ISSN: 2456-8686, Volume 4, Issue 2, 2020:44-49
DOI : http://doi.org/10.26524/cm79

Journal of Computational Mathematica

Open Access

A Study on Fuzzy Normal Subnearings

A. Mohamed Ali*, S. Soundararajan and G. Devaraji
[1,2,3]Department of Mathematics, Islamiah College (Autonomous), Vaniyambadi, Tirupattur District, Tamil Nadu, India.

Abstract

In this paper, we made an attempt to study the algebraic nature of fuzzy normal subnearring and its properties.

Key words: Fuzzy set, Fuzzy subnearring, Anti-Fuzzy normal subnearring.

AMS classification:

Introduction

There are many concepts of universal algebras generalizing an associative ring $(R;+,.)$. Some of them in particular, near rings and several kinds of semi rings have been proven very useful. An algebra $(R;+,.)$ is said to be a near ring if $(R,+)$ is a group and $(R,.)$ are semi group satisfying $a.(b+c) = a.b + a.c$, for all a, b and c in R. After the introduction of fuzzy sets by L.A.Zadeh, several researchers explored on the generalization of the concept of fuzzy sets. The notion of Fuzzy subnear rings and ideals was introduced by S.Abou Zaid. In this paper, we introduce fuzzy normal subnearring of a near ring and prove some properties.

Properties of fuzzy normal subnearrings

Definition 2.1 Let R be a nearring. A fuzzy subset A of R is said to be a fuzzy subnearring (FSNR) of R if it satisfies the following conditions:

(i) $\mu_A(x-y) \geq \min\{\mu_A(x), \mu_A(y)\}$,
(ii) $\mu_A(xy) \geq \min\{\mu_A(x), \mu_A(y)\}$, for all x and y in R.

Definition 2.2 Let R be a nearring. A fuzzy subnearring A of R is said to be a

[1*]mohamedalihashim@gmail.com and [2]soundar.rajan2002@yahoo.com

ISSN: 2456-8686, Volume 4, Issue 2, 2020:44-49
DOI : http://doi.org/10.26524/cm79

fuzzy normal subnearring (FNSNR) of R if it satisfies the following conditions:

(i) $\mu_A(x+y) = \mu_A(y+x)$,
(ii) $\mu_A(xy) = \mu_A(yx)$, for all x and y in R.

Theorem 2.3 Let $(R, +, \cdot)$ be a near ring. If A and B are two fuzzy normal subnearrings of R, then their intersection $A \cap B$ is a fuzzy normal subnearring of R.
Proof: Let x and y in R. Let $A = \{\langle x, \mu_A(x)\rangle \,/\, x \in R\}$ and $B = \{\langle x, \mu_B(x)\rangle \,/\, x \in R\}$ be a fuzzy normal subnearring of a nearring R. Let $C = A \cap B$ and $C = \{\langle x, \mu_C(x)\rangle \,/\, x \in R\}$. Then, clearly C is a fuzzy subnearring of a nearring R, Since A and B are two fuzzy subnearring of a nearring R.
And, (i)

$$\begin{aligned} \mu_C(x+y) &= \min\{\,\mu_A(x+y), \mu_B(x+y)\,\}, \\ &= \min\{\,\mu_A(y+x), \mu_B(y+x)\,\}, \\ &= \mu_C(y+x), \end{aligned}$$

Therefore, $\mu_C(x+y) = \mu_C(y+x)$, for all x and y in R.
(ii)

$$\begin{aligned} \mu_C(xy) &= \min\{\,\mu_A(xy), \mu_B(xy)\,\}, \\ &= \min\{\,\mu_A(yx), \mu_B(yx)\,\}, \\ &= \mu_C(yx), \end{aligned}$$

Therefore, $\mu_C(xy) = \mu_C(yx)$, for all x and y in R. Hence $A \cap B$ is a fuzzy normal subnearring of a nearring R.

Theorem 2.4 Let R be a nearring. The intersection of a family of fuzzy normal subnearring of R is a fuzzy normal subnearring of R.
Proof: Let $\{A_i\}_{i \in I}$ be a family of fuzzy normal subnearrings of a nearring R and let $A = \bigcap_{i \in I} A_i$.
Then for x and y in R. Clearly the intersection of a family of fuzzy subnearrings of a nearring R is a fuzzy subnearring of a nearring R.

