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Pairwise Fuzzy D-Baire Spaces

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Abstract

In this paper we introduce the concept of D-Baire bitopological spaces and several properties are investigated.

Key words: Pairwise fuzzy dense, Pairwise fuzzy open, Pairwise fuzzy closed, Pairwise fuzzy nowhere dense, Pairwise fuzzy first category, Pairwise fuzzy residual, Pairwise fuzzy Baire, Pairwise fuzzy D-Baire spaces.

AMS classification: 54 A 40, 03 E 72.

1. Introduction

The theory of fuzzy sets was initiated by by L.A.Zadeh in his classical paper [12] in the year 1965 as an attempt to develop a mathematically precise framework in which to treat systems or phenomena which cannot themselves be characterized precisely. The potential of fuzzy notion was realized by the researchers and has successfully been applied for investigations in all the branches of Science and Technology. The paper of C.L.Chang [3] in 1968 paved the way for the subsequent tremendous growth of the numerous fuzzy topological concepts. In 1989, A.Kandil [4] introduced the concept of fuzzy bitopological space as an extension and generalization of fuzzy topological space. Rene Baire introduced the concept of first and second category in topology. To define first category Baire, relied on Cantor's definition of dense sets and P.du Bois-Reymond's definition of nowhere dense sets.Denjoy introduced the concept residual as the sets which are complements of first category sets around 1912.

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The concept of Baire spaces in fuzzy setting was introduced and studied by G.Thangaraj and S.Anjalmose in [7]. The concept of Baire spaces in fuzzy bitopological setting was introduced and studied by the authors in [9]. In this paper, we introduce the concept of D-Baire bitopological spaces in fuzzy setting and investigate several characterizations of pairwise fuzzy D-Baire spaces.

2. Preliminaries

Now we give some basic notions and results used in the sequel. In this work by (X, T_1, T_2) or simply by X, we will denote a fuzzy bitopological space due to Kandil [4]. By a fuzzy Bitopological space we mean an ordered triple (X, T_1, T_2) where T_1 and T_2 are fuzzy topologies on the non-empty set X.

Definition 2.1 Let λ and μ be any two fuzzy sets in a fuzzy topological space (X, T). Then we define:

(i) $\lambda \vee \mu$: X $\rightarrow$ [0,1] as follows: $(\lambda \vee \mu)(x) =$ max $\{\lambda(\text{x}), \mu(\text{x})\}$;
(ii) $\lambda \wedge \mu$: X $\rightarrow [0, 1]$ as follows: $(\lambda \wedge \mu)(x) =$ min $\{\lambda(\text{x}), \mu(\text{x})\}$;
(iii) $\mu = \lambda^c \Leftrightarrow \mu(x) = 1 - \lambda(x)$.

For a family $\{\lambda_i / i \in I\}$ of fuzzy sets in (X, T), the union $\psi = \vee_i \lambda_i$ and intersection $\delta = \wedge_i \lambda_i$ are defined respectively as $\psi(x) = sup_i\{\lambda_i(x), x \in X\}$ and $\delta(x) = inf_i\{\lambda_i(x), x \in X\}$.

Definition 2.2 [1] Let (X, T) be a fuzzy topological space. For a fuzzy set λ of X, the interior and the closure of λ are defined respectively as $int(\lambda) = \vee\{\mu/\mu \leq \lambda, \mu \in T\}$ and $cl(\lambda) = \wedge\{\mu/\lambda \leq \mu, 1 - \mu \in T\}$.

Definition 2.3 [11] Let (X, T_1, T_2) be a fuzzy bitopological space. A fuzzy set λ in (X, T_1, T_2) is called a pairwise fuzzy open set if $\lambda \in T_1$ and $\lambda \in T_2$.

Definition 2.4 [11] Let (X, T_1, T_2) be a fuzzy bitopological space. A fuzzy set λ in (X, T_1, T_2) is called a pairwise fuzzy closed set if $1 - \lambda \in T_1$ and $1 - \lambda \in T_2$.

Definition 2.5 [5] Let (X, T_1, T_2) be a fuzzy bitopological space. A fuzzy set λ in (X, T_1, T_2) is called a pairwise fuzzy dense set if $cl_{T_1}(cl_{T_2}(\lambda)) = cl_{T_2}(cl_{T_1}(\lambda)) = 1$.

Definition 2.6 [8] A fuzzy set λ in a fuzzy bitopological space (X, T_1, T_2) is called pairwise fuzzy nowhere dense if $int_{T_1}(cl_{T_2}(\lambda)) = int_{T_2}(cl_{T_1}(\lambda)) = 0$.

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Definition 2.7 [8] Let (X, T_1, T_2) be a fuzzy bitopological space. A fuzzy set λ in (X, T_1, T_2) is called pairwise fuzzy first category set if $\lambda = \bigvee_{i=1}^{\infty} \lambda_i$ where λ_i's are pairwise fuzzy nowhere dense sets in (X, T_1, T_2). A fuzzy set which is not pairwise fuzzy first category set is called a pairwise fuzzy second category set in (X, T_1, T_2).

Definition 2.8 [8] Let (X, T_1, T_2) be a fuzzy bitopological space. A fuzzy set λ in (X, T_1, T_2) is called a pairwise fuzzy residual set if its complement is a pairwise fuzzy first category set.

