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Pairwise Fuzzy Semi-Baire Spaces

G.Thangaraj[1*] and S.Sethuraman[2]
[1]Department of Mathematics, Thiruvalluvar University, Vellore-632115,
Tamil Nadu, INDIA.
[2]Department of Mathematics, T.K.Govt. Arts College, Vriddhachalam-606001,
Tamil Nadu, INDIA.

Abstract

In this paper, the concepts of pairwise fuzzy semi-Baire bitopological spaces are introduced and characterizations of pairwise fuzzy semi-Baire bitopological spaces are studied.

Key words: Pairwise fuzzy semi-dense, pairwise fuzzy semi-nowhere dense, pairwise fuzzy semi-first category, pairwise fuzzy semi-second category, pairwise fuzzy semi-residual and pairwise fuzzy semi-Baire spaces.

AMS classification: 54 A 40, 03 E 72.

1. Introduction

The concept of fuzzy sets and fuzzy set operations were first introduced by L.A.Zadeh in his classical paper [13] in the year 1965. Thereafter the paper of C.L.Chang [3] in 1968 paved the way for the subsequent tremendous growth of the numerous fuzzy topological concepts. Since then much attention has been paid to generalize the basic concepts of General Topology in fuzzy setting and thus a modern theory of fuzzy topology has been developed. In 1989, A.Kandil [4] introduced the concept of fuzzy bitopological spaces. Levine [5] introduced the concepts of semi-open sets and semicontinuous mappings in topological spaces. K.K.Azad [2] carried out this into fuzzification in 1981 and presented fuzzy semi-open (resp., fuzzy semi-closed), fuzzy regular open (resp., closed) sets. The concept of Baire spaces in fuzzy setting was introduced and studied by G.Thangaraj and S.Anjalmose in [7]. The concept of pairwise Baire spaces have been studied by the authors in [1]. In this paper, the concepts of pairwise fuzzy semi-Baire spaces are introduced and characterizations of pairwise fuzzy semi-Baire spaces are studied.

[1*]g.thangaraj@rediffmail.com,[2]koppan60@gmail.com

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2. Preliminaries

Now we give some basic notions and results used in the sequel. In this work by (X,T) or simply by X, we will denote a fuzzy topological space due to (Chang, 1968). By a fuzzy bitopological space (Kandil, 1989) we mean an ordered triple (X,T_1,T_2) where T_1 and T_2 are fuzzy topologies on the non-empty set X. The complement λ' of a fuzzy set λ is defined by $\lambda'(x) = 1 - \lambda(x) \;\; x \in X$.

Definition 2.1 Let λ and μ be any two fuzzy sets in (X,T). Then we define $\lambda \vee \mu : X \to [0,1]$ as follows : $(\lambda \vee \mu)(x) = Max\{\lambda(x), \mu(x)\}$. Also we define $\lambda \wedge \mu : X \to [0,1]$ as follows : $(\lambda \wedge \mu)(x) = Min\{\lambda(x), \mu(x)\}$.

Definition 2.2 [4] Let (X,T) be a fuzzy topological space and λ be any fuzzy set in (X,T). We define $int(\lambda) = \vee\{\mu/\mu \leq \lambda, \mu \in T\}$ and $cl(\lambda) = \wedge\{\mu/\lambda \leq \mu, 1-\mu \in T\}$.

For any fuzzy set λ in a fuzzy topological space (X,T), it is easy to see that $1 - cl(\lambda) = int(1-\lambda)$ and $1 - int(\lambda) = cl(1-\lambda)$[**4**].

Definition 2.3 [6] A fuzzy set λ in a fuzzy bitopological space (X,T_1,T_2) is called a pairwise fuzzy dense set if $cl_{T_1}cl_{T_2}(\lambda) = cl_{T_2}cl_{T_1}(\lambda) = 1$.

Definition 2.4 [8] A fuzzy set λ in a fuzzy bitopological space (X,T_1,T_2) is called a pairwise fuzzy semi-dense set if $scl_{T_1}scl_{T_2}(\lambda) = scl_{T_2}scl_{T_1}(\lambda) = 1$.

Definition 2.5 [9] A fuzzy set λ in a fuzzy bitopological space (X,T_1,T_2) is called a pairwise fuzzy nowhere dense set if $int_{T_1}cl_{T_2}(\lambda) = int_{T_2}cl_{T_1}(\lambda) = 0$, in (X,T_1,T_2).

Definition 2.6 [2] Let (X,T_1,T_2) be a fuzzy bitopological space. A fuzzy set λ in (X,T_1,T_2) is called a pairwise fuzzy semi-closed set if $int_{T_1}(cl_{T_2}(\lambda)) \leq \lambda$ and $int_{T_2}(cl_{T_1}(\lambda)) \leq \lambda$.

Definition 2.7 [2] Let (X,T_1,T_2) be a fuzzy bitopological space. A fuzzy set λ in (X,T_1,T_2) is called a pairwise fuzzy semi-open set if $1-\lambda$ is a pairwise fuzzy semi-closed set.

Definition 2.8 [1] Let (X,T_1,T_2) be a fuzzy bitopological space. A fuzzy set λ in (X,T_1,T_2) is called a pairwise fuzzy semi-closed set if $\lambda = scl(\lambda)$ and pairwise fuzzy semi-open set if $\lambda = sint(\lambda)$.