ISSN: 2456-8686, Volume 4, Issue 2, 2020:44-49
DOI : http://doi.org/10.26524/cm79

(i)

$$\begin{aligned} \mu_A(x+y) &= \inf_{i\in I} \mu_{A_i}(x+y) \\ &= \inf_{i\in I} \mu_{A_i}(y+x) \\ &= \mu_A(y+x) \end{aligned}$$

Therefore, $\mu_A(x+y) = \mu_A(y+x)$ for all x and y in R.
(ii)

$$\begin{aligned} \mu_A(xy) &= \inf_{i\in I} \mu_{A_i}(xy) \\ &= \inf_{i\in I} \mu_{A_i}(yx) \\ &= \mu_A(yx) \end{aligned}$$

Therefore, $\mu_A(xy) = \mu_A(yx)$ for all x and y in R. Hence the intersection of a family of fuzzy normal subnearrings of a nearring R is a fuzzy normal subnearring of a nearring R.

Theorem 2.5 Let A and B be fuzzy subnearring of the nearrings G and H, respectively. If A and B are fuzzy normal subnearrings, then $A \times B$ is a fuzzy normal subnearring of $G \times H$.
Proof: Let A and B be fuzzy normal subnearrings of the nearrings G and H respectively. Clearly, $A \times B$ is a fuzzy subnearrings of $G \times H$.
Let x_1 and x_2 be in G, y_1 and y_2 be in H. Then (x_1, y_1) and (x_2, y_2) are in $G \times H$.
Now,

$$\begin{aligned} \mu_{A\times B}[(x_1,y_1)+(x_2,y_2)] &= \mu_{A\times B}(x_1+x_2, y_1+y_2) \\ &= \min\{\mu_A(x_1+x_2), \mu_B(y_1+y_2)\} \\ &= \min\{\mu_A(x_2+x_1), \mu_B(y_2+y_1)\} \end{aligned}$$

$$\begin{aligned} &= \mu_{A\times B}(x_2+x_1, y_2+y_1) \\ &= \mu_{A\times B}[(x_2,y_2)+(x_1,y_1)]. \end{aligned}$$

Therefore, $\mu_{A\times B}[(x_1,y_1)+(x_2,y_2)] = \mu_{A\times B}[(x_2,y_2)+(x_1,y_1)]$.
And

$$\begin{aligned}\mu_{A\times B}[(x_1,y_1)(x_2,y_2)] &= \mu_{A\times B}(x_1x_2,y_1y_2)\\ &= \min\{\,\mu_A(x_1x_2),\mu_B(y_1y_2)\}\\ &= \min\{\,\mu_A(x_2x_1),\mu_B(y_2y_1)\}\\ &= \mu_{A\times B}(x_2x_1,y_2y_1)\\ &= \mu_{A\times B}[(x_2,y_2)(x_1,y_1)].\end{aligned}$$

Therefore, $\mu_{A\times B}[(x_1,y_1)(x_2,y_2)] = \mu_{A\times B}[(x_2,y_2)(x_1,y_1)]$. Hence $A\times B$ is a fuzzy normal subnearring of $G\times H$.

Theorem 2.6 Let A and B be fuzzy subsets of the nearrings R and H, respectively and $A\times B$ is a fuzzy normal subnearring of $R\times H$. Then the following are true:

(i) if $\mu_A(x)\leq\mu_B(e')$, then A is a fuzzy normal subnearring of R.
(ii) if $\mu_B(x)\leq\mu_A(e)$, then B is a fuzzy normal subnearring of H.
(iii) either A is a fuzzy normal subnearring of R or B is a fuzzy normal subnearring of H.