Definition 2.9 [8] A fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy Baire if $int(\bigvee_{i=1}^{\infty} \lambda_i) = 0$ where λ_i's are pairwise fuzzy nowhere dense sets in (X, T_1, T_2).

Theorem 2.10 [8] Let (X, T_1, T_2) be a fuzzy bitopological space. Then the following are equivalent:
(i)(X, T_1, T_2) is a pairwise fuzzy Baire space.
(ii)$int_{T_j}(\lambda) = 0$,(j=1,2) for every pairwise fuzzy first category set λ in (X, T_1, T_2).
(iii)$cl_{T_j}(\mu) = 1$, (j=1,2) for every pairwise fuzzy residual set μ in (X, T_1, T_2).

Theorem 2.11 [10] If $\lambda \leq \mu$ and μ is a pairwise fuzzy first category set in a fuzzzy bitopological space (X, T_1, T_2) then λ is also a pairwise fuzzy first category set.

Theorem 2.12 [8] If λ is a pairwise fuzzy nowhere dense set in a fuzzy bitopological space (X, T_1, T_2), then $1 - \lambda$ is a pairwise fuzzy dense set in (X, T_1, T_2).

Theorem 2.13 [9] If the fuzzy bitopological space (X, T_1, T_2) is a pairwise fuzzy Baire space, then each pairwise fuzzy residual set is a pairwise fuzzy dense set in (X, T_1, T_2).

Theorem 2.14 [8] If the fuzzy bitopological space (X, T_1, T_2) is a pairwise fuzzy Baire space, then(X, T_1, T_2) is a pairwise fuzzy second category space.

Theorem 2.15 [10] Let (X, T_1, T_2) be a pairwise fuzzy strongly irresolvable space. Then λ is a pairwise fuzzy dense set in (X, T_1, T_2) if and only if $1 - \lambda$ is a pairwise fuzzy nowhere dense set.

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Theorem 2.16 [10] If every pairwise fuzzy G_δ-set is fuzzy pairwise dense in a pairwise fuzzy submaximal and pairwise fuzzy strongly irresolvable space (X, T_1, T_2), then (X, T_1, T_2) is a pairwise fuzzy Baire space.

Theorem 2.17 [10] If every pairwise fuzzy G_δ-set is a pairwise fuzzy dense set in a pairwise fuzzy strongly irresolvable and pairwise fuzzy nodec space (X, T_1, T_2), then (X, T_1, T_2) is a pairwise fuzzy Baire space.

3. Pairwise fuzzy D-Baire spaces

Definition 3.1 A fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy D-Baire space if $int_{T_1}(cl_{T_2}(\bigvee_{i=1}^{\infty}(\lambda_i))) = int_{T_2}(cl_{T_1}(\bigvee_{i=1}^{\infty}(\lambda_i))) = 0$, where (λ_i)'s are pairwise fuzzy nowhere dense sets in (X, T_1, T_2).

Example 3.2 Let $X = \{a, b, c\}$. The fuzzy sets λ, μ and ν are defined on X as follows:

$\lambda : X \longrightarrow$ [0, 1] is defined as λ (a) = 0.5; λ (b) = 0.7; λ (c) = 0.6.

$\mu : X \longrightarrow$[0, 1] is defined as μ(a) = 0.4; μ (b) = 0.6; μ(c) = 0.5.

$\nu : X \longrightarrow$ [0,1] is defined as ν(a) = 0.6; ν(b) = 0.5; ν(c) = 0.4.

Clearly $T_1 = \{0, \lambda, \mu, \nu, \lambda \vee \nu, \mu \vee \nu, \lambda \wedge \nu, \mu \wedge \nu, \lambda \wedge (\mu \vee \nu), 1\}$ and $T_2 = \{0, \lambda, \mu, 1\}$ are fuzzy topologies on X and (X, T_1, T_2) is a fuzzy Bitopological space. Clearly $1 - \lambda$, $1 - \mu$, $1 - (\lambda \vee \nu)$, $1 - (\mu \vee \nu)$, and $1 - (\lambda \wedge (\mu \vee \nu))$ are pairwise fuzzy nowhere dense sets in (X, T_1, T_2). Also $1 - \mu = (1 - \lambda) \vee (1 - \mu) \vee (1 - (\lambda \vee \nu)) \vee (1 - (\mu \vee \nu)) \vee (1 - (\lambda \wedge (\mu \vee \nu)))$ is a pairwise fuzzy first category set in (X, T_1, T_2). Also $int_{T_1}(cl_{T_2}(1 - \mu)) = int_{T_2}(cl_{T_1}(1 - \mu)) = 0$. Hence the bitopological space (X, T_1, T_2) is a pairwise fuzzy D-Baire space.

Example 3.3 Let $X = \{a, b, c\}$. The fuzzy sets (λ_i)(i=1,2,3) ,μ_j(j=1,2,3), are defined on X as follows :

$\lambda_1 : X \longrightarrow$ [0, 1] is defined as λ_1 (a) = 0.5; λ_1 (b) = 0.7; λ_1 (c) = 0.6.

$\lambda_2 : X \longrightarrow$ [0, 1] is defined as λ_2 (a) = 0.4; λ_2 (b) = 0.6; λ_2 (c) = 0.5.

$\lambda_3 : X \longrightarrow$ [0, 1] is defined as λ_3 (a) = 0.6; λ_3 (b) = 0.5; λ_3 (c) = 0.4.