[1*]g.thangaraj@rediffmail.com,[2]koppan60@gmail.com

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Theorem 2.9 [1] If λ is a fuzzy set in a fuzzy topological space, then

1. $int(cl(\lambda)) \leq sint(scl(\lambda))$.
2. $scl(\lambda) \leq cl(\lambda)$.
3. $int(\lambda) \leq sint(\lambda)$.

Theorem 2.10 [11] If λ is a pairwise fuzzy semi-closed set with $sint_{T_j}(\lambda) = 0, \quad (j = 1, 2)$, in a fuzzy bitopological space (X, T_1, T_2) , then λ is a pairwise fuzzy semi-nowhere dense set in (X, T_1, T_2).

3. Pairwise fuzzy semi-nowhere dense set

Definition 3.1 [8] A fuzzy set λ in a fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy semi-nowhere dense set if $sint_{T_1} scl_{T_2}(\lambda) = sint_{T_2} scl_{T_1}(\lambda) = 0$.

Proposition 3.2 If λ is a pairwise fuzzy semi-closed set with $sint_{T_j}(\lambda) = 0, \quad (j = 1, 2)$, in a fuzzy bitopological space (X, T_1, T_2), then λ is a pairwise fuzzy semi-nowhere dense set in (X, T_1, T_2).
Proof: Let λ be a pairwise fuzzy semi-closed set in (X, T_1, T_2). Then $scl_{T_1}(\lambda) = \lambda$ and $scl_{T_2}(\lambda) = \lambda$. Also $sint_{T_1}(\lambda) = 0$ and $sint_{T_2}(\lambda) = 0$. Then $sint_{T_1}(scl_{T_2}(\lambda)) = 0$ and $sint_{T_2}(scl_{T_1}(\lambda)) = 0$ implies that $sint_{T_1} scl_{T_2}(\lambda) = sint_{T_2} scl_{T_1}(\lambda) = 0$. Therefore λ is a pairwise fuzzy semi-nowhere dense set in (X, T_1, T_2).

Proposition 3.3 If λ is a pairwise fuzzy semi-nowhere dense set in a fuzzy bitopological space (X, T_1, T_2), then $sint_{T_i}(\lambda) = 0, \quad (i = 1, 2)$.
Proof: Let λ be a pairwise fuzzy semi-nowhere dense set in (X, T_1, T_2). Then $sint_{T_1} scl_{T_2}(\lambda) = sint_{T_2} scl_{T_1}(\lambda) = 0$. Now $\lambda \leq scl_{T_2}(\lambda)$ implies that $sint_{T_1}(\lambda) \leq sint_{T_1} scl_{T_2}(\lambda)$. Then $sint_{T_1}(\lambda) = 0$. Also $\lambda \leq scl_{T_1}(\lambda)$ implies that $sint_{T_2}(\lambda) \leq sint_{T_2} scl_{T_1}(\lambda)$. Then $sint_{T_2}(\lambda) = 0$ and hence $sint_{T_i}(\lambda) = 0, \quad (i = 1, 2)$.

Proposition 3.4 If λ is a pairwise fuzzy semi-nowhere dense set in a fuzzy bitopological space (X, T_1, T_2), then $1 - \lambda$ is a pairwise fuzzy semi-dense set in (X, T_1, T_2).
Proof: Let λ be a pairwise fuzzy semi-nowhere dense set in (X, T_1, T_2). Then $sint_{T_1} scl_{T_2}(\lambda) = sint_{T_2} scl_{T_1}(\lambda) = 0$. Now $1 - sint_{T_1} scl_{T_2}(\lambda) = 1 - 0 = 1$. Then $scl_{T_1}(1 - scl_{T_2}(\lambda)) = 1$ which implies that $scl_{T_1} sint_{T_2}(1 - \lambda) = 1$. But $scl_{T_1} sint_{T_2}(1 - \lambda) \leq scl_{T_1} scl_{T_2}(1 - \lambda)$. Hence $1 \leq scl_{T_1} scl_{T_2}(1 - \lambda)$. That is, $scl_{T_1} scl_{T_2}(1 - \lambda) = 1$.

[1*]g.thangaraj@rediffmail.com,[2]koppan60@gmail.com

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Now $1 - sint_{T_2} scl_{T_1}(\lambda) = 1 - 0 = 1$. Then $scl_{T_2}(1 - scl_{T_1}(\lambda)) = 1$ which implies that $scl_{T_2} sint_{T_1}(1 - \lambda) = 1$. But $scl_{T_2} sint_{T_1}(1 - \lambda) \leq scl_{T_2} scl_{T_1}(1 - \lambda)$. Hence $1 \leq scl_{T_2} scl_{T_1}(1 - \lambda)$. That is., $scl_{T_2} scl_{T_1}(1 - \lambda) = 1$. Hence $scl_{T_1} scl_{T_2}(1 - \lambda) = scl_{T_2} scl_{T_1}(1 - \lambda) = 1$. Therefore, $1 - \lambda$ is a pairwise fuzzy semi-dense set in (X, T_1, T_2).

