



A Study on Continuous Inspection Of Markov Process With a Clearance Interval

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Abstract

Dodge's continuous sampling plan-1 (CSP-1) with clearance interval zero may be inefficient if there is serial correlation between successive units which are Markov dependent and a clearance interval greater than zero is appropriate. For such a situation, the average outgoing quality limit (AOQL) expression has been obtained and, when the serial correlation coefficient of the Markov chain is assumed to be known a priori, it is numerically demonstrated that smaller AOQL values are achieved numerically for values of the clearance interval from 1 to 4, by improving the performance of CSP-1.

Key words: Continuous Sampling Plan, Average Outgoing Quality, Bernoulli, Markovian

AMS classification:

1. INTRODUCTION

The first Continuous Sampling Plan (CSP) was devised by Dodge (1943). He assumed that, the production process is under statistical control, that is, the probability of finding a non-conforming unit is constant over the time axis. That is, the production process follows an i.i.d. Bernoulli pattern and obtained the Average Outgoing Quality (AOQ) and AOQL contours. Such an assumption him may be basic and it is quite conceivable that there could be some fluctuation pattern in the output quality that might induce correlation between successive units. At the same time assumption of total lack of control in a CSP is also unrealistic as any automat mass production is unlikely to follow such a scheme. For total lack of control

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situation Lieberman (1953) obtained the unrestricted AOQL (UAOQL) for CSP-1 as $(k-1)/(r+k)$. Hence, we consider a two-state time-homogeneous MC model to study the effectiveness of CSP-1 especially when the clearance number greater than zero seems to be appropriate rather than Dodge's basic CSP-1 with clearance interval zero and illustrate that clearance intervals from 1 to 4 give smaller AOQL values.

2. THE MODEL AND ASSUMPTIONS

The produced units are indexed by n . Let $X_n = 0$ or 1 depending on whether the n -th unit produced is conforming or otherwise.

Assumption 1 $\{X_n, n \geq 0\}$ follows a two-stage time-homogeneous MC Markov Chain (MC) with transition probabilities.

$$\begin{aligned} p_{00} &= 1 - \alpha, & p_{01} &= \alpha \\ p_{10} &= \beta, & p_{11} &= 1 - \beta. \end{aligned} \tag{1}$$

Assumption 2 The zeroth unit is assumed to be non conforming and

$$P[X_0 = 1] = 1.$$

Assumption 3 The inspected unit that is found to be nonconforming is replaced by conforming unit.

Let $\alpha + \beta = \delta$. Then $p = \alpha^{-1}$ is the long run proportion of nonconforming units. In fact (p, q) (where $q = \beta\delta^{-1}$) is the stationary distribution in for the transition probabilities in (1). The permanent $\phi = 1 - \delta$ is the serial correlation coefficient between X_n and X_{n+1} ($n \geq 0$) provided the stationary distribution is taken as the initial distribution. With the assumption that $P[X_0 = 1] = 1$, together with the strong Markov property of the MC essentially implies that the completion of an implementation of a CSP is a recurrent event. That is, the point at which $P[X_0 = 1] = 1$ is a regenerative point where renewal takes place. Observe that a renewal process is regenerative. We make it a convention that, the zeroth unit not counted in the computation of AOQ.

3. FORMULATION

A CSP is imposed on the production line. The CSP starts at item $X_0 = 1$ with full inspection until a success run of length r of conforming units are observed and then the manufacturer switches to fractional sampling. Let T_1 be the number of

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units produced during the first inspection period. We have

$$T_1 = \min\{n \geq r; X_{n-r+1} = \dots = X_n = 0\}.$$

Similarly let M_1 be the number of units produced during the subsequent fractional sampling. The stopping time under fractional sampling varies from one sampling plan to another. CSPs are used when the production is continuous and the formation of inspection lots for lot-by-lot inspection is artificial or impractical as in manufacturing industries like (i) ammunition loading and component manufacture and (ii) confectionery and food industries. The objective of CSPs is to guarantee a limiting value of AOQ called Average Outgoing Quality Limit (AOQL). The concept of continuous sampling inspection and its mathematical basis for CSP-1 were first presented by Dodge(1943). He studied the behaviour of CSP-1 under the assumption of statistical control. The procedure of CSP-1 is as follows:

- (a) At the start, inspect 100 percent of the units (screening) until r consecutive units in succession are found to be conforming.
- (b) When such a run of length r of conforming units are observed, discontinue 100 percent inspection and inspect every k -th unit from the flow of products in the production line and
- (c) When a nonconforming unit is observed under fractional sampling, revert immediately to 100 percent inspection of succeeding units as per the above procedure and correct or replace all nonconforming units found.

