1}(x), \frac{t}{2a} \right) \\
&= \max \left\{ \nu \left(f^{k-1}(x) - f^k(x), \frac{t}{2a} \right), \nu \left(f^{k+1}(x) - f^k(x), \frac{t}{2a} \right) \right\} \\
&= \nu \left(f^{k-1}(x) - f^k(x), \frac{t}{2a} \right) \\
&\leq \nu \left(f^{k-2}(x) - f^k(x), \frac{t}{2a^2} \right) \Diamond \nu \left(f^{k-1}(x) - f^{k-1}(x), \frac{t}{2a^2} \right) \\
&= \nu \left(f^{k-2}(x) - f^k(x), \frac{t}{2a^2} \right) \Diamond 0 = \nu \left(f^{k-2}(x) - f^k(x), \frac{t}{2a^2} \right) \\
&\leq \nu \left(f^{k-2}(x) - f^{k-1}(x), \frac{t}{4a^2} \right) \Diamond \nu \left(f^{k-1}(x) - f^k(x), \frac{t}{4a^2} \right) \\
&= \max \left\{ \nu \left(f^{k-2}(x) - f^{k-1}(x), \frac{t}{4a^2} \right), \nu \left(f^k(x) - f^{k-1}(x), \frac{t}{4a^2} \right) \right\} \\
&= \nu \left(f^{k-2}(x) - f^{k-1}(x), \frac{t}{4a^2} \right) \leq \dots \leq \nu \left(f^{k-(k-2)}(x) - f^{k-(k-3)}(x), \frac{t}{2^{k-2}a^{k-2}} \right) \\
&= \nu \left(f^2(x) - f^3(x), \frac{t}{2^{k-2}a^{k-2}} \right) \\
&\leq \nu \left(f(x) - f^3(x), \frac{t}{2^{k-2}a^{k-1}} \right) \Diamond \nu \left(f^2(x) - f^3(x), \frac{t}{2^{k-2}a^{k-1}} \right) \\
&= \nu \left(f(x) - f^3(x), \frac{t}{2^{k-2}a^{k-1}} \right) \Diamond 0 = \nu \left(f(x) - f^3(x), \frac{t}{2^{k-2}a^{k-1}} \right)
\end{aligned}$$

$$\begin{aligned}
&\leq \nu \left(f(x) - f^2(x), \frac{t}{2^{k-1}a^{k-1}} \right) \Diamond \nu \left(f^2(x) - f^3(x), \frac{t}{2^{k-1}a^{k-1}} \right) \\
&= \max \left\{ \nu \left(f(x) - f^2(x), \frac{t}{2^{k-1}a^{k-1}} \right), \nu \left(f^3(x) - f^2(x), \frac{t}{2^{k-1}a^{k-1}} \right) \right\} \\
&= \nu \left(f(x) - f^2(x), \frac{t}{2^{k-1}a^{k-1}} \right) \\
&\leq \nu \left(x - f^2(x), \frac{t}{2^{k-1}a^k} \right) \Diamond \nu \left(f(x) - f(x), \frac{t}{2^{k-1}a^k} \right) \\
&= \nu \left(x - f^2(x), \frac{t}{2^{k-1}a^k} \right) \Diamond 0 = \nu \left(x - f^2(x), \frac{t}{2^{k-1}a^k} \right) \\
&\leq \nu \left(x - f(x), \frac{t}{2^k a^k} \right) \Diamond \nu \left(f(x) - f^2(x), \frac{t}{2^k a^k} \right) \\
&= \max \left\{ \nu \left(x - f(x), \frac{t}{2^k a^k} \right), \nu \left(f^2(x) - f(x), \frac{t}{2^k a^k} \right) \right\} \\
&= \nu \left(x - f(x), \frac{t}{2^k a^k} \right) \text{ and} \\
&\omega \left(f^{k+1}(x) - f^k(x), t \right) \leq \omega \left(f^k(x) - f^k(x), \frac{t}{a} \right) \otimes \omega \left(f^{k-1}(x) - f^{k+1}(x), \frac{t}{a} \right) \\
&= 0 \otimes \omega \left(f^{k-1}(x) - f^{k+1}(x), \frac{t}{a} \right) = \omega \left(f^{k-1}(x) - f^{k+1}(x), \frac{t}{a} \right) \\
&\leq \omega \left(f^{k-1}(x) - f^k(x), \frac{t}{2a} \right) \otimes \omega \left(f^k(x) - f^{k+1}(x), \frac{t}{2a} \right) \\
&= \max \left\{ \omega \left(f^{k-1}(x) - f^k(x), \frac{t}{2a} \right), \omega \left(f^{k+1}(x) - f^k(x), \frac{t}{2a} \right) \right\} \\
&= \omega \left(f^{k-1}(x) - f^k(x), \frac{t}{2a} \right) \\
&\leq \omega \left(f^{k-2}(x) - f^k(x), \frac{t}{2a^2} \right) \otimes \omega \left(f^{k-1}(x) - f^{k-1}(x), \frac{t}{2a^2} \right) \\
&= \omega \left(f^{k-2}(x) - f^k(x), \frac{t}{2a^2} \right) \otimes 0 = \omega \left(f^{k-2}(x) - f^k(x), \frac{t}{2a^2} \right) \\
&\leq \omega \left(f^{k-2}(x) - f^{k-1}(x), \frac{t}{4a^2} \right) \otimes \omega \left(f^{k-1}(x) - f^k(x), \frac{t}{4a^2} \right) \\
&= \max \left\{ \omega \left(f^{k-2}(x) - f^{k-1}(x), \frac{t}{4a^2} \right), \omega \left(f^k(x) - f^{k-1}(x), \frac{t}{4a^2} \right) \right\} \\
&= \omega \left(f^{k-2}(x) - f^{k-1}(x), \frac{t}{4a^2} \right) \leq \dots \leq \omega \left(f^{k-(k-2)}(x) - f^{k-(k-3)}(x), \frac{t}{2^{k-2}a^{k-2}} \right)
\end{aligned}$$