$\mu_1 : X \longrightarrow$[0, 1] is defined as μ_1(a) = 0.8; μ_1 (b) = 0.5; μ_1(c) = 0.7.

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$\mu_2 : X \longrightarrow [\ 0,\ 1]$ is defined as μ_2(a) = 0.6; μ_1 (b) = 0.9; μ_1(c) = 0.4.

$\mu_3 : X \longrightarrow [\ 0,\ 1]$ is defined as μ_3(a) = 0.4; μ_3 (b) = 0.7; μ_3(c) = 0.8.

Clearly $T_1 = \{0, \lambda_1, \lambda_2, \lambda_3, \lambda_1 \vee \lambda_3, \lambda_2 \vee \lambda_3, \lambda_1 \wedge \lambda_3, \lambda_2 \wedge \lambda_3, \lambda_2 \wedge (\lambda_1 \wedge \lambda_3), 1\}$ and $T_2 = \{0, \mu_1, \mu_2, \mu_3, \mu_1 \vee \mu_2, \mu_1 \vee \mu_3, \mu_2 \vee \mu_3, \mu_1 \wedge \mu_2, \mu_1 \wedge \mu_3, \mu_2 \wedge \mu_3, \mu_1 \vee (\mu_2 \wedge \mu_3), \mu_2 \vee (\mu_1 \wedge \mu_3), \mu_3 \vee (\mu_1 \wedge \mu_2), \mu_1 \wedge (\mu_2 \vee \mu_3), \mu_2 \wedge (\mu_1 \vee \mu_3), \mu_3 \wedge (\mu_1 \vee \mu_2), (\mu_1 \vee \mu_2 \vee \mu_3), 1\}$ are fuzzy topologies on X and (X, T_1, T_2) is a fuzzy bitopological space. α, β and ν are defined on X as follows:

$\alpha : X \longrightarrow [\ 0,\ 1]$ is defined as α(a) = 0.6; α (b) = 0.3; α(c) = 0.4.

$\beta : X \longrightarrow [\ 0,\ 1]$ is defined as β(a) = 0.4; β (b) = 0.3; β(c) = 0.6.

$\nu : X \longrightarrow [\ 0{,}1]$ is defined as ν(a) = 0.6; ν(b) = 0.5; ν(c) = 0.6.

Clearly α, β, $1 - \lambda_1$, $1 - \mu_1$, $1 - \mu_3$ and $1 - (\mu_1 \vee \mu_2)$ are pairwise fuzzy nowhere dense sets in (X, T_1, T_2). Hence $\nu = \alpha \vee \beta \vee (1-\lambda_1) \vee (1-\mu_1) \vee (1-\mu_3) \vee (1-(\mu_1 \vee \mu_2))$ is a pairwise fuzzy first category set in (X, T_1, T_2). But $int_{T_2}(cl_{T_1}(\nu)) = \mu_1 \wedge \mu_2 \neq 0$. Hence the bitopological space (X, T_1, T_2) is not a pairwise fuzzy D-Baire space.

Proposition 3.4 Let (X, T_1, T_2) be a fuzzy bitopological space. Then the following are equivalent:
(i)(X, T_1, T_2) is a pairwise fuzzy D-Baire space.
(ii)$int_{T_1}(cl_{T_2}(\lambda)) = int_{T_2}(cl_{T_1}(\lambda)) = 0$, for every pairwise fuzzy first category set λ in (X, T_1, T_2)
(iii)$cl_{T_1}(int_{T_2}(\mu)) = cl_{T_2}(int_{T_1}(\mu)) = 1$, for every pairwise fuzzy residual set μ in (X, T_1, T_2).
Proof: (i)$\Longrightarrow (ii)$: Let λ be a pairwise fuzzy first category set in the pairwise fuzzy D-Baire space (X, T_1, T_2). Then $\lambda = \bigvee_{i=1}^{\infty}(lambda_i)$ where (λ_i)'s are pairwise fuzzy nowhere dense sets in (X, T_1, T_2). Sine (X, T_1, T_2) is a pairwise fuzzy D-Baire space, $int_{T_1}(cl_{T_2}(\bigvee_{i=1}^{\infty}((\lambda_i)))) = int_{T_2}(cl_{T_1}(\bigvee_{i=1}^{\infty}((\lambda_i)))) = 0$. Hence $int_{T_1}(cl_{T_2}(\lambda)) = int_{T_2}(cl_{T_1}(\lambda)) = 0$, for every pairwise fuzzy first category set λ in (X, T_1, T_2).

(ii)$\Longrightarrow (iii)$: Let μ be a pairwise fuzzy residual set in (X, T_1, T_2). Then $1 - \mu$ is a pairwise fuzzy first category set and hence, by hypothesis, $int_{T_1}(cl_{T_2}(1 - \mu)) = int_{T_2}(cl_{T_1}(1 - \mu)) = 0$. This implies that, $cl_{T_1}(int_{T_2}(\mu)) = cl_{T_2}(int_{T_1}(\mu)) = 1$. Hence $cl_{T_1}(int_{T_2}(\mu)) = cl_{T_2}(int_{T_1}(\mu)) = 1$, for every pairwise fuzzy residual set μ in (X, T_1, T_2).