Proof: Let $A\times B$ be a fuzzy normal subnearring of $R\times H$ and $x,yinR$ and e' in H. Then (x,e') and (y,e') are in $R\times H$.
Now, using the property that $\mu_A(x)\leq\mu_A(e')$, for all x in R. Clearly A is a fuzzy subnearring of R. Now,

$$\begin{aligned}\mu_A(x+y) &= \min\{\mu_A(x+y),\mu_B(e'+e')\}\\ &= \mu_{A\times B}((x+y),(e'+e'))\\ &= \mu_{A\times B}[(x,e')+(y,e')]\\ &= \mu_{A\times B}[(y,e')+(x,e')]\\ &= \mu_{A\times B}[(y+x),(e'+e')]\\ &= \min\{\mu_A(y+x),\mu_B(e'+e')\}\\ &= \mu_A(y+x).\end{aligned}$$

Therefore,

$$\mu_A(x+y)=\mu_A(y+x),$$

for all x and y in R. Also,

ISSN: 2456-8686, Volume 4, Issue 2, 2020:44-49
DOI : http://doi.org/10.26524/cm79

$$\begin{aligned}
\mu_A(xy) &= \min\{\mu_A(xy), \mu_B(e'e')\} \\
&= \mu_{A\times B}((xy), (e'e')) \\
&= \mu_{A\times B}[(x, e')(y, e')] \\
&= \mu_{A\times B}[(y, e')(x, e')] \\
&= \mu_{A\times B}[(yx), (e'e')] \\
&= \min\{\mu_A(yx), \mu_B(e'e')\} \\
&= \mu_A(yx).
\end{aligned}$$

Therefore,

$$\mu_A(xy) = \mu_A(yx),$$

for all x and y in R. Hence A is a fuzzy normal subnearring of R.
Thus (i) is proved.
Now, using the property that $\mu_B(x) \leq \mu_A(e)$, for all x in H, let x and y in H and e in R. Then (e, x) and (e, y) are in $R \times H$. Clearly B is a fuzzy subnearring of H.
Now,

$$\begin{aligned}
\mu_B(x+y) &= \min\{\mu_B(x+y), \mu_A(e+e)\} \\
&= \min\{\mu_A(e+e), \mu_B(x+y)\} \\
&= \mu_{A\times B}((e+e), (x+y)) \\
&= \mu_{A\times B}[(e, x) + (e, y)] \\
&= \mu_{A\times B}[(e, y) + (e, x)] \\
&= \mu_{A\times B}[(e+e), (y+x)] \\
&= \min\{\mu_A(e+e), \mu_B(y+x)\} \\
&= \mu_B(y+x).
\end{aligned}$$

Therefore,

$$\mu_B(x+y) = \mu_B(y+x),$$

for all x and y in H.
Also,

$$\begin{aligned}
\mu_B(xy) &= \min\{\mu_B(xy), \mu_A(ee)\} \\
&= \min\{\mu_A(ee), \mu_B(xy)\} \\
&= \mu_{A\times B}((ee), (xy)) \\
&= \mu_{A\times B}[(e, x)(e, y)]
\end{aligned}$$

[1*]mohamedalihashim@gmail.com and [2]soundar.rajan2002@yahoo.com

ISSN: 2456-8686, Volume 4, Issue 2, 2020:44-49
DOI : http://doi.org/10.26524/cm79

$$
\begin{aligned}
&= \mu_{A\times B}[(e,y)(e,x)] \\
&= \mu_{A\times B}[(ee),(yx)] \\
&= \min\{\mu_A(ee),\mu_B(yx)\} \\
&= \mu_B(yx).
\end{aligned}
$$

Therefore,

$$\mu_B(xy) = \mu_B(yx),$$

for all x and y in H. Hence B is a fuzzy normal subnearring of H.
Thus (ii) is proved. (iii) is clear.

Conclusion

In this paper, we have defined Fuzzy normal subnearring and proved its properties

References

[1] Mustafa Akgul, Some properties of fuzzy groups, Journal of mathematical analysis and applications, 133, 93-100 (1988).

[2] Prabir Bhattacharya, Fuzzy Subgroups: Some Characterizations, Journal of Mathematical Analysis and Applications, 128, 241-252 (1987).

[3] Salah Abou-Zaid, On generalized characteristic fuzzy subgroups of a finite group, fuzzy sets and systems, 235-241 (1991).

[4] Zadeh. L.A., Fuzzy Sets, Information and control, Vol.8, 338-353 (1965).

[5] Biswas.R, Fuzzy subgroups and Anti-fuzzy subgroups, Fuzzy sets and systems, 35, 121-124 (1990).

[6] Sung Min Hong, Young Bae Jun and Hee Sik Kim, Fuzzy ideals in Nearrings, Bull. Korean Math. Soc. 35, No.3, 455-464 (1998).

[1*]mohamedalihashim@gmail.com and [2]soundar.rajan2002@yahoo.com