$$\begin{aligned}
 &= \omega \left(f^2(x) - f^3(x), \frac{t}{2^{k-2}a^{k-2}} \right) \\
 &\leq \omega \left(f(x) - f^3(x), \frac{t}{2^{k-2}a^{k-1}} \right) \otimes \omega \left(f^2(x) - f^3(x), \frac{t}{2^{k-2}a^{k-1}} \right) \\
 &= \omega \left(f(x) - f^3(x), \frac{t}{2^{k-2}a^{k-1}} \right) \otimes 0 = \omega \left(f(x) - f^3(x), \frac{t}{2^{k-2}a^{k-1}} \right) \\
 &\leq \omega \left(f(x) - f^2(x), \frac{t}{2^{k-1}a^{k-1}} \right) \otimes \omega \left(f^2(x) - f^3(x), \frac{t}{2^{k-1}a^{k-1}} \right) \\
 &= \max \left\{ \omega \left(f(x) - f^2(x), \frac{t}{2^{k-1}a^{k-1}} \right), \omega \left(f^3(x) - f^2(x), \frac{t}{2^{k-1}a^{k-1}} \right) \right\} \\
 &= \omega \left(f(x) - f^2(x), \frac{t}{2^{k-1}a^{k-1}} \right) \\
 &\leq \omega \left(x - f^2(x), \frac{t}{2^{k-1}a^k} \right) \otimes \omega \left(f(x) - f(x), \frac{t}{2^{k-1}a^k} \right) \\
 &= \omega \left(x - f^2(x), \frac{t}{2^{k-1}a^k} \right) \otimes 0 = \omega \left(x - f^2(x), \frac{t}{2^{k-1}a^k} \right) \\
 &\leq \omega \left(x - f(x), \frac{t}{2^ka^k} \right) \otimes \omega \left(f(x) - f^2(x), \frac{t}{2^ka^k} \right) \\
 &= \max \left\{ \omega \left(x - f(x), \frac{t}{2^ka^k} \right), \omega \left(f^2(x) - f(x), \frac{t}{2^ka^k} \right) \right\} \\
 &= \omega \left(x - f(x), \frac{t}{2^ka^k} \right).
 \end{aligned}$$

Since $\frac{t}{(2a)^k} \rightarrow \infty$ for $k \rightarrow \infty$, we have

$$\begin{aligned}
 \lim_{k \rightarrow \infty} \mu(f^k(x) - f^{k+1}(x), t) &\geq \lim_{k \rightarrow \infty} \mu \left(x - f(x), \frac{t}{(2a)^k} \right) = 1, \\
 \lim_{k \rightarrow \infty} \nu(f^k(x) - f^{k+1}(x), t) &\leq \lim_{k \rightarrow \infty} \nu \left(x - f(x), \frac{t}{(2a)^k} \right) = 0 \text{ and} \\
 \lim_{k \rightarrow \infty} \omega(f^k(x) - f^{k+1}(x), t) &\leq \lim_{k \rightarrow \infty} \omega \left(x - f(x), \frac{t}{(2a)^k} \right) = 0,
 \end{aligned}$$

by means of (NNS-8), (NNS-14), (NNS-20) properties of NN. We see that neutrosophic Chatterjea operator has approximate fixed point property by Theorem (3.11).