Proposition 3.5 If μ is a pairwise fuzzy semi-nowhere dense set in a fuzzy bitopological space (X, T_1, T_2) and if $\lambda \leq \mu$ for a fuzzy set λ in (X, T_1, T_2), then λ is also pairwise fuzzy semi-nowhere dense set in (X, T_1, T_2).
Proof: Let μ be a pairwise fuzzy semi-nowhere dense set in (X, T_1, T_2). Then, $sint_{T_1}(scl_{T_2}(\mu)) = sint_{T_2}(scl_{T_1}(\mu)) = 0$. Now $\lambda \leq \mu$, implies that $sint_{T_1}(scl_{T_2}(\lambda)) \leq sint_{T_1}(scl_{T_2}(\mu))$ and $sint_{T_2}(scl_{T_1}(\lambda)) \leq sint_{T_2}(scl_{T_1}(\mu))$. Hence $sint_{T_1}(scl_{T_2}(\lambda)) \leq 0$ and $sint_{T_2}(scl_{T_1}(\lambda)) \leq 0$. That is., $sint_{T_1}(scl_{T_2}(\lambda)) = sint_{T_2}(scl_{T_1}(\lambda)) = 0$. Therefore λ is a pairwise fuzzy semi-nowhere dense set in (X, T_1, T_2).

4. Pairwise fuzzy nowhere dense and pairwise fuzzy semi-nowhere dense sets

Proposition 4.1 Every pairwise fuzzy semi-nowhere dense set in (X, T_1, T_2) is a pairwise fuzzy nowhere dense set in (X, T_1, T_2).
Proof: Let λ be a pairwise fuzzy semi-nowhere dense set in (X, T_1, T_2). Then $sint_{T_1}(scl_{T_2}(\lambda)) = sint_{T_2}(scl_{T_1}(\lambda)) = 0$. By theorem 2.9 (1), $int_{T_1}(cl_{T_2}(\lambda)) \leq sint_{T_1}(scl_{T_2}(\lambda))$ and $int_{T_2}(cl_{T_1}(\lambda)) \leq sint_{T_2}(scl_{T_1}(\lambda))$. Hence $int_{T_1}(cl_{T_2}(\lambda)) \leq 0$ and $int_{T_2}(cl_{T_1}(\lambda)) \leq 0$. Therefore $int_{T_1}(cl_{T_2}(\lambda)) = 0$ and $int_{T_2}(cl_{T_1}(\lambda)) = 0$ and hence λ is a pairwise fuzzy nowhere dense set in (X, T_1, T_2).

Proposition 4.2 Every pairwise fuzzy semi-dense set in (X, T_1, T_2) is a pairwise fuzzy dense set in (X, T_1, T_2).
Proof: Let λ be a pairwise fuzzy semi dense set in (X, T_1, T_2). Then $scl_{T_1}(scl_{T_2}(\lambda)) = scl_{T_2}(scl_{T_1}(\lambda)) = 1$. By theorem 2.9 (2), $scl_{T_1}(\lambda) \leq cl_{T_1}(\lambda)$ and $scl_{T_2}(\lambda) \leq cl_{T_2}(\lambda)$. Hence $scl_{T_2}(scl_{T_1}(\lambda)) \leq scl_{T_2}(cl_{T_1}(\lambda))$ and $scl_{T_1}(scl_{T_2}(\lambda)) \leq scl_{T_1}(cl_{T_2}(\lambda))$. Again using theorem 2.9 (2), $scl_{T_2}(scl_{T_1}(\lambda)) \leq cl_{T_2}(cl_{T_1}(\lambda))$ and $scl_{T_1}(scl_{T_2}(\lambda)) \leq cl_{T_1}(cl_{T_2}(\lambda))$. But $scl_{T_1}(scl_{T_2}(\lambda)) = scl_{T_2}(scl_{T_1}(\lambda)) = 1$. Hence $1 \leq cl_{T_2}(cl_{T_1}(\lambda))$ and $1 \leq cl_{T_1}(cl_{T_2}(\lambda))$. Therefore $cl_{T_1}(cl_{T_2}(\lambda)) = cl_{T_2}(cl_{T_1}(\lambda)) = 1$ and λ is a pairwise fuzzy dense set in (X, T_1, T_2).

Proposition 4.3 Let (X, T_1, T_2) be a fuzzy bitopological space. If a non-zero pairwise fuzzy set λ in (X, T_1, T_2) is a pairwise fuzzy nowhere dense set with $sint_{T_j}(\lambda) = 0, \quad (j = 1, 2)$, then λ is a pairwise fuzzy semi-nowhere dense set in (X, T_1, T_2).

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Proof: Let λ be a pairwise fuzzy nowhere dense set in (X, T_1, T_2). Then $int_{T_1}(cl_{T_2}(\lambda)) = int_{T_2}(cl_{T_1}(\lambda)) = 0$ and therefore $int_{T_1}(cl_{T_2}(\lambda)) \leq \lambda$ and $int_{T_2}(cl_{T_1}(\lambda)) \leq \lambda$. Hence λ is pairwise fuzzy semi-closed set with $sint_{T_j}(\lambda) = 0, \ (j = 1, 2)$, in (X, T_1, T_2). Hence by theorem 2.10, λ is a pairwise fuzzy semi-nowhere dense set in (X, T_1, T_2).

