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Pythagorean Fuzzy α - Continuity

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Abstract

In this research article, we present the new concept of Pythagorean fuzzy $\alpha-$ continuous function between Pythagorean fuzzy topological spaces and establish their corresponding characterizations.

Key words: Pythagorean fuzzy topology, continuous, Pythagorean fuzzy α closure, Pythagorean α continuous, Pythagorean α open, Pythagorean α closed.

1. Introduction

L.A. Zadeh's idea of fuzzy set [1] is a generalization of the usual set where each element has a membership in the range [0,1]. Following this idea, Atanassov [2] extended the concept of fuzzy to intuitionistic set with elements comprising membership and non-membership degree.

Chang [3] in 1968 defined fuzzy topological spaces with few results as continuity, closed and open sets. The different notion and definition for fuzzy topological spaces were given by Lowen [4]. Subsequently the fuzzy topological spaces were developed into many advancements such as compactness, convergence, separation axioms, fuzzy soft topological spaces, fuzzy metric spaces along with applications [19-23]. The concepts as fuzzy alpha open and closed sets and open map and continuous functions was developed by Rajvanshi and Singal [12,13]. The idea of intuitionistic topological spaces were developed by Coker[5] and the concepts Intuitionistic fuzzy alpha sets and continuity were studied [9,10,11].

Yager [6,7] presented the concept of Pythagorean fuzzy subset which is a typical fuzzy set. After the introduction of Pythagorean fuzzy sets, it was widely used in

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the field of decision making and was applied for the real life applications. Following the definition of Chang, Olgun [8] introduced Pythagorean fuzzy topology along with the definition of continuity in Pythagorean fuzzy topological spaces and their characterizations.Thus it was further developed with connected, compactness of Pythagorean topological spaces and Pythagorean nano fuzzy topologies [14-18]. In this article, Pythagorean alpha open sets and closed sets and Pythagorean alpha continuity are introduced along with their characterizations.

2. Preliminaries

Definition 2.1:
A subset of a non-void set X is a fuzzy subset if it is as $A = \{\langle a, \rho_A(a)\rangle, a \in X\}$ with $\rho_A : X \to [0,1]$ (membership degree).

Definition 2.2:
A subset $B = \{\langle b, \vartheta_B(b), \mu_B(b)\rangle, b \in X\}$ with $\vartheta_B : X \to [0,1], \mu_B : X \to [0,1]$ (membership and nonmembership degree) is Intuitionistic fuzzy subset satisfying $\vartheta(b) + \mu(b) \leq 1$.

Definition 2.3:
A non-empty subset C, $C = \{\langle c, \vartheta_C(c), \mu_C(c)\rangle, c \in X\}$ of X with $\vartheta_C : X \to [0,1], \mu_C : X \to [0,1]$ as membership and non-membership degree satisfying $\vartheta^2(c) + \mu^2(c) \leq 1$ is named as Pythagorean fuzzy subset.

Definition 2.4:
An Intuitionistic fuzzy set $I =< x, \mu_I, \gamma_I >$ of a PFTS (X, τ) is called an intuitionistic fuzzy α open set if $I \subseteq int(cl(int(I)))$. An Intuitionisitc fuzzy set whose complement is a intuitionistic fuzzy α open set $(IF\alpha OS)$ is called a intuitionistic fuzzy α closed set $(IF\alpha CS)$.

Definition 2.5:
The Intuitionistic fuzzy α closure of a IFS P in a IFTS (X, τ) represented as $cl_\alpha(I)$ and defined by $cl_\alpha(I) = \cap\{A_i/A_i \ is \ a \ PF\alpha C \ set \ and \ I \subseteq A_i\}$

3. Pythagorean Fuzzy α - Continuity

In this section, we define the Pythagorean α open, closed and Pythagorean α closure and continuity in Pythagorean topological spaces.

Definition 3.1:
A Pythagorean fuzzy topology (PFT) on X is a collection τ of PFSs in X satisfying:

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(1) $\underline{0}, \underline{1} \in \tau$
(2) $B_1 \cap B_2 \in \tau$ for any $B_1, B_2 \in \tau$,
(3) $\cup B_i \in \tau$ for any family $\{B_i/i \in I\} \subseteq \tau$.
The duo (X, τ) is called a Pythagorean Fuzzy Topological Space ($PFTS$) and any PFS in τ is pythagorean fuzzy open set ($PFOS$) in X. The complement of an PFOS in PFTS (X, τ) is Pythagorean fuzzy closed set (PFCS) in X.

Definition 3.2:
Let (X, τ) be a PFTS and $P =< x, \mu_P, \gamma_P >$ be a PFS in X. Then the Pythagorean fuzzy interior and closure of P are given by
$int(P) = \cup\{E/E \ is \ a \ PFOS \ in \ X \ and \ E \subseteq P\}$
$cl(P) = \cap\{K/K \ is \ a \ PFCS \ in \ X \ and \ P \subseteq K\}$
for any PFS in (X, τ), we have
$cl(\bar{P}) = int\bar{(}P)$ and $int(\bar{P}) = cl\bar{(}A)$.

Definition 3.3:
A PFS $P =< x, \mu_P, \gamma_P >$ of a PFTS (X, τ) is called a Pythagorean fuzzy α open set if $P \subseteq int\,(cl(int(P)))$. A PFS whose complement is a pythagorean fuzzy αopen set ($PF\alpha OS$) is called a pythagorean fuzzy α closed set ($PF\alpha CS$).