(iii)$\Longrightarrow (i)$: Let (λ_i)'s be pairwise fuzzy nowhere dense sets in (X, T_1, T_2). Then

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$\lambda = \bigvee_{i=1}^{\infty}((\lambda_i))$ is a pairwise fuzzy first category set and hence, $1-\lambda$ is a pairwise fuzzy residual set in (X, T_1, T_2). By hypothesis $cl_{T_1}(int_{T_2}(1-\lambda)) = cl_{T_2}(int_{T_1}(1-\lambda)) = 1$. This implies $int_{T_1}(cl_{T_2}(\lambda)) = int_{T_2}(cl_{T_1}(\lambda)) = 0$. That is, $int_{T_1}(cl_{T_2}(\bigvee_{i=1}^{\infty}((\lambda_i)))) = int_{T_2}(cl_{T_1}(\bigvee_{i=1}^{\infty}((\lambda_i)))) = 0$. Hence (X, T_1, T_2) is a pairwise fuzzy D-Baire space.

Proposition 3.5 If (X, T_1, T_2) is a pairwise fuzzy D-Baire space then (X, T_1, T_2) is a pairwise fuzzy Baire space.
Proof: Let λ be a pairwise fuzzy first category set in a pairwise fuzzy D-Baire space (X, T_1, T_2). By proposition 3.4, $int_{T_1}(cl_{T_2}(\lambda)) = int_{T_2}(cl_{T_1}(\lambda)) = 0$. Now $int_{T_1}(\lambda) \leq int_{T_1}(cl_{T_2}(\lambda))$ and $int_{T_2}(\lambda) \leq int_{T_2}(cl_{T_1}(\lambda))$ implies that $int_{T_1}(\lambda) = int_{T_2}(\lambda) = 0$, and by theorem 2.10, (X, T_1, T_2) is a pairwise fuzzy Baire space.

Proposition 3.6 If the fuzzy bitopological space (X, T_1, T_2) is a pairwise fuzzy D-Baire space , then no nonzero pairwise fuzzy open set is a pairwise fuzzy first category set in (X, T_1, T_2).
Proof: Suppose that the nonzero pairwise fuzzy open set λ is a pairwise fuzzy first category set in (X, T_1, T_2). Since (X, T_1, T_2) is a pairwise fuzzy D-Baire space and λ is a pairwise fuzzy first category set implies $int_{T_1}(cl_{T_2}(\lambda)) = int_{T_2}(cl_{T_1}(\lambda)) = 0$. But λ is a pairwise fuzzy open set in (X, T_1, T_2), $int_{T_i}(\lambda) = \lambda$ (i=1,2). This gives $int_{T_1}(\lambda) \leq int_{T_1}(cl_{T_2}(\lambda))$ and $int_{T_2}(\lambda) \leq int_{T_2}(cl_{T_1}(\lambda))$. This implies that $int_{T_1}(\lambda) = int_{T_2}(\lambda) = 0$ and so $\lambda = 0$, a contradiction to λ, being a nonzero pairwise fuzzy open set. Hence no nonzero pairwise fuzzy open set is a pairwise fuzzy first category set in a pairwise fuzzy D-Baire space (X, T_1, T_2).

Proposition 3.7 If (X, T_1, T_2) is a pairwise fuzzy D-Baire space and if $\vee_{i=1}^{\infty}((\lambda_i))$ =1 then there is exists atleast one fuzzy set (λ_i) such that either $int_{T_1}(cl_{T_2}((\lambda_i))) \neq 0$ or $int_{T_2}(cl_{T_1}((\lambda_i))) \neq 0$.
Proof: Suppose $int_{T_1}(cl_{T_2}((\lambda_i))) = 0$ and $int_{T_2}(cl_{T_1}((\lambda_i))) = 0$ for all i, then (λ_i)'s are pairwise fuzzy nowhere dense sets in (X, T_1, T_2). Then $\vee_{i=1}^{\infty}((\lambda_i)) = 1$ implies that $int_{T_1} cl_{T_2}(\vee_{i=1}^{\infty}((\lambda_i))) = int_{T_1} cl_{T_2}(1) = 1 \neq 0$, a contradiction to (X, T_1, T_2) being a pairwise fuzzy D-Baire space in which $int_{T_1} cl_{T_2}(\vee_{i=1}^{\infty}((\lambda_i))) = int_{T_2} cl_{T_1}(\vee_{i=1}^{\infty}((\lambda_i))) = 0$. Hence either $int_{T_1}(cl_{T_2}((\lambda_i))) \neq 0$ or $int_{T_2}(cl_{T_1}((\lambda_i))) \neq 0$ for atleast one i in (X, T_1, T_2).

Proposition 3.8 If $int_{T_1}(cl_{T_2}(\bigvee_{i=1}^{\infty}((\lambda_i)))) = int_{T_2}(cl_{T_1}(\bigvee_{i=1}^{\infty}((\lambda_i)))) = 0$ where $int_{T_j}((\lambda_i)) = 0$,(j=1,2) and (λ_i)'s are pairwise fuzzy closed sets in (X, T_1, T_2), then the fuzzy bitopological space (X, T_1, T_2) is a pairwise fuzzy D-Baire space.
Proof: Let (λ_i)'s be pairwise fuzzy closed sets. Then $cl_{T_j}((\lambda_i)) = (\lambda_i)$ (j=1,2).