Proposition 4.4 If λ is a pairwise fuzzy open and T_j (j=1,2) fuzzy dense set in (X, T_1, T_2) and $\mu \leq 1 - \lambda$ with $sint_{T_j}(\lambda) = 0, (j = 1, 2)$, then μ is a pairwise fuzzy semi-nowhere dense set in (X, T_1, T_2).
Proof: Let λ be a pairwise fuzzy open and T_j (j=1,2) fuzzy dense set in (X, T_1, T_2). Then, $1 - \lambda$ is a pairwise fuzzy nowhere dense set and $int_{T_1}(cl_{T_2}(1 - \lambda)) = int_{T_2}(cl_{T_1}(1 - \lambda)) = 0$. Now $\mu \leq 1 - \lambda$ implies that $int_{T_1}(cl_{T_2}(\mu)) \leq int_{T_1}(cl_{T_2}(1 - \lambda)$ and $int_{T_2}(cl_{T_1}(\mu)) \leq int_{T_2}(cl_{T_1}(1 - \lambda)$. Therefore $int_{T_1}(cl_{T_2}(\mu)) = int_{T_2}(cl_{T_1}(\mu)) = 0$ and μ is a pairwise fuzzy nowhere dense set with $sint_{T_j}(\lambda) = 0, \ (j = 1, 2)$, in (X, T_1, T_2). Therefore by proposition 4.3, μ is a pairwise fuzzy semi-nowhere dense set in (X, T_1, T_2).

Proposition 4.5 If μ is a pairwise fuzzy nowhere dense set in a fuzzy bitopological space (X, T_1, T_2) and if $\lambda \leq \mu$ for a fuzzy set λ with $sint_{T_j}(\lambda) = 0, \ (j = 1, 2)$, in (X, T_1, T_2), then λ is a pairwise fuzzy semi-nowhere dense set in (X, T_1, T_2).
Proof: Let μ be a pairwise fuzzy nowhere dense set in (X, T_1, T_2). Then, $int_{T_1}(cl_{T_2}(\mu)) = int_{T_2}(cl_{T_1}(\mu)) = 0$. Now $\lambda \leq \mu$, implies that $int_{T_1}(cl_{T_2}(\lambda)) \leq int_{T_1}(cl_{T_2}(\mu))$ and $int_{T_2}(cl_{T_1}(\lambda)) \leq int_{T_2}(cl_{T_1}(\mu))$. Hence $int_{T_1}(cl_{T_2}(\lambda)) \leq 0$ and $int_{T_2}(cl_{T_1}(\lambda)) \leq 0$. That is., $int_{T_1}(cl_{T_2}(\lambda)) = int_{T_2}(cl_{T_1}(\lambda)) = 0$ .Therefore λ is a pairwise fuzzy nowhere dense set with $sint_{T_j}(\lambda) = 0, \ (j = 1, 2)$ and by proposition 4.3, λ is a pairwise fuzzy semi-nowhere dense set in (X, T_1, T_2).

5. Pairwise fuzzy semi-first category sets

Definition 5.1 Let (X, T_1, T_2) be a fuzzy bitopological space. A fuzzy set λ in (X, T_1, T_2) is called a pairwise fuzzy semi-first category set if $\lambda = \vee_{k=1}^{\infty}(\lambda_k)$, where (λ_k)'s are pairwise fuzzy semi-nowhere dense sets in (X, T_1, T_2). Any other fuzzy set in (X, T_1, T_2) is said to be a pairwise fuzzy semi-second category set in (X, T_1, T_2).

Definition 5.2 If λ is a pairwise fuzzy semi-first category set in a fuzzy bitopological space (X, T_1, T_2), then the fuzzy set $1 - \lambda$ is called a pairwise fuzzy semi-residual set in (X, T_1, T_2).

Definition 5.3 A fuzzy bitopological space(X, T_1, T_2) is called pairwise fuzzy

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semi-first category if the fuzzy set 1_X is a pairwise fuzzy semi-first category set in (X, T_1, T_2). That is., $1_X = \vee_{k=1}^{\infty}(\lambda_k)$, where (λ_k)'s are pairwise fuzzy semi-nowhere dense sets in (X, T_1, T_2). Otherwise, (X, T_1, T_2) will be called a pairwise fuzzy second category space.

Proposition 5.4 If λ is a pairwise fuzzy semi-first category set in (X, T_1, T_2), then $1 - \lambda = \wedge_{k=1}^{\infty}(\mu_k)$, where (μ_k)'s are pairwise fuzzy semi-dense sets.
Proof: Let λ be a pairwise fuzzy semi-first category set in (X, T_1, T_2). Then $\lambda = \vee_{k=1}^{\infty}(\lambda_k)$, where (λ_k)'s are pairwise fuzzy semi-nowhere dense sets in (X, T_1, T_2). Now $1 - \lambda = 1 - \vee_{k=1}^{\infty}(\lambda_k) = \wedge_{k=1}^{\infty}(1 - \lambda_k)$. Since λ_k is a pairwise fuzzy semi-nowhere dense set in (X, T_1, T_2) and by Proposition 3.4, $1 - \lambda_k$ is a pairwise fuzzy semi-dense set in (X, T_1, T_2). Let us put $\mu_k = 1 - \lambda_k$. Then $1 - \lambda = \wedge_{k=1}^{\infty}(\mu_k)$, where $cl_{T_j}(\mu_k) = 1$, $(j = 1, 2)$.

