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An Additive-Quadratic Functional Equation in Intuitionistic Fuzzy Normed Spaces

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Abstract

In this work, we investigate the stability of additive-quadratic (AQ) functional equation in intuitionistic fuzzy normed spaces.

Key words: Stability, fixed point, intuitionistic fuzzy normed space, additive-quadratic functional equation.

AMS classification: 39B52, 39B72, 39B82.

1. Section

Stability problem of a functional equation was first posed in [14] which was answered in [4] and then generalized in [1, 10] for additive mappings and linear mappings respectively. A generalization of the Th. M. Rassias theorem was obtained in [3]. On the other hand, Radu et al. [6] noticed that a fixed point method is very important for the solution of the Ulam problem.

The notion of fuzzy sets was introduced by Zadeh [16]. Recently, fuzzy version is discussed in [7, 8]. The concept of intuitionistic fuzzy normed spaces, initially has been introduced by Saadati and Park in [11]. Also, the generalized Hyers-Ulam stability of various types of functional equations in intuitionistic fuzzy normed space has been studied by number of authors [9, 12, 13, 15].

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In this paper, we investigate the following functional equation

$$g(-p+2q)+2[g(3p-2q)+g(2p+q)-g(q)-g(q-p)]=4g(2p-q) \\ +3[g(p+q)+g(p-q)+g(-p)] \tag{1}$$

in intuitionistic fuzzy normed spaces. The above equation (1) is called additive-quadratic functional equation if the function $g(p)=ap^2+b$ is its solution. Indeed, general solution of the above functional equation (1) was found in [5].

2. Stability of Additive-Quadratic functional equation (1)

Throughout this section, let us consider X be a linear space, $(Z,\mathcal{P'}_{\mu,\nu},\mathcal{T}')$ be an intuitionistic fuzzy normed space, $(Y,\mathcal{P'}_{\mu,\nu},\mathcal{T}')$ be a complete intuitionistic fuzzy normed space.

In this section, we prove the Generalized Hyers-Ulam stability for the functional equation (1) in intuitionistic fuzzy normed space by using the fixed point method. This section is divided into two subsections. We use the operator $\mathcal{H}$ for the given mapping $g: X\to Y$ as

$$\mathcal{H}g(p,q)=g(-p+2q)+2[g(3p-2q)+g(2p+q)-g(q)-g(q-p)] \\ -3[g(p+q)+g(p-q)+g(-p)]-4g(2p-q), \quad \text{for all } p,q\in X.$$

Generalized Hyers-Ulam stability of (1): odd case

In this subsection, we prove the generalized Hyers-Ulam stability of the functional equation (1) in intuitionistic fuzzy normed space for odd case.

Theorem 2.1 Let $l=\pm 1$ be fixed and let $\psi: X^2\to Z$ be a function such that there exists a $\tau\neq 2$ with $\left(\frac{\tau}{2}\right)^l<1$ and

$$\mathcal{P'}_{\mu,\nu}(\psi(2^l p,2^l)q,t)\geq \mathcal{P'}_{\mu,\nu}(\psi(p,q),\tau^{-l}t) \tag{2}$$

for all $p,q\in X$ and $t>0$. Let $g: X\to Y$ be an odd mapping satisfying $g(0)=0$ and

$$\mathcal{P'}_{\mu,\nu}(\mathcal{H}g(p,q),t)\geq \mathcal{P'}_{\mu,\nu}(\psi(p,q),t) \tag{3}$$

for all $p,q\in X$ and $t>0$. Then there exists a unique additive mapping $\mathcal{A}: X\to Y$ such that

$$\mathcal{P'}_{\mu,\nu}(g(p)-\mathcal{A}(p),t)\geq \mathcal{P'}_{\mu,\nu}(\psi(0,p),|2-\tau|\,t) \tag{4}$$

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for all $p, q \in X$ and $t > 0$.
proof: For the cases $l = 1$ and $l = -1$, we consider $\tau < 2$ and $\tau > 2$, respectively.
Put $n = 1$ in (3) , we get

$$\mathcal{P}'_{\mu,\nu}(\mathcal{H}g(p,q),t) \geq \mathcal{P}'_{\mu,\nu}(\psi(p,q),t) \tag{5}$$

$\forall\, p \in X$ and $t > 0$. Let $p = 0$ and $q = p$ in (5), we have

$$\mathcal{P}'_{\mu,\nu}(g(2p) - 2g(p),t) \geq \mathcal{P}'_{\mu,\nu}(\psi(0,p),t) \tag{6}$$

$\forall\, p \in X$ and $t > 0$. Hence

$$\mathcal{P}'_{\mu,\nu}\left(g(p) - \frac{1}{2^l}g(2^l p),t\right) \geq \mathcal{P}'_{\mu,\nu}\left(\frac{\tau^{\frac{l-1}{2}}}{|2|^{\left(\frac{1+l}{2}\right)}}\psi(0,p),t\right) \tag{7}$$

$\forall\, p \in X$ and $t > 0$. Let $\mathcal{S} = \{g : X \to Y\}$ and introduce the generalized metric ρ on $\mathcal{S}$ as follows:

$$\rho(f,g) = \inf\left\{\iota \in \mathbb{R}_+ : \mathcal{P}'_{\mu,\nu}(f(p) - g(p),\iota t) \geq \mathcal{P}'_{\mu,\nu}(\psi(0,p),t) \quad \forall p \in X, t > 0\right\}.$$