Corollary 3.15 In the Theorem (3.14), if $x \succeq f(x)$ for neutrosophic Chatterjea operator f and $\mu(., t)$ is non-increasing, $\nu(., t)$ is non-decreasing and $\omega(., t)$ is non-decreasing for every $t \in (0, \infty)$, $x \succeq \theta$, f has still NAFPP.

Definition 3.16 Let X be an NNS. A mapping $f : X \rightarrow X$ is called neutrosophic Zamfirescu operator if there exists at least $a \in (0, 1)$, $k \in (0, \frac{1}{2})$, $c \in (0, \frac{1}{2})$ such that atleast one of the followings is true for every $x, y \in X$ and $t > 0$:

(i)

$$\begin{aligned}\mu(f(x) - f(y), at) &\geq \mu(x - y, t), \\ \nu(f(x) - f(y), at) &\leq \nu(x - y, t) \text{ and} \\ \omega(f(x) - f(y), at) &\leq \omega(x - y, t).\end{aligned}$$

(ii)

$$\begin{aligned}\mu(f(x) - f(y), kt) &\geq \mu(x - f(x), t) * \mu(y - f(y), t), \\ \nu(f(x) - f(y), kt) &\leq \nu(x - f(x), t) \diamond \nu(y - f(y), t) \text{ and} \\ \omega(f(x) - f(y), kt) &\leq \omega(x - f(x), t) \otimes \omega(y - f(y), t)\end{aligned}$$

(iii)

$$\begin{aligned}\mu(f(x) - f(y), ct) &\geq \mu(x - f(y), t) * \mu(y - f(x), t), \\ \nu(f(x) - f(y), ct) &\leq \nu(x - f(y), t) \diamond \nu(y - f(x), t) \text{ and} \\ \omega(f(x) - f(y), ct) &\leq \omega(x - f(y), t) \otimes \omega(y - f(x), t).\end{aligned}$$

Theorem 3.17 Let $(X, \mu, \nu, \omega, *, \diamond, \otimes)$ be a NNS having partial order relation denoted by $\preceq$, where $a * b = \min\{a, b\}$ and $a \diamond b = \max\{a, b\}$, and $f : X \rightarrow X$ be a neutrosophic Zamfirescu operator satisfying $x \preceq f(x)$ for every $x \in X$. Assume that $\preceq \subset X \times X$ holds one of the following conditions:

- (i) $\preceq$ is subvector space,
- (ii) X is a totally ordered space.

If $\mu(., t)$ is non-decreasing, $\nu(., t)$ is non-increasing and $\omega(., t)$ is non-increasing, for every $t \in (0, \infty)$, $x \succeq \theta$ (θ is unit element in vector space X), then $F_{\epsilon}^{(\mu, \nu, \omega)}(f) \neq \emptyset$

for every $\epsilon \in (0, 1)$.

Proof: The proof is clear from Theorem (3.11) and Theorem (3.14).

Definition 3.18 Let X be a NNS. If there exists $a \in (0, 1)$ and $L \geq 0$ such that

$$\begin{aligned}\mu(f(x) - f(y), t) &\geq \mu\left(x - f(y), \frac{t}{a}\right) * \mu\left(y - f(x), \frac{t}{L}\right), \\ \nu(f(x) - f(y), at) &\leq \nu\left(x - f(y), \frac{t}{a}\right) \diamond \nu\left(y - f(x), \frac{t}{L}\right) \text{ and} \\ \omega(f(x) - f(y), at) &\leq \omega\left(x - f(y), \frac{t}{a}\right) \otimes \omega\left(y - f(x), \frac{t}{L}\right),\end{aligned}$$

for every $x, y \in X$ and $t > 0$, then $f : X \rightarrow X$ is called neutrosophic weak contraction operator.