Proposition 5.5 If μ is a pairwise fuzzy semi-first category set in a fuzzy bitopological space (X, T_1, T_2) and if $\lambda \leq \mu$ for a fuzzy set λ in (X, T_1, T_2), then λ is also pairwise fuzzy semi-first category set in (X, T_1, T_2).
Proof: Let μ be a pairwise fuzzy semi-first category set in (X, T_1, T_2). Then, $\mu = \vee_{i=1}^{\infty}(\mu_i)$, where (μ_i)'s are pairwise fuzzy semi-nowhere dense sets in (X, T_1, T_2). Now $\lambda \wedge \mu = \lambda \wedge [\vee_{i=1}^{\infty}(\mu_i)] = \vee_{i=1}^{\infty}(\lambda \wedge \mu_i)$. Also $\lambda \leq \mu$, implies that $\lambda \wedge \mu = \lambda$. Therefore $\lambda = \vee_{i=1}^{\infty}(\lambda \wedge \mu_i)$. Since $\lambda \wedge \mu_i \leq \mu_i$ and (μ_i)'s are pairwise fuzzy semi-nowhere dense sets in (X, T_1, T_2) and by proposition 3.5, $(\lambda \wedge \mu_i)$'s are pairwise fuzzy semi-nowhere dense sets in (X, T_1, T_2). Hence $\lambda = \vee_{i=1}^{\infty}(\lambda \wedge \mu_i)$, where $(\lambda \wedge \mu_i)$'s are pairwise fuzzy semi-nowhere dense sets in (X, T_1, T_2), implies that λ is a pairwise fuzzy semi-first category set in (X, T_1, T_2).

Proposition 5.6 If λ is a pairwise fuzzy semi-residual set in a fuzzy bitopological space (X, T_1, T_2) and if $\lambda \leq \mu$ for a fuzzy set μ in (X, T1,T2), then μ is a pairwise fuzzy semi-residual set in (X, T_1, T_2).
Proof: Let λ be a pairwise fuzzy semi-residual set in (X, T_1, T_2). Then, $1 - \lambda$ is a pairwise fuzzy semi-first category set in (X, T_1, T_2). Now $\lambda \leq \mu$ for a fuzzy set μ in (X, T_1, T_2), implies that $1 - \lambda \geq 1 - \mu$. Then, by proposition 5.5, $1 - \mu$ is a pairwise fuzzy semi-first category set in (X, T_1, T_2). Hence μ is a pairwise fuzzy semi-residual set in (X, T_1, T_2).

Proposition 5.7 If λ is a pairwise fuzzy semi-first category set in a fuzzy bitopological space (X, T_1, T_2), then λ is a pairwise fuzzy first category set in (X, T_1, T_2).

[1*]g.thangaraj@rediffmail.com,[2]koppan60@gmail.com

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Proof: Let λ be a pairwise fuzzy semi-first category set in (X, T_1, T_2). Then $\lambda = \bigvee_{i=1}^{\infty}(\lambda_i)$, where (λ_i)'s are pairwise fuzzy semi-nowhere dense sets in (X, T_1, T_2). By proposition 4.1, the pairwise fuzzy semi-nowhere dense sets (λ_i)'s are pairwise fuzzy nowhere dense sets in (X, T_1, T_2) and hence $\lambda = \bigvee_{i=1}^{\infty}(\lambda_i)$, where (λ_i)'s are pairwise fuzzy nowhere dense sets and λ is a pairwise fuzzy first category set in (X, T_1, T_2)

Proposition 5.8 If (X, T_1, T_2) is a pairwise fuzzy semi-Baire space, then (X, T_1, T_2) is a pairwise fuzzy Baire space.
Proof: Let λ be a pairwise fuzzy semi-first category set in a pairwise fuzzy semi-Baire space (X, T_1, T_2). Then $sint_{T_j}(\lambda) = 0, (j = 1, 2)$. By proposition 5.7, the pairwise fuzzy semi-first category set is a pairwise fuzzy first category set in (X, T_1, T_2). By theorem 2.9 (3) $int_{T_j}(\lambda)) \leq sint_{T_j}(\lambda))$,(j = 1, 2). Since $sint_{T_j}(\lambda) = 0, (j = 1, 2)$, $int_{T_j}(\lambda)) \leq 0$,(j = 1, 2). Therefore $int_{T_j}(\lambda) = 0, (j = 1, 2)$, and (X, T_1, T_2) is a pairwise fuzzy Baire space.

Proposition 5.9 If $\lambda = \bigvee_{i=1}^{\infty}(\lambda_i)$, with $sint_{T_j}(\lambda_i) = 0, \quad (j = 1, 2)$, is a pairwise fuzzy first category set in (X, T_1, T_2), then λ is a pairwise fuzzy semi-first category set in (X, T_1, T_2).
Proof: Let λ be a pairwise fuzzy first category set in (X, T_1, T_2). Then $\lambda = \bigvee_{i=1}^{\infty}(\lambda_i)$, where (λ_i)'s are pairwise fuzzy nowhere dense sets with $sint_{T_j}(\lambda) = 0, \quad (j = 1, 2)$, in (X, T_1, T_2). By proposition 4.5, the pairwise fuzzy nowhere dense sets (λ_i)'s are pairwise fuzzy semi-nowhere dense sets in (X, T_1, T_2) and hence $\lambda = \bigvee_{i=1}^{\infty}(\lambda_i)$, where (λ_i)'s are pairwise fuzzy semi-nowhere dense sets and λ is a pairwise fuzzy semi-first category set in (X, T_1, T_2).