It is easy to check that $(\mathcal{S}, \rho)$ is a complete generalized metric (see also [5]).
Define the mapping $\mathcal{E} : \mathcal{S} \to \mathcal{S}$ by

$$\mathcal{E}(p) = \frac{1}{2^l}f(2^l p)$$

for all $f \in \mathcal{S}$ and $p \in X$.
Thus,

$$\mathcal{P}'_{\mu,\nu}(\mathcal{E}f(p) - \mathcal{E}g(p),t) = \mathcal{P}'_{\mu,\nu}\left(\frac{1}{2^l}f(2^l p) - \frac{1}{2^l}g(2^l p),t\right)$$
$$\mathcal{P}'_{\mu,\nu}(f(p) - g(p),t) \geq \mathcal{P}'_{\mu,\nu}\left(\left(\frac{\tau}{2}\right)^l \psi(0,p),t\right)$$

for all $p \in X$ and $t > 0$.
This means that $\mathcal{E}$ is a contractive mapping with lipschitz constant $L = (\frac{\tau}{2})^l < 1$.
It follows from (7) that

$$\rho(g, \mathcal{E}g) \leq \frac{\tau^{\left(\frac{l-1}{2}\right)}}{|2|^{\left(\frac{1+l}{2}\right)}}.$$

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By using (Theorem 2.2 in [2]), there exists a mapping $\mathcal{A} : X \to Y$ which satisfying:

1. $\mathcal{A}$ is a unique fixed point of $\mathcal{E}$, which is satisfied

$$\mathcal{A}(2^l p) = 2^l \mathcal{A}(p) \tag{8}$$

for all $p \in X$.

2. $\rho(\mathcal{E}^k g, \mathcal{A}) \to 0$ as $k \to \infty$. This implies the equality

$$\lim_{k\to\infty} \frac{1}{2^{kl}} g(2^{kl} p) = \mathcal{A}(p)$$

for all $p \in X$.

3. $\rho(f, \mathcal{A}) \leq \dfrac{1}{1-L}\rho(g, \mathcal{E}g)$, which implies the inequality $\rho(g, \mathcal{A}) \leq \dfrac{1}{|2-\tau|}$. So

$$\mathcal{P'}_{\mu,\nu}(g(p) - A(p), t) \geq \mathcal{P'}_{\mu,\nu}(\psi(0,p), |2-\tau|\, t)$$

for all $p, q \in X$ and $t > 0$.

By (3) , we have

$$\begin{aligned} \mathcal{P}'_{\mu,\nu}(\mathcal{H}\mathcal{A}(p,q), t) &= \lim_{k\to\infty} \mathcal{P}'_{\mu,\nu}(2^{-kl}\mathcal{H}g(2^{kl}p, 2^{kl}q), t) \\ &\geq \lim_{k\to\infty} \mathcal{P}'_{\mu,\nu}(2^{-kl}\psi(2^{kl}p, 2^{kl}q), t) \end{aligned}$$

Thus the function $\mathcal{A}$ is additive. Therefore, $\mathcal{A} : X \to Y$ is a unique additive mapping satisfies (4).

Corollary 2.2 Let $l = \pm 1$ be fixed and let r, ς be non-negative real numbers with $r \neq 1$. Let $g : X \to Y$ be a mapping such that

$$\mathcal{P'}_{\mu,\nu}(\mathcal{H}g(p,q), t) \geq \mathcal{P'}_{\mu,\nu}(\varsigma(\|p\|^r + \|q\|^r, t) \tag{9}$$

for all $p, q \in X$ and $t > 0$.
Then there exists a unique additive mapping $\mathcal{A} : X \to Y$ such that

$$\mathcal{P'}_{\mu,\nu}(g(p) - A(p), t) \geq \mathcal{P'}_{\mu,\nu}(\varsigma \|p\|^r, |2-2^r|\, t) \tag{10}$$

for all $p, q \in X$ and $t > 0$.
proof: The proof is resembling to the proof of Theorem 2.1.

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Generalized Hyers-Ulam stability of (1): even case

In this Subsection, we prove the generalized Hyers-Ulam stability of the functional equation (1) in intuitionistic fuzzy normed space for even case.

Theorem 2.3 Let $l = \pm 1$ be fixed and let $\psi : X^2 \to [0, \infty)$ be a function such that there exists a $\tau \neq 4$ with $\left(\frac{\tau}{4}\right)^l < 1$

$$\mathcal{P'}_{\mu,\nu}(\psi(2^l p, 2^l)q, t) \geq \mathcal{P'}_{\mu,\nu}(\psi(p, q), \tau^{-l} t) \tag{11}$$

for all $p, q \in X$ and $t > 0$. Let $g : X \to Y$ be an even mapping satisfying $g(0) = 0$ and (3). Then there exists a unique quadratic mapping $\mathcal{Q} : X \to Y$ such that

$$\mathcal{P'}_{\mu,\nu}(g(p) - \mathcal{Q}(p), t) \geq \mathcal{P'}_{\mu,\nu}(\psi(0, p), |4 - \tau|\, t) \tag{12}$$

for all $p, q \in X$ and $t > 0$.
proof: The rest of the proof is identical to the proof of Theorem 2.1.

Corollary 2.4 Let $l = \pm 1$ be fixed and let r, ς be non-negative real numbers with $r \neq 2$. Let $g : X \to Y$ be a mapping satisfying (9). Then there exists a unique quadratic mapping $\mathcal{Q} : X \to Y$ such that

$$\mathcal{P'}_{\mu,\nu}(g(p) - Q(p), t) \geq \mathcal{P'}_{\mu,\nu}(\varsigma \left\|p\right\|^r, |4 - 2^r|\, t) \tag{13}$$

for all $p, q \in X$ and $t > 0$.
proof: The proof is resembling to the proof of Theorem 2.3.

3. Conclusion

This work presents the generalized Hyers-Ulam-Rassias stability of an additive-quadratic (AQ) functional equation in intuitionistic fuzzy normed spaces.

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