The striking features of this plan are (i) its heavy dependence on the occurrence of a single nonconforming unit which may be isolated and (ii) the assumption of statistical control which is basic.

The abrupt change between 100 percent inspection and fractional sampling may lead to difficulties in personel assignments in the administration of the inspection process. For example, in the production of a very complicated and expensive item such as an aircraft engine, this transition may require major readjustments. Hence we have modified the rule of action under fractional sampling of the procedure of Dodge's CSP-1 as follows:

- (b') When such a run of length r of conforming units is observed terminate screening inspection and begin to inspect every k -th unit from the flow of products in the production line until $(c + 1)$ nonconforming units are observed (when $c = 0$ we easily recognize Dodge's CSP-1).
- (c') As soon as $(c + 1)$ nonconforming units are observed, revert from fractional sampling to screening inspection and start the procedure from paragraph (a) of the procedure of Dodge's CSP-1.

Continuous sampling of the units produces renewal cycles (cycle is the period

between two consecutive epochs when 100 percent screening is instituted). In each cycle we observe a pair of random variables (T_j, M_j) for $j = 1, 2, \dots$. Let $W_j = T_j + M_j$. Note that, W_j is the number of units produced in the j -th renewal cycle. It is also observed that there is an unobservable random variable V_j which is associated with W_j ; where V_j is the number of uninspected outgoing nonconforming units in j -th renewal cycle. Let t be the length of a production run and N_t be the number of renewal inspection cycles completed in the production run of length t . Then $\{N_t, t \geq 0\}$ forms a discrete renewal process. Divide the discrete interval $[0, t]$ into N_t renewal intervals and a possible incomplete $(N_t + 1)$ st interval $[S_{N_t}, 1]$ where

$$S_{N_t} = \sum_{j=1}^{N_t} W_j.$$

Let V_t be the number of uninspected outgoing nonconforming units in $[S_{N_t}, 1]$. V_t is also unobservable like V_j . It is necessary to distinguish a natural renewal interval and the last incomplete one, because of the different probability structures of the two. We now define

$$AOQ(t) = E \left(\sum_{j=1}^{N_t} V_j \right) t^{-1} \quad \text{for } t = 1, 2, \dots \quad (2)$$

For simplicity, let $E(V) = E(V_j)$ and $E(W) = E(W_j)$. Likewise drop the subscripts in the other variables. By the strong Markov property of $\{X_n, n \geq 0\}$ we note that $\{V_j, j \geq 1\}$, $\{T_j, j \geq 1\}$, $\{M_j, j \geq 1\}$, $\{W_j, j \geq 1\}$ and $\{V_t, t \geq 1\}$ are said i.i.d. sequences. Hence, by strong law of large numbers.

$$AOQ = \limsup_{t \rightarrow \infty} AOQ(t) = E(V)/E(W).$$

It must be noted that, under Markovian assumption, the AOQ expression of CSP-1 would depend on the type of fractional sampling procedure used (such as inspecting every k -th unit or adopting probability sampling procedures). It should be pointed out that, random sampling in CSPs for Markovian production processes seems absolutely intractable for any mathematical discussion.

For the modified CSP-1 noted above, we have

$$\begin{aligned}
 E(T) &= [1 - q(1 - p\delta)^{r-1}] / p\delta q(1 - p\delta)^{r-1} \\
 E(M) &= \left\{ \sum_{s=c+1}^{\infty} sk [p_{00}^{(k)}]^{s-1} p_{01}^{(k)} \right\} / \sum_{s=c+1}^{\infty} [p_{00}^{(k)}]^{s-1} p_{01}^k \\
 &= k [p_{00}^{(k)}]^c [1 + cp_{01}^{(k)}] / p_{01}^{(k)} \text{ and} \\
 E(V) &= \left\{ [p_{00}^{(k)}]^c [1 + cp_{01}^{(k)}] / p_{01}^{(k)} \right\} \left[\sum_{h=1}^{k-1} p_{01}^{(h)} \right]
 \end{aligned}$$

Hence Hence

$$\begin{aligned}
 AOQ &= E(V) / [E(T) + E(M)] \\
 &= \frac{[p_{00}^{(k)}]^c [1 + cp_{01}^{(k)}] \left(\sum_{h=1}^{k-1} p_{01}^{(h)} \right) p\delta q(1 - p\delta)^{r-1}}{\left\{ p_{01}^{(k)} [1 - q(1 - p\delta)^{r-1}] + kp\delta q(1 - p\delta)^{r-1} [p_{00}^{(k)}]^c [p_{00}^{(k)}]^c [1 + cp_{01}^{(k)}] \right\}}
 \end{aligned}$$

where $p_{00}^{(k)} = q + (1 - \delta)^k p$, $p_{01}^{(k)} = p [1 - (1 - \delta)^k]$
 $p_{10}^{(k)} = q [1 - (1 - \delta)^k]$ and $p_{11}^{(k)} = p + (1 - \delta)^k q$.