Proposition 5.10 If $\mu = \vee_{i=1}^{\infty}(\mu_i)$ is a pairwise fuzzy first category set with $sint_{T_j}(\mu_i) = 0, \quad (j = 1, 2)$, in a fuzzy bitopological space (X, T_1, T_2) and if $\lambda \leq \mu$ for a fuzzy set λ in (X, T_1, T_2), then λ is a pairwise fuzzy semi-first category set in (X, T_1, T_2).
Proof: Let μ be a pairwise fuzzy first category set in (X, T_1, T_2). Then, $\mu = \vee_{i=1}^{\infty}(\mu_i)$, where (μ_i)'s are pairwise fuzzy nowhere dense sets in (X, T_1, T_2). Now $\lambda \wedge \mu = \lambda \wedge (\vee_{i=1}^{\infty}(\mu_i)) = \vee_{i=1}^{\infty}(\lambda \wedge (\mu_i))$. Also $\lambda \leq \mu$, implies that $\lambda \wedge \mu = \lambda$. Therefore $\lambda = \vee_{i=1}^{\infty}(\lambda \wedge (\mu_i))$. Let $\nu_i = \lambda \wedge (\mu_i)$. Since $\nu_i = \lambda \wedge (\mu_i) \leq (\mu_i)$ and (μ_i)'s are pairwise fuzzy nowhere dense sets with $sint_{T_j}((\mu_i)) = 0, \quad (j = 1, 2)$, in (X, T_1, T_2) and by proposition 4.5, (ν_i)'s are pairwise fuzzy semi-nowhere dense sets in (X, T_1, T_2). Hence λ is a pairwise fuzzy semi-first category set in (X, T_1, T_2).

[1*]g.thangaraj@rediffmail.com,[2]koppan60@gmail.com

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6. Pairwise fuzzy semi-Baire spaces

Definition 6.1 A fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy semi-Baire space if $sint_{T_j}(\vee_{k=1}^{\infty}(\lambda_k)) = 0$, $(j = 1, 2)$, where (λ_k)'s are pairwise fuzzy semi-nowhere dense sets in (X, T_1, T_2).

Proposition 6.2 Let (X, T_1, T_2) be a fuzzy bitopological space. Then the following are equivalent:

(1). (X, T_1, T_2) is a pairwise fuzzy semi-Baire space.
(2). $sint_{T_j}(\lambda) = 0$, $(j = 1, 2)$, for every pairwise fuzzy semi-first category set λ in (X, T_1, T_2).
(3). $scl_{T_j}(\mu) = 1$, $(j = 1, 2)$, for every pairwise fuzzy semi-residual set μ in (X, T_1, T_2).

Proof: $(\mathbf{1}) \Rightarrow (\mathbf{2})$ Let λ be a pairwise fuzzy semi-first category set in (X, T_1, T_2). Then $\lambda = \vee_{k=1}^{\infty}(\lambda_k)$, where (λ_k)'s are pairwise fuzzy semi-nowhere dense sets in (X, T_1, T_2). Now $sint_{T_j}(\lambda) = sint_{T_j}(\vee_{k=1}^{\infty}(\lambda_k)) = 0$, $(j = 1, 2)$, [since (X, T_1, T_2) is a pairwise fuzzy semi-Baire space]. Therefore $sint_{T_j}(\lambda) = 0$, where (λ_k)'s are pairwise fuzzy semi-nowhere dense fuzzy sets in (X, T_1, T_2).

$(\mathbf{2}) \Rightarrow (\mathbf{3})$ Let μ be a pairwise fuzzy semi-residual set in (X, T_1, T_2). Then $1 - \mu$ is a pairwise fuzzy semi-first category set in (X, T_1, T_2). By hypothesis, $sint_{T_j}(1 - \mu) = 0$, $(j = 1, 2)$, which implies that $1 - scl_{T_j}(\mu) = 0$. Hence $scl_{T_j}(\mu) = 1$, $(j = 1, 2)$.

$(\mathbf{3}) \Rightarrow (\mathbf{1})$ Let λ be a pairwise semi-fuzzy first category set in (X, T_1, T_2). Then $\lambda = \vee_{k=1}^{\infty}(\lambda_k)$, where (λ_k)'s are pairwise fuzzy semi-nowhere dense sets in (X, T_1, T_2). Now λ is a pairwise fuzzy semi-first category set in (X, T_1, T_2) implies that $1 - \lambda$ is a pairwise fuzzy semi-residual set in (X, T_1, T_2). By hypothesis, $scl_{T_j}(1 - \lambda) = 1$, $(j = 1, 2)$, which implies that $1 - sint_{T_j}(\lambda) = 0$, $(j = 1, 2)$. Then $sint_{T_j}(\lambda) = 1$. That is., $sint_{T_j}(\vee_{k=1}^{\infty}(\lambda_k)) = 0$, where (λ_k)'s are pairwise fuzzy semi-nowhere dense sets in (X, T_1, T_2). Hence (X, T_1, T_2) is a pairwise fuzzy semi-Baire space.