Proposition 3.4:
Let (X, τ) be a PFTS. Then arbitrary union of $PF\alpha OS$ is a $PF\alpha OS$ and arbitrary intersection of $PF\alpha C$ sets is an $PF\alpha CS$.
Proof:
Let $\{P_i =< x, \mu_P, \gamma_P > /i \in I\}$ be a family of $PF\alpha O$ sets. Then for each $i \in I$, $P_i \subseteq int(cl(int(P_i)))$. Thus
$\cup P_i \subseteq \cup int\,(cl(int(P_i)))$
$\subseteq int\,(\cup cl(int(P_i)))$
$= int\,(cl(\cup int(P_i))$
$\subseteq int\,(cl(int(\cup P_i)))$
Hence $\cup P_i$ is a $PF\alpha O$ set. If we take complement of this part, the consecutive will proved (ie. arbitrary intersection of $PF\alpha C$ is also a $PF\alpha C$).
Every PFOS is a Pythagorean fuzzy α - open set and every PFCS is a $PF\alpha C$ but the converse is not true.

Definition 3.5:
The Pythagorean fuzzy α closure of a PFS P in a PFTS (X, τ) represented as $cl_\alpha(P)$ and defined by $cl_\alpha(P) = \cap\{C_i/C_i \ is \ a \ PF\alpha C \ set \ and \ P \subseteq C_i\}$

Proposition 3.6:

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In a PFTS (X, τ), a PFS P is $PF\alpha C$ if and only if $P = cl_\alpha(P)$. **Proof:**
Assume that P is a $PF\alpha C$ set. Then
$P \in \{C_i / C_i \textit{ is a } PF\alpha C \textit{ set and } P \subseteq C_i\}$
so $P = \cap \{C_i / C_i \textit{ is a } PF\alpha C \textit{ and } P \subseteq C_i\}$
$= cl_\alpha(P)$
Conversely, consider $P = cl_\alpha(P)$,
$P \in \{C_i / C_i \textit{ is a } PF\alpha C \textit{ set and } P \subseteq C_i\}$
Thus P is a Pythagorean fuzzy α - closed set.

Proposition 3.7:
In a PFTS (X, τ), the following hold for pythagorean fuzzy α - closure:
(1) $cl_\alpha(\underline{0}) = \underline{0}$
(2) $cl_\alpha(P)$ is a $PF\alpha C$ in (X, τ) for every PFS P in X.
(3) $cl_\alpha(P) \subseteq cl_\alpha(R)$ whenever $P \subseteq R$ for every P and R in X.
(4) $cl_\alpha(cl_\alpha(P)) = cl_\alpha(P)$ for every PFS P in X.
Proof:
(1) The proof is obvious
(2) By preposition, P is $PF\alpha C$ iff $P = cl_\alpha(P)$ we get $cl_\alpha(P)$ is a $PF\alpha C$ for every P in X.
(3) By same preposition, we get $P = cl_\alpha(P)$ and $R = cl_\alpha(R)$. whenever $P \subseteq R$, we have $cl_\alpha(P) \subseteq cl_\alpha(R)$.
(4) Let P be a PFS in X. We know that $P = cl_\alpha(P)$
$cl_\alpha(P) = cl_\alpha(cl_\alpha(P))$.
Thus $cl_\alpha(cl_\alpha(P)) = cl_\alpha(P)$ for every P in X.

Definition 3.8:
Let (X, τ_X) and (Y, τ_Y) be PFTSs. A mapping $f : X \to Y$ is named as Pythagorean fuzzy α - continuous $(PF\alpha CN)$ if the inverse image of each PFOS of Y is a $PF\alpha O$ set in X.

Theorem 3.9:
Let $f : (X, \tau_X) \to (Y, \tau_Y)$ be a mapping from a PFTS (X, τ_X) to a PFTS (Y, τ_Y). If f is Pythagorean fuzzy α - continous, then
(1) $f\,(cl(int(cl(P)))) \subseteq cl(f(P))$ for all PFS P in X.
(2) $cl\,(int(f^{-1}(B))) \subseteq f^{-1}(cl(B))$ for all B in Y.
Proof:
Assume that f is a $PF\alpha CN$ mapping. Let P be a PFS in X. Then $cl(f(P))$ is a PFCS in Y, and thus $f^{-1}\,(cl(f(P)))$ is a $PF\alpha C$ set in X. Thus
$cl\,(int(cl(P))) = cl\,(int(cl(cl(P))))$
$\subseteq cl\,(int(cl(f^{-1}(cl(f(P))))))$

$\subseteq f^{-1}(cl(f(P)))$
So $f\,(cl(int(cl(P)))) \subseteq cl(f(P))$.
Now let B be a PFS in Y. Then $f^{-1}(B)$ an PFS in X. By (i),
$f\,(cl\,(int\,(cl\,(f^{-1}(B))))) \subseteq cl\,(f\,(f^{-1}(B)))$
$\subseteq cl(B)$
Thus $cl\,(int\,(cl\,(f^{-1}(B)))) \subseteq f^{-1}(cl(B))$.

4. Conclusion

Herein, the Pythagorean fuzzy alpha open and closed, pythagorean alpha closure and pythagorean alpha continuinity have been introduced. Furthermore this study can be extended to Pythagorean semi open, pre-open sets.

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