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Now $int_{T_j}((\lambda_i)) = 0$ (j=1,2) implies that $int_{T_1}(cl_{T_2}((\lambda_i))) = int_{T_2}(cl_{T_1}((\lambda_i))) = 0$. Therefore (λ_i)'s are pairwise fuzzy nowhere dense sets in (X, T_1, T_2). Hence $int_{T_1}(cl_{T_2}(\bigvee_{i=1}^{\infty}((\lambda_i)))) = int_{T_2}(cl_{T_1}(\bigvee_{i=1}^{\infty}((\lambda_i)))) = 0$ where (λ_i)'s are pairwise fuzzy nowhere dense sets in (X, T_1, T_2) implies the pairwise fuzzy D-Baire space.

Proposition 3.9 If μ is any fuzzy set such that $\mu \leq \lambda$, where λ is any pairwise fuzzy first category set in a pairwise fuzzzy D-Baire space (X, T_1, T_2) then μ is a pairwise fuzzy nowhere dense set.
Proof: Let λ be a pairwise fuzzy first category set in (X, T_1, T_2) and μ be any fuzzy set in (X, T_1, T_2) such that $\mu \leq \lambda$. By theorem 2.11, μ is also a pairwise fuzzy first category set. Since μ is a pairwise fuzzy first category set in the pairwise fuzzy D-Baire space (X, T_1, T_2), by proposition 3.4, $int_{T_1}(cl_{T_2}(\lambda)) = int_{T_2}(cl_{T_1}(\lambda)) = 0$. Hence μ is a pairwise fuzzy nowhere dense set.

Proposition 3.10 If (X, T_1, T_2) is a pairwise fuzzy D-Baire space then every pairwise fuzzy residual set in (X, T_1, T_2) is a pairwise fuzzy dense set.
Proof: Let λ be a pairwise fuzzy residual set in (X, T_1, T_2). Then $1 - \lambda$ is a pairwise fuzzy first category set. Since (X, T_1, T_2) is a pairwise fuzzy D-Baire space, $1 - \lambda$ is a pairwise fuzzy nowhere dense set. By theorem 2.12, $\lambda = 1 - (1 - \lambda)$ is a pairwise fuzzy dense set.

Proposition 3.11 If μ is any fuzzy set such that $\lambda \leq \mu$, where λ is any pairwise fuzzy residual set in a pairwise fuzzzy D-Baire space (X, T_1, T_2), then μ is a pairwise fuzzy dense set.
Proof: Let λ be a pairwise fuzzy residual set in (X, T_1, T_2) and μ be any fuzzy set in (X, T_1, T_2) such that $\lambda \leq \mu$. Now $1 - \mu \leq 1 - \lambda$ and $1 - \lambda$ is a pairwise fuzzy first category set. Hence by theorem 2.11, $1 - \mu$ is a pairwise fuzzy first category set in pairwise fuzzy D-Baire space (X, T_1, T_2). Then μ is a pairwise fuzzy residual set and hence by proposition 3.10, μ is a pairwise fuzzy dense set.

Proposition 3.12 If the pairwise fuzzy first category set λ , is a pairwise fuzzy closed set, in a pairwise fuzzy Baire space (X, T_1, T_2), then (X, T_1, T_2) is a pairwise fuzzy D-Baire space.
Proof: Let λ be a pairwise fuzzy first category set in a pairwise fuzzy Baire space (X, T_1, T_2) and $cl_{T_i}(\lambda) = \lambda$ ...(1) (i = 1,2) By theorem 2.10, $int_{T_i}(\lambda) = 0$ ...(2) (i = 1, 2) , for the pairwise fuzzy first category set λ in (X, T_1, T_2). Then, from (1) and (2), $int_{T_1}(cl_{T_2}(\lambda)) = int_{T_2}(cl_{T_1}(\lambda)) = 0$. Hence, by proposition 3.4, (X, T_1, T_2) is a pairwise fuzzy D-Baire space.

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Proposition 3.13 If the pairwise fuzzy residual set μ , is a pairwise fuzzy open set, in a pairwise fuzzy Baire space (X, T_1, T_2), then (X, T_1, T_2) is a pairwise fuzzy D-Baire space.

Proof: Let λ be a pairwise fuzzy first category set in a pairwise fuzzy Baire space (X, T_1, T_2). Then $1 - \lambda$ is a pairwise fuzzy residual set. By hypothesis $1 - \lambda$ is a pairwise fuzzy open set in (X, T_1, T_2). Hence λ is a pairwise fuzzy closed set. This implies that the pairwise fuzzy first category set λ , is a pairwise fuzzy closed set, in the pairwise fuzzy Baire space (X, T_1, T_2). By proposition 3.12, (X, T_1, T_2) is a pairwise fuzzy D-Baire space.

Proposition 3.14 If the fuzzy bitopological space (X, T_1, T_2) is a pairwise fuzzy first category space then (X, T_1, T_2) is not a pairwise fuzzy D-Baire space.