Proposition 6.3 If the fuzzy bitopological space (X, T_1, T_2) is a pairwise fuzzy semi-Baire space, then (X, T_1, T_2) is a pairwise fuzzy semi-second category space.
Proof: Let (X, T_1, T_2) be a pairwise fuzzy semi-Baire space. Then $sint_{T_j}(\vee_{n=1}^{\infty}(\lambda_k)) = 0$, $(j = 1, 2)$, where (λ_k)'s are pairwise fuzzy semi-nowhere dense sets in (X, T_1, T_2). Now we claim that $\vee_{k=1}^{\infty}(\lambda_k) \neq 1_X$, $(j = 1, 2)$. Suppose that $\vee_{k=1}^{\infty}(\lambda_k) = 1_X$. Then $int_{T_j}(\vee_{k=1}^{\infty}(\lambda_k)) = int 1_X = 1_X$, $(j = 1, 2)$, which implies that $0 = 1$, a

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contradiction. Hence we must have $\vee_{k=1}^{\infty}(\lambda_k) \neq 1_X$. Therefore (X, T_1, T_2) is a pairwise fuzzy semi-second category space.

Proposition 6.4 If $sint_{T_j}(\vee_{k=1}^{\infty}(\lambda_k)) = 0, \quad (j = 1, 2)$, where $sint_{T_j}(\lambda_k) = 0$ and $1 - \lambda_k \in T_j, \quad (j = 1, 2)$, then (X, T_1, T_2) is a pairwise fuzzy semi-Baire space.
Proof: Now $1 - \lambda_k \in T_j, \quad (j = 1, 2 \ \ and \ \ n \geq 1)$, implies that $sint_{T_j}(1 - \lambda_k) = 1 - \lambda_k$. Then $1 - scl_{T_j}(\lambda_k) = 1 - \lambda_k$ and hence $scl_{T_j}(\lambda_k) = \lambda_n, \quad (j = 1, 2 \ \ and \ \ n \geq 1)$. Now $sint_{T_j}(\lambda_k) = 0$ and $cl_{T_j}(\lambda_k) = \lambda_k$ implies that $sint_{T_j}(scl_{T_j}(\lambda_k)) = 0, \quad (j = 1, 2 \ \ and \ \ n \geq 1)$. In particular, $sint_{T_1}(scl_{T_2}(\lambda_k)) = 0$ and $sint_{T_2}(scl_{T_1}(\lambda_k)) = 0, n \geq 1$. Hence (λ_k)'s are pairwise fuzzy semi-nowhere dense sets in (X, T_1, T_2). Therefore $sint_{T_j}(\vee_{k=1}^{\infty}(\lambda_k)) = 0, \quad n \geq 1$, where (λ_k)'s are pairwise fuzzy semi-nowhere dense sets in (X, T_1, T_2). Hence (X, T_1, T_2) is a pairwise fuzzy semi-Baire space.

Proposition 6.5 If the fuzzy bitopological space (X, T_1, T_2) is a pairwise fuzzy semi-Baire space, then no non-zero pairwise fuzzy semi-open set is a pairwise fuzzy semi-first category set in (X, T_1, T_2).
Proof: Let λ be a non-zero pairwise fuzzy semi-open set in (X, T_1, T_2). Then, $sint_{T_j}(\lambda) = \lambda$ (j=1,2). Suppose that λ is a pairwise fuzzy semi-first category set in (X, T_1, T_2). Since (X, T_1, T_2) is a pairwise fuzzy semi-Baire space and by proposition 6.2, $sint_{T_j}(\lambda) = 0$ (j = 1, 2). This implies that $\lambda = 0$, a contradiction. Hence no non-zero pairwise fuzzy semi-open set is a pairwise fuzzy semi-first category set in (X, T_1, T_2).

Proposition 6.6 If the fuzzy bitopological space (X, T_1, T_2) is a pairwise fuzzy semi-Baire space, then each pairwise fuzzy semi-residual set is a pairwise fuzzy semi-dense set in (X, T_1, T_2).
Proof: Let λ be a pairwise fuzzy semi-residual set in a pairwise fuzzy semi-Baire space (X, T_1, T_2). Then, by proposition 6.2, $scl_{T_j}(\lambda) = 1$ (j = 1, 2) in (X, T_1, T_2). Hence $scl_{T_1}(scl_{T_2}(\lambda)) = scl_{T_2}(scl_{T_1}(\lambda)) = 1$. Therefore λ is a pairwise fuzzy semi-dense set in (X, T_1, T_2).

Proposition 6.7 If the fuzzy bitopological space (X, T_1, T_2) is a pairwise fuzzy semi-Baire space and if $\vee_{i=1}^{\infty}(\lambda_i) = 1$, where the λ_i's are fuzzy sets defined on X, then there is atleast one fuzzy set λ_i such that either $sint_{T_1}(scl_{T_2}(\lambda_i)) \neq 0$ or $sint_{T_2}(scl_{T_1}(\lambda_i)) \neq 0$.
Proof: Suppose $sint_{T_1}(scl_{T_2}(\lambda_i)) = 0$ and $sint_{T_2}(scl_{T_1}(\lambda_i)) = 0$ for all i∈N. Then λ_i's are pairwise fuzzy semi-nowhere dense sets in (X, T_1, T_2). Then $\vee_{i=1}^{\infty}(\lambda_i)$ is a pairwise fuzzy semi-first category set in a pairwise fuzzy semi-Baire space (X, T_1, T_2).