Theorem 3.19 Let X be a NNS and $f : X \rightarrow X$ be neutrosophic weak contraction. Then $F_{\epsilon}^{(\mu, \nu, \omega)}(f) \neq \emptyset$, for every $\epsilon \in (0, 1)$.

Proof: Let $x \in X$ and $\epsilon \in (0, 1)$.

$$\begin{aligned}\mu(f^k(x) - f^{k+1}(x), t) &= \mu(f(f^{k-1}(x)) - f(f^k(x)), t) \\ &\geq \mu\left(f^{k-1}(x) - f^k(x), \frac{t}{a}\right) * \mu\left(f^k(x) - f^k(x), \frac{t}{L}\right) \\ &= \mu\left(f^{k-1}(x) - f^k(x), \frac{t}{a}\right) * 1 \\ &= \mu\left(f^{k-1}(x) - f^k(x), \frac{t}{a}\right) \\ &\geq \mu\left(f^{k-2}(x) - f^{k-1}(x), \frac{t}{a^2}\right) * \mu\left(f^{k-1}(x) - f^{k-1}(x), \frac{t}{L}\right) \\ &= \mu\left(f^{k-2}(x) - f^{k-1}(x), \frac{t}{a^2}\right) * 1 \geq \mu\left(f^{k-2}(x) - f^{k-1}(x), \frac{t}{a^2}\right) \geq \dots \\ &= \mu\left(f^{k-(k-1)}(x) - f^{k-(k-2)}(x), \frac{t}{a^{k-1}}\right) \\ &= \mu\left(f(x) - f^2(x), \frac{t}{a^{k-1}}\right)\end{aligned}$$



$$\begin{aligned}
&\geq \mu \left(x - f(x), \frac{t}{a^k} \right) * \mu \left(f(x) - f(x), \frac{t}{L} \right) \geq \mu \left(x - f(x), \frac{t}{a^k} \right) * 1 \\
&= \mu \left(x - f(x), \frac{t}{a^k} \right). \\
\nu(f^k(x) - f^{k+1}(x), t) &= \nu(f(f^{k-1}(x)) - f(f^k(x)), t) \\
&\leq \nu \left(f^{k-1}(x) - f^k(x), \frac{t}{a} \right) \diamond \nu \left(f^k(x) - f^k(x), \frac{t}{L} \right) = \nu \left(f^{k-1}(x) - f^k(x), \frac{t}{a} \right) \diamond 0 \\
&= \nu \left(f^{k-1}(x) - f^k(x), \frac{t}{a} \right) \\
&\leq \nu \left(f^{k-2}(x) - f^{k-1}(x), \frac{t}{a^2} \right) \diamond \nu \left(f^{k-1}(x) - f^{k-1}(x), \frac{t}{L} \right) \\
&= \nu \left(f^{k-2}(x) - f^{k-1}(x), \frac{t}{a^2} \right) \diamond 0 \leq \nu \left(f^{k-2}(x) - f^{k-1}(x), \frac{t}{a^2} \right) \leq \dots \\
&= \nu \left(f^{k-(k-1)}(x) - f^{k-(k-2)}(x), \frac{t}{a^{k-1}} \right) = \nu \left(f(x) - f^2(x), \frac{t}{a^{k-1}} \right) \\
&\leq \nu \left(x - f(x), \frac{t}{a^k} \right) \diamond \nu \left(f(x) - f(x), \frac{t}{L} \right) \\
&\leq \nu \left(x - f(x), \frac{t}{a^k} \right) \diamond 0 = \nu \left(x - f(x), \frac{t}{a^k} \right) \text{ and} \\
\omega(f^k(x) - f^{k+1}(x), t) &= \omega(f(f^{k-1}(x)) - f(f^k(x)), t) \\
&\leq \omega \left(f^{k-1}(x) - f^k(x), \frac{t}{a} \right) \otimes \omega \left(f^k(x) - f^k(x), \frac{t}{L} \right) \\
&= \omega \left(f^{k-1}(x) - f^k(x), \frac{t}{a} \right) \otimes 0 \\
&= \omega \left(f^{k-1}(x) - f^k(x), \frac{t}{a} \right) \\
&\leq \omega \left(f^{k-2}(x) - f^{k-1}(x), \frac{t}{a^2} \right) \otimes \omega \left(f^{k-1}(x) - f^{k-1}(x), \frac{t}{L} \right) \\
&= \omega \left(f^{k-2}(x) - f^{k-1}(x), \frac{t}{a^2} \right) \otimes 0 \\
&\leq \omega \left(f^{k-2}(x) - f^{k-1}(x), \frac{t}{a^2} \right) \leq \dots \\
&= \omega \left(f^{k-(k-1)}(x) - f^{k-(k-2)}(x), \frac{t}{a^{k-1}} \right) \\
&= \omega \left(f(x) - f^2(x), \frac{t}{a^{k-1}} \right)
\end{aligned}$$