Proof: Let (X, T_1, T_2) be a pairwise fuzzy first category space. Then $\vee_{i=1}^{\infty}\lambda_i = 1_X$ where λ_i's are pairwise fuzzy nowhere dense sets in (X, T_1, T_2). Then $int_{T_1}(cl_{T_2}(\vee_{i=1}^{\infty}(\lambda_i)) = int_{T_1}(cl_{T_2}(1)) = int_{T_1}(1) = 1 \neq 0$ and $int_{T_2}(cl_{T_1}(\vee_{i=1}^{\infty}(\lambda_i)) = int_{T_2}(cl_{T_1}(1)) = int_{T_2}(1) = 1 \neq 0$. Hence (X, T_1, T_2) is not a pairwise fuzzy D-Baire space.

4. Inter-relations between pairwise fuzzy strongly irresolvable spaces, pairwise fuzzy submaximal spaces and pairwise fuzzy Baire spaces

Proposition 4.1 If (X, T_1, T_2) is a pairwise fuzzy submaximal space ,then (X, T_1, T_2) is not a pairwise fuzzy D-Baire space.

Proof: Let (X, T_1, T_2) be a pairwise fuzzy submaximal space. Suppose that (X, T_1, T_2) is a pairwise fuzzy D-Baire space. Let $\lambda = \vee_{i=1}^{\infty}\lambda_i$, be a pairwise fuzzy first category set in (X, T_1, T_2). Then λ_i's are pairwise fuzzy nowhere dense sets. This implies $int_{T_1}(cl_{T_2}((\lambda_i)) = 0$ and $int_{T_2}(cl_{T_1}((\lambda_i)) = 0$. Now $int_{T_1}((\lambda_i)) \leq int_{T_1}(cl_{T_2}((\lambda_i)))$ and $int_{T_2}((\lambda_i)) \leq int_{T_2}(cl_{T_1}((\lambda_i)))$, implies that $int_{T_1}((\lambda_i)) = 0$ and $int_{T_2}((\lambda_i)) = 0$. Then $1 - int_{T_1}((\lambda_i)) = 1$ and $1 - int_{T_2}((\lambda_i)) = 1$ implies that $cl_{T_1}(1 - (\lambda_i)) = 1$ and $cl_{T_2}(1-(\lambda_i)) = 1$. This implies that $cl_{T_1}(cl_{T_2}(1-(\lambda_i))) = 1$ and $cl_{T_2}(cl_{T_1}(1-(\lambda_i))) = 1$. Hence $1 - \lambda_i$'s are pairwise fuzzy dense sets in (X, T_1, T_2). Now $int_{T_1}(1 - (\lambda_i)) = 1 - (cl_{T_1}((\lambda_i))) < (1 - (\lambda_i))$ and $int_{T_2}(1 - (\lambda_i)) = 1 - (cl_{T_2}((\lambda_i))) < (1 - (\lambda_i))$. Hence $int_{T_1}(1-(\lambda_i)) \neq (1-(\lambda_i))$ and $int_{T_2}(1-(\lambda_i)) \neq (1-(\lambda_i))$ and therefore $(1-\lambda_i)$'s are not pairwise fuzzy open sets in (X, T_1, T_2). But this is a contradiction to (X, T_1, T_2), being a pairwise fuzzy submaximal space, in which each pairwise fuzzy dense set is pairwise fuzzy open set in (X, T_1, T_2). Hence our assumption that (X, T_1, T_2) is a pairwise fuzzy D-Baire space does not hold. Thus every pairwise fuzzy submaximal space is not a pairwise fuzzy D-Baire space.

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Under what conditions, a pairwise fuzzy submaximal space is a pairwise fuzzy D-Baire space? The answer, for this question,is given in the following proposition.

Proposition 4.2 If the fuzzy bitopological space (X, T_1, T_2) is a pairwise fuzzy submaximal space and pairwise fuzzy Baire space, in which every pairwise fuzzy residual set is a pairwise fuzzy dense set in (X, T_1, T_2), then (X, T_1, T_2) is a pairwise fuzzy D-Baire space.
Proof: Let (X, T_1, T_2) be a pairwise fuzzy submaximal Baire space and λ be a pairwise fuzzy residual set in (X, T_1, T_2). By hypothesis, λ is a pairwise fuzzy dense set. Also since (X, T_1, T_2) is a pairwise fuzzy submaximal space, for the pairwise fuzzy dense set λ, $\lambda \in T_i$ (i = 1,2). Hence the pairwise fuzzy residual set λ is a pairwise fuzzy open set in (X, T_1, T_2). Then, by proposition 3.14, (X, T_1, T_2) is a pairwise fuzzy D-Baire space.

Definition 4.3 [2] A fuzzy bitopological space (X, T_1, T_2) is said to be a <u>pairwise fuzzy strongly irresolvable space</u> if for each pairwise fuzzy dense set λ in (X, T_1, T_2), $cl_{T_1}(int_{T_2}(\lambda)) = cl_{T_2}(int_{T_1}(\lambda)) = 1$.

Proposition 4.4 If the fuzzy bitopological space (X, T_1, T_2) is a pairwise fuzzy strongly irresolvable Baire space, then (X, T_1, T_2) is a pairwise fuzzy D-Baire space.
Proof: Let (X, T_1, T_2) be a pairwise fuzzy strongly irresolvable Baire space and λ be a pairwise fuzzy residual set in (X, T_1, T_2). Since (X, T_1, T_2) is a pairwise fuzzy Baire space, by theorem 2.13, λ is a pairwise fuzzy dense set. Also since (X, T_1, T_2) is a pairwise fuzzy strongly irresolvable space, for the pairwise fuzzy dense set λ, $cl_{T_1}(int_{T_2}(\lambda)) = cl_{T_2}(int_{T_1}(\lambda)) = 1$. Then by proposition 3.4, (X, T_1, T_2) is a pairwise fuzzy D-Baire space.