[1*]g.thangaraj@rediffmail.com,[2]koppan60@gmail.com

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By proposition 6.2, $sint_{T_j}(\vee_{i=1}^{\infty}(\lambda_i)) = 0$ (j = 1, 2). But, this is a contradiction, since $\vee_{i=1}^{\infty}(\lambda_i) = 1$, implies that $sint_{T_j}(\vee_{i=1}^{\infty}(\lambda_i)) = sint_{T_j}(1) = 1$ (j = 1,2). Hence, there is atleast one fuzzy set λ_i defined on X such that either $sint_{T_1}(scl_{T_2}(\lambda_i)) \neq 0$ or $sint_{T_2}(scl_{T_1}(\lambda_i)) \neq 0$.

Proposition 6.8 If $\wedge_{i=1}^{\infty}(\lambda_i) \neq 0$, for pairwise fuzzy semi-dense sets (λ_i)'s in a fuzzy bitopological space (X, T_1, T_2), then (X, T_1, T_2) is a pairwise fuzzy semi-second category space.
Proof: Suppose that (X, T_1, T_2) is a pairwise fuzzy semi-first category space. Then, $\vee_{i=1}^{\infty}(\mu_i) = 1$, where (μ_i)'s are pairwise fuzzy semi-nowhere dense sets in (X, T_1, T_2). This implies that $\wedge_{i=1}^{\infty}(1-\mu_i) = 0$. Since (μ_i)'s are pairwise fuzzy semi-nowhere dense sets in (X, T_1, T_2) and by proposition 3.4, $(1-\mu_i)$'s are pairwise fuzzy semi-dense sets in (X, T_1, T_2). Let $\lambda_i = 1 - \mu_i$. Thus, $\wedge_{i=1}^{\infty}(\lambda_i) = 0$, for pairwise fuzzy semi-dense sets λ_i's in (X, T_1, T_2). But this is a contradiction to the hypothesis. Therefore (X, T_1, T_2) is not a pairwise fuzzy semi-first category space and hence (X, T_1, T_2) is a pairwise fuzzy semi-second category space.

Proposition 6.9 If the pairwise fuzzy first category set λ , is a pairwise fuzzy semi-closed set, in a pairwise fuzzy semi-Baire space (X, T_1, T_2), then λ is a pairwise semi-fuzzy nowhere dense set in (X, T_1, T_2).
Proof: Let λ be a pairwise fuzzy semi-first category set in a pairwise fuzzy semi-Baire space (X, T_1, T_2) and $scl_{T_j}(\lambda) = \lambda$ ...(1) (j = 1,2) By proposition 6.2, $sint_{T_j}(\lambda) = 0$ ...(2) (j = 1, 2) ,for the pairwise fuzzy semi-first category set λ in (X, T_1, T_2). Then, from (1) and (2), $sint_{T_1}(scl_{T_2}(\lambda)) = sint_{T_2}(scl_{T_1}(\lambda)) = 0$. Hence, λ is a pairwise fuzzy semi-nowhere dense set in (X, T_1, T_2).

Proposition 6.10 If $scl_{T_j}(\wedge_{k=1}^{\infty}(\lambda_k)) = 1$, where (λ_k)'s, $(j = 1, 2)$, are T_j- fuzzy dense and pairwise fuzzy semi-open sets in (X, T_1, T_2), then (X, T_1, T_2) is a pairwise fuzzy semi-Baire space.
Proof: Let (λ_k)'s, $(j = 1, 2 \ \ and \ \ n \geq 1)$, be T_j-fuzzy semi-dense and pairwise fuzzy semi-open sets in (X, T_1, T_2). Now $scl_{T_j}(\wedge_{k=1}^{\infty}(\lambda_k)) = 1$ implies that $1 - scl_{T_j}(\wedge_{k=1}^{\infty}(\lambda_k)) = 0$, $(j = 1, 2)$. Then $sint_{T_j}(1 - \wedge_{k=1}^{\infty}(\lambda_k)) = 0$, $(j = 1, 2)$ and hence $sint_{T_i}(\vee_{n=1}^{\infty}(1 - \lambda_k)) = 0. \Longrightarrow (1)$

Since (λ_k)'s are T_j-fuzzy semi-dense sets in (X, T_1, T_2), $cl_{T_j}(\lambda_k) = 1$, $(j = 1, 2 \ \ and \ \ n \geq 1)$. Then $1 - scl_{T_j}(\lambda_k) = 0$, which implies that $sint_{T_j}(1 - \lambda_k) = 0$ and $\lambda_k = 1 - (1 - \lambda_k) \in T_j$, $(j = 1, 2 \ \ and \ \ n \geq 1)$. Hence from (1), by Proposition 5.2, we have (X, T_1, T_2) is a pairwise fuzzy semi-Baire space.

[1*]g.thangaraj@rediffmail.com,[2]koppan60@gmail.com

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7. Conclusion

The concepts of pairwise fuzzy semi-Baire spaces have been introduced and characterizations of pairwise fuzzy semi-Baire spaces were studied.

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[1*]g.thangaraj@rediffmail.com,[2]koppan60@gmail.com