$$\begin{aligned} &\leq \omega\left(x - f(x), \frac{t}{a^k}\right) \otimes \omega\left(f(x) - f(x), \frac{t}{L}\right) \leq \omega\left(x - f(x), \frac{t}{a^k}\right) \otimes 0 \\ &= \omega\left(x - f(x), \frac{t}{a^k}\right). \end{aligned}$$

Since $\frac{t}{a^k} \rightarrow \infty$ for $k \rightarrow \infty$, by means of (NNS-8), (NNS-14) and (NNS-20) properties of NN, we see neutrosophic weak contraction map has approximate fixed point property by Theorem (3.7).

In the following, we give definition of approximate fixed point property of a set. Furthermore, we prove that a dense set of neutrosophic Banach space has approximate fixed point property.

Definition 3.20 Let X be NNS and let K be subset of X . Then K is said to have NAFPP if every neutrosophic nonexpansive map $f : K \rightarrow K$ satisfies the property that $\sup\{\mu(x - f(x), t) : x \in K\} = 1$, $\inf\{\nu(x - f(x), t) : x \in K\} = 0$ and $\inf\{\omega(x - f(x), t) : x \in K\} = 0$.

Theorem 3.21 Let X be a NNS having NAFPP, K be dense subset of X . Then K has NFAFPP.

Proof: Let $f : X \rightarrow X$ be a neutrosophic nonexpansive mapping. We prove that

$$\begin{aligned} \sup\{\mu(x - f(x), t) : x \in K\} &= \sup\{\mu(y - f(y), s) : y \in X\}, \\ \inf\{\nu(x - f(x), t) : x \in K\} &= \inf\{\nu(y - f(y), s) : y \in X\} \text{ and} \\ \inf\{\omega(x - f(x), t) : x \in K\} &= \inf\{\omega(y - f(y), s) : y \in X\}, \text{ for } t, s > 0. \end{aligned}$$

Since $K \subset X$,

$$\begin{aligned} \sup\{\mu(y - f(y), s) : y \in X\} &\geq \sup\{\mu(x - f(x), t) : x \in K\}, \\ \inf\{\nu(y - f(y), s) : y \in X\} &\leq \inf\{\nu(x - f(x), t) : x \in K\} \quad \text{and} \\ \inf\{\omega(y - f(y), s) : y \in X\} &\leq \inf\{\omega(x - f(x), t) : x \in K\}. \end{aligned}$$

Let $y \in X$. There exists a sequence (y_k) in K such that $y_k \xrightarrow{(u,v,\omega)} y$ for all $y \in X$ because of K is dense. We know that for each $k \in \mathbb{N}$ and $t, s > 0$,

$$\sup\{\mu(x - f(x), t) : x \in K\} \geq \mu(y_k - f(y_k), t)$$



$$\begin{aligned}
 &\geq \mu(y_k - y + y - f(y) + f(y) - f(y_k), t) \\
 &\geq \mu\left(y_k - y, \frac{t}{3}\right) * \mu\left(y - f(y), \frac{t}{3}\right) * \mu\left(y_k - f(y_k), \frac{t}{3}\right), \\
 \inf\{\nu(x - f(x), t) : x \in K\} &\leq \nu(y_k - f(y_k), t) \\
 &\leq \nu(y_k - y + y - f(y) + f(y) - f(y_k), t) \\
 &\leq \nu\left(y_k - y, \frac{t}{3}\right) \diamond \nu\left(y - f(y), \frac{t}{3}\right) \diamond \nu\left(y_k - f(y_k), \frac{t}{3}\right) \text{ and} \\
 \inf\{\omega(x - f(x), t) : x \in K\} &\leq \omega(y_k - f(y_k), t) \\
 &\leq \omega(y_k - y + y - f(y) + f(y) - f(y_k), t) \\
 &\leq \omega\left(y_k - y, \frac{t}{3}\right) \otimes \omega\left(y - f(y), \frac{t}{3}\right) \otimes \omega\left(y_k - f(y_k), \frac{t}{3}\right).
 \end{aligned}$$

Since f is neutrosophic nonexpansive mapping, it is neutrosophic continuous.