4. NUMERICAL EXAMPLE

We now proceed to show the practical application of the work by considering an example. Though other values could have been chosen for (r, k) , let us consider $r = 89$ and $k = 7$ and illustrate how $c > 0$ plans are useful when the items are serially correlated as compared to Dodge ($c = 0$) plan.

Table- 1 compares the AOQL values in percentages for different values of c and δ . We first of all note that Dodge's CSP-1 is easily recognized at $c = 0$ and $d = 1$. We find that when $\delta > 1$ (or $1 - \delta = \phi$ the serial correlation coefficient is positive), the

AOQL values are large as compared to the case when $\delta \leq 1$ (or ϕ the serial correlation coefficient is negative or equal to zero). Also, we observe that the values of c from 1 to 4 (rather than Dodge $c = 0$) the Markov-dependant CSP-1 performs better. But when c is increased (above 4) there is only a marginal decrease in the AOQL values of c . Thus, for values of c from 1 to 4 seems to be of practical interest with desired AOQL guaranteed for $\phi \leq 0$ and not for $\phi > 0$. For this example we also note further that, the AOQL values obtained under Markovian setup are far less than the UAOQL values. It may be remarked further that for small values of r and large values of k , CSP-1 with a clearance interval is robust for small departures

of the serial correlation coefficient from zero.

Table 1
Comparison of AOQL values in percentage for $(r, k) = (89, 7)$ and for values of clearance interval c and dependence parameter δ in CSP-1.

c	δ							
	0.0001	0.0900	0.1500	0.2600	0.5400	0.6900	0.9100	0.9750
0	0.007465	1.1184	1.1825	1.2209	1.1784	1.1231	1.0338	1.0085
1	0.006314	1.1069	1.1619	1.1976	1.1367	1.0938	1.0116	1.0063
2	0.005943	1.0962	1.1576	1.1859	1.1259	1.0861	1.0094	1.0061
3	0.005812	1.0851	1.1406	1.1757	1.1158	1.0716	1.0076	1.0056
4	0.005611	1.0849	1.1405	1.1656	1.1047	1.0632	1.0069	1.0055
5	0.005610	1.0848	1.1404	1.1656	1.1046	1.0631	1.0068	1.0054
6	0.005609	1.0847	1.1403	1.1655	1.1046	1.0630	1.0067	1.0054
7	0.005608	1.0847	1.1403	1.1655	1.1046	1.0630	1.0067	1.0054

c	δ				
	1.0000	1.0250	1.0900	1.1300	1.1800
0	1.0000	0.95020	0.01115	0.00029	0.00020
1	0.9432	0.86051	0.00962	0.00016	0.00014
2	0.8468	0.78612	0.00811	0.00009	0.00008
3	0.7693	0.74011	0.00762	0.00008	0.00004
4	0.7464	0.72016	0.00758	0.00007	0.00003
5	0.7463	0.72015	0.00757	0.00007	0.00003
6	0.7462	0.72015	0.00756	0.00007	0.00003
7	0.7462	0.72015	0.00756	0.00007	0.00003

5.CONCLUSION

In this paper, it is suggested that the quality assurance practitioner may carry out the run test (Klotz(1973)) to check for Markov dependence quite frequently to decide whether to use CSP-1 devised under Markovian scheme with clearance interval taking values from 1 to 4.

References

- [1] Dodge HF, Sampling Inspection Plan for Continuous Production. Ann.Math.Stat.14, 1943, 264-279(1943).
- [2] Klotz J, Statistical Inference for Bernoulli trials with dependence. Ann. Statist. 1, 373-379(1973).
- [3] Lieberman GJ, A note on Dodge's Continuous Inspection Plan. Ann. Math. Statist. 24, 480-484(1953).
- [4] Sampath Kumar VS Some Studies in Continuous Sampling Plans for Markovian Production Processes. Ph.D., thesis, 1983, University of Poona.