Definition 4.5 [9] A fuzzy bitopological space (X, T_1, T_2) is said to be a <u>pairwise fuzzy almost resolvable space</u> , if $\vee_{k=1}^{\infty}(\lambda_k) = 1$, where the fuzzy sets λ_k's in (X, T_1, T_2) are such that $int_{T_i}(\lambda_k) = 0$,(i=1,2).

Proposition 4.6 If the fuzzy bitopological space (X, T_1, T_2) is a pairwise fuzzy D-Baire space then (X, T_1, T_2) is not a pairwise fuzzy almost resolvable space.
Proof: Let (X, T_1, T_2) be a pairwise fuzzy D-Baire space. Then, by proposition 3.5, (X, T_1, T_2) is a pairwise fuzzy Baire space. By theorem 2.14, (X, T_1, T_2) is a pairwise fuzzy second category space and hence (X, T_1, T_2) is not a pairwise fuzzy first category space. This implies that $\bigvee_{i=1}^{\infty}(\lambda_k) \neq 1$, where (λ_k)'s (k = 1 to ∞) are pairwise fuzzy

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nowhere dense sets in (X, T_1, T_2). Since (λ_k)'s (k = 1 to ∞) are pairwise fuzzy nowhere dense sets in (X, T_1, T_2), $int_{T_1}(cl_{T_2}(\lambda_k)) = int_{T_2}(cl_{T_1}(\lambda_k)) = 0$. Also, since $int_{T_1}(\lambda_k) \leq int_{T_1}(cl_{T_2}(\lambda_k))$ and $int_{T_2}(\lambda_k) \leq int_{T_2}(cl_{T_1}(\lambda_k))$,$int_{T_i}(\lambda_k) = 0$ (i=,2). Hence $\bigvee_{i=1}^{\infty}(\lambda_k) \neq 1$,where $int_{T_i}(\lambda_k) = 0$, (i = 1,2). Therefore (X, T_1, T_2) is not a pairwise fuzzy almost irresolvable space.

Definition 4.7 [9] A fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy nodec space if every non - zero pairwise fuzzy nowhere dense set in (X, T_1, T_2), is a pairwise fuzzy closed set in (X, T_1, T_2). That is., if λ is a pairwise fuzzy nowhere dense set in a fuzzy bitopological space (X, T_1, T_2), then $1 - \lambda \in T_i$ (i = 1,2).

Proposition 4.8 If (X, T_1, T_2) is a pairwise fuzzy nodec space, then (X, T_1, T_2) is not a pairwise fuzzy D-Baire space.
Proof: Let $\lambda = \vee_{i=1}^{\infty}\lambda_i$, be a pairwise fuzzy first category set in (X, T_1, T_2). Then λ_i's are pairwise fuzzy nowhere dense sets in (X, T_1, T_2). But (X, T_1, T_2) is a pairwise fuzzy nodec space, hence (λ_i)'s are pairwise fuzzy closed sets and $cl_{T_j}((\lambda_i)) = (\lambda_i)$, j=1,2. Now $int_{T_1}(\lambda) = int_{T_1}(\vee_{i=1}^{\infty}((\lambda_i))) = int_{T_1}(\vee_{i=1}^{\infty}cl_{T_2}((\lambda_i)) > \vee_{i=1}^{\infty}int_{T_1}(cl_{T_2}((\lambda_i))$. Since (λ_i)'s are pairwise fuzzy nowhere dense sets in $(X, T_1, T_2))$, $int_{T_1}(cl_{T_2}((\lambda_i)) = 0$. Hence $int_{T_1}(\lambda) \neq 0$ and $0 \neq int_{T_1}(\lambda) \leq int_{T_1}(cl_{T_2}(\lambda))$ implies that $int_{T_1}(cl_{T_2}(\lambda)) \neq 0$. Therefore by proposition 3.4, (X, T_1, T_2) is not a pairwise fuzzy D-Baire space.

Proposition 4.9 Let (X, T_1, T_2) be a pairwise fuzzy strongly irresolvable space. Then (X, T_1, T_2) is a pairwise fuzzy D-Baire space if and only if (X, T_1, T_2) is a pairwise fuzzy Baire space.
Proof: Let (X, T_1, T_2) be a pairwise fuzzy D-Baire space. By proposition 3.5, (X, T_1, T_2) is a pairwise fuzzy Baire space.
Conversely (X, T_1, T_2) is a pairwise fuzzy Baire space and pairwise fuzzy strongly irresolvable space. Let λ be a pairwise fuzzy first category set in (X, T_1, T_2). Then $1 - \lambda$ is a pairwise fuzzy residual set in (X, T_1, T_2). Since (X, T_1, T_2) is a pairwise fuzzy Baire space, by theorem 2.13, $1 - \lambda$ is a pairwise fuzzy dense set in (X, T_1, T_2). Since (X, T_1, T_2), is a pairwise fuzzy strongly irresolvable space, $cl_{T_1}(int_{T_2}(1-\lambda)) = 1$ and $cl_{T_2}(int_{T_1}(1 - \lambda)) = 1$. Then $int_{T_1}(cl_{T_2}(\lambda)) = 0$ and $int_{T_2}(cl_{T_1}(\lambda)) = 0$, hence by proposition 3.4, (X, T_1, T_2) is a pairwise fuzzy D-Baire space.