Because, if $y_k \xrightarrow{(u,v,\omega)} y$, then

$$\begin{aligned}
 \mu(f(y_k) - f(y), t) &\geq \mu(y_k - y, t) \rightarrow 1, \\
 \nu(f(y_k) - f(y), t) &\leq \nu(y_k - y, t) \rightarrow 0 \quad \text{and} \\
 \omega(f(y_k) - f(y), t) &\leq \omega(y_k - y, t) \rightarrow 0.
 \end{aligned}$$

So $f(y_k) \xrightarrow{(u,v,\omega)} f(y)$ when $y_k \xrightarrow{(u,v,\omega)} y$. If we take limit above inequalities, we get

$$\begin{aligned}
 \sup\{\mu(x - f(x), t) : x \in K\} &\geq \mu\left(y - f(y), \frac{t}{3}\right), \\
 \inf\{\nu(x - f(x), t) : x \in K\} &\leq \nu\left(y - f(y), \frac{t}{3}\right) \quad \text{and} \\
 \inf\{\omega(x - f(x), t) : x \in K\} &\leq \omega\left(y - f(y), \frac{t}{3}\right), \text{ for all } y \in X \text{ and } t > 0.
 \end{aligned}$$

Thus, if we take $\frac{t}{3} = s'$,

$$\begin{aligned}
 \sup\{\mu(x - f(x), t) : x \in K\} &\geq \sup\{\mu(y - f(y), s') : y \in X\}, \\
 \inf\{\nu(x - f(x), t) : x \in K\} &\leq \inf\{\nu(y - f(y), s') : y \in X\} \text{ and} \\
 \inf\{\omega(x - f(x), t) : x \in K\} &\leq \inf\{\omega(y - f(y), s') : y \in X\}.
 \end{aligned}$$

Therefore our claim is proved. Now, consider any neutrosophic nonexpansive mapping $f_k : K \rightarrow K$. Since K is dense, there exists a sequence (y_k) in K such that $y_k \xrightarrow{(u,v,\omega)} y$ for any $y \in X$. Since a neutrosophic nonexpansive mapping is continuous, $f_k : K \rightarrow K$ is neutrosophic continuous and it can be extended by defining $f(x) = \lim(\mu, \nu, \omega) - f(x_k)$ on X . Hence we can consider f as a neutrosophic nonexpansive mapping on X . Because, using Lemma (2.9) we get

$$\begin{aligned}\mu(f(x) - f(y), t) &= \lim_{k \rightarrow \infty} \sup \mu(f(x_k) - f(y_k), t) \geq \lim_{k \rightarrow \infty} \sup \mu(x_k - y_k, t) = \mu(x, y, t), \\ \nu(f(x) - f(y), t) &= \lim_{k \rightarrow \infty} \inf \nu(f(x_k) - f(y_k), t) \leq \lim_{k \rightarrow \infty} \inf \nu(x_k - y_k, t) = \nu(x, y, t), \\ \omega(f(x) - f(y), t) &= \lim_{k \rightarrow \infty} \inf \omega(f(x_k) - f(y_k), t) \leq \lim_{k \rightarrow \infty} \inf \omega(x_k - y_k, t) = \omega(x, y, t),\end{aligned}$$

for all $x, y \in X$ and $t > 0$. Then since X has NAFPP,

$$\begin{aligned}\sup\{\mu(x - f(x), t) : x \in K\} &= \sup\{\mu(y - f(y), s) : y \in X\} = 1, \\ \inf\{\nu(x - f(x), t) : x \in K\} &= \inf\{\nu(y - f(y), s) : y \in X\} = 0 \text{ and} \\ \inf\{\omega(x - f(x), t) : x \in K\} &= \inf\{\omega(y - f(y), s) : y \in X\} = 0.\end{aligned}$$

That is, for given any neutrosophic nonexpansive mapping f on K we have

$$\begin{aligned}\sup\{\mu(x - f(x), t) : x \in K\} &= 1, \\ \inf\{\nu(x - f(x), t) : x \in K\} &= 0 \quad \text{and} \\ \inf\{\omega(x - f(x), t) : x \in K\} &= 0 \quad \text{and } K \text{ has NAFPP.}\end{aligned}$$

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