Proposition 4.10 Let (X, T_1, T_2) be a pairwise fuzzy strongly irresolvable space. Then (X, T_1, T_2) is a pairwise fuzzy D-Baire space if and only if $\lambda = \wedge_{i=1}^{\infty}\lambda_i$, where (λ_i)'s are pairwise fuzzy dense sets, is a pairwise fuzzy dense set in (X, T_1, T_2).
Proof: Let (X, T_1, T_2) be a pairwise fuzzy strongly irresolvable and pairwise fuzzy

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D-Baire space. Let $\lambda = \wedge_{i=1}^{\infty}\lambda_i$, where (λ_i)'s are pairwise fuzzy dense sets in (X, T_1, T_2). We have to prove that λ is pairwise fuzzy dense set. Now $1 - \lambda = \vee_{i=1}^{\infty}(1 - \lambda_i)$ and since (X, T_1, T_2) is a pairwise fuzzy strongly irresolvable space by theorem 2.15, $(1 - \lambda_i)'s$ are pairwise fuzzy nowhere dense sets in (X, T_1, T_2). Hence $1 - \lambda$ is a pairwise fuzzy first category set. Since (X, T_1, T_2) is a pairwise fuzzy D-Baire space, by proposition 3.4, $int_{T_1}(cl_{T_2}(1 - \lambda)) = 0$ and $int_{T_2}(cl_{T_1}(1 - \lambda)) = 0$. This implies that $1 - int_{T_1}(cl_{T_2}(1 - \lambda)) = 1$ and $1 - int_{T_2}(cl_{T_1}(1 - \lambda)) = 1$. Hence $cl_{T_1}(int_{T_2}(\lambda)) = 1$ and $cl_{T_2}(int_{T_1}(\lambda)) = 1$. Since $cl_{T_1}(int_{T_2}(\lambda)) \leq cl_{T_1}(cl_{T_2}(\lambda))$ and $cl_{T_2}(int_{T_1}(\lambda)) \leq cl_{T_2}(cl_{T_1}(\lambda))$, $cl_{T_1}(cl_{T_2}(\lambda)) = cl_{T_2}(cl_{T_1}(\lambda)) = 1$ and λ is pairwise fuzzy dense set. Conversely suppose $\lambda = \wedge_{i=1}^{\infty}\lambda_i$, where (λ_i)'s are pairwise fuzzy nowhere dense sets, is a pairwise fuzzy dense set in (X, T_1, T_2). We have to prove that (X, T_1, T_2) is a pairwise fuzzy D-Baire space. Let μ be a pairwise fuzzy first category set. Then $\mu = \bigvee_{i=1}^{\infty}\mu_i$, where μ_i's are pairwise fuzzy nowhere dense sets in (X, T_1, T_2). Now $1 - \mu = \wedge_{i=1}^{\infty}(1 - \mu_i)$. Since (X, T_1, T_2) is a pairwise fuzzy strongly irresolvable space and μ_i's are pairwise fuzzy nowhere dense sets, by theorem 2.15, $(1 - \mu_i)'s$ are pairwise fuzzy dense sets. Therefore, by hypothesis, $1 - \mu$ is a pairwise fuzzy dense set in a pairwise fuzzy strongly irresolvable space (X, T_1, T_2). Hence $cl_{T_1}(int_{T_2}(1 - \mu)) = cl_{T_2}(int_{T_1}((1 - \mu)) = 1$. This implies that $int_{T_1}(cl_{T_2}(\mu)) = int_{T_2}(cl_{T_1}(\mu)) = 0$. Hence, by proposition 3.4, (X, T_1, T_2) is a pairwise fuzzy D-Baire space.

Remark 4.11 In view of proposition 4.9 and proposition 4.10, the following result. Let (X, T_1, T_2) be a pairwise fuzzy strongly irresolvable space. Then the following are equivalent.
(i)(X, T_1, T_2) is a pairwise fuzzy Baire space.
(ii)(X, T_1, T_2) is a pairwise fuzzy D-Baire space.
(iii) $\lambda = \wedge_{i=1}^{\infty}\lambda_i$, where (λ_i)'s are pairwise fuzzy dense sets in (X, T_1, T_2), is a pairwise fuzzy dense set in (X, T_1, T_2).

Proposition 4.12 If every pairwise fuzzy G_δ-set is fuzzy pairwise dense in a pairwise fuzzy submaximal and pairwise fuzzy strongly irresolvable space (X, T_1, T_2), then (X, T_1, T_2) is a pairwise fuzzy D-Baire space.
Proof: Proof follows from remark 4.11 and theorem 2.16.

Proposition 4.13 If every pairwise fuzzy G_δ-set is a pairwise fuzzy dense set in a pairwise fuzzy strongly irresolvable and pairwise fuzzy nodec space (X, T_1, T_2), then (X, T_1, T_2) is a pairwise fuzzy D-Baire space.
Proof: Proof follows from remark 4.11 and theorem 2.17.

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5. Conclusion

We have introduced the concept of D-Baire bitopological spaces in fuzzy setting and investigated several characterizations of pairwise fuzzy D-Baire spaces.

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