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A Study on Stochastic Maximal Regularity for Rough Time - Dependent Problems

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Abstract

We unify and extend the semigroup and PDE approaches to stochastic maximal regularity of time-dependent semilinear parabolic problems with noise given by a cylindrical Brownian motion. We treat random coefficients that are only progressively measurable in the time variable. For 2m-th order systems with VMO regularity in space, we obtain $L^p(L^q)$ estimates for all $p > 2$ and $q \geq 2$, leading to optimal space-time regularity results. For second order systems with continuous coefficients in space, we also include a first order linear term, under a stochastic parabolicity condition, and obtain $L^p(L^p)$ estimates together with optimal space-time regularity. For linear second order equations in divergence form with random coefficients that are merely measurable in both space and time, we obtain estimates in the tent spaces $T^{p,2}_\sigma$ of Coifman-Meyer-Stein. This is done in the deterministic case under no extra assumption, and in the stochastic case under the assumption that the coefficients are divergence free.

Key words: Stochastic PDEs, Maximal Regularity, VMO Coefficients, Measurable Coefficients.

AMS classification: 39B52, 39B72, 39B82.

1 Introduction

On X_0 (typically $X_0 = L^r(\mathcal{O}; \mathbb{C}^N)$ where $r \in [2, \infty)$), we consider the following stochastic evolution equation:

$$\begin{cases} dU(t) + A(t)U(t)dt = F(t, U(t))dt + \big(B(t)U(t) + G(t, U(t))\big)dW_H(t), \\ \qquad\qquad U(0) = u_0, \end{cases} \tag{1}$$

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where H is a Hilbert space, W_H a cylindrical Brownian motion, $A : \mathbb{R}_+ \times \Omega \to \mathcal{L}(X_1, X_0)$ (for some Banach space X_1 such that $X_1 \hookrightarrow X_0$, typically a Sobolev space) and $B : \mathbb{R}_+ \times \Omega \to \mathcal{L}(X_1, \gamma(H, X_{\frac{1}{2}}))$ are progressively measurable (and satisfy a suitable stochastic parabolic estimate), the functions F and G are suitable nonlinearities, and the initial value $u_0 : \Omega \to X_0$ is $\mathcal{F}_0$-measurable (see Chapter 2 for precise definitions). We are interested in maximal L^p-regularity results for (1). This means that we want to investigate well-posedness together with sharp L^p-regularity estimates given the data F, G and u_0.

Knowing these sharp regularity results for equations such as (1), gives a priori estimates to nonlinear equations involving suitable nonlinearities $F(t, U(t))dt$ and $G(t, U(t))dW_H(t)$. Well-posedness of such non-linear equations follows easily from these a priori estimates.

2 Preliminaries

Definition 2.1 (Measurability) Let (S, Σ, μ) be a measure space. A function $f : S \to X$ is called strongly measurable if it can be approximated by μ-simple functions a.e. An operator valued function $f : S \to \mathcal{L}(X, Y)$ is called X-strongly measurable if for every $x \in X, s \mapsto f(s)x$ is strongly measurable. Let $(\Omega, \mathbb{P}, \mathcal{A})$ be a probability space with filtration $(\mathcal{F}_t)_{t \geq 0}$. A process $\phi : \mathbb{R}_+ \times \Omega \to X$ is called progressively measurable if for every fixed $T \geq 0, \phi$ restricted to $[0, T] \times \Omega$ is strongly $\mathcal{B}([0, T]) \times \mathcal{F}_T$-measurable An operator valued process $\phi : \mathbb{R}_+ \times \Omega \to \mathcal{L}(X, Y)$ will be called X-strongly progressively measurable if for every $x \in X$, ϕx is progressively measurable. Let $\triangle := \{(s, t) : 0 \leq s \leq t < \infty\}$ and $\triangle_T = \triangle \cap [0, T]^2$. Let $\mathcal{B}_T$ denotes the Borel σ-algebra on $\triangle_T$. A two-parameter process $\phi : \triangle \times \Omega \to X$ will be called progressively measurable if for every fixed $T \geq 0$, ϕ restricted to $\triangle_T \times \Omega$ is strongly $\mathcal{B}_T \times \mathcal{F}_T$-measurable.

Definition 2.2 (Functional calculus) For $\sigma \in (0, \pi)$ let $\Sigma_\sigma = \{z \in \mathbb{C} : |\arg(z)| < \sigma\}$. A closed and densely defined operator $(A, D(A))$ on a Banach space X is called sectorial of type $(M, \sigma) \in \mathbb{R}_+ \times (0, \pi)$ if A is injective, has dense range, $\sigma(A) \subseteq \overline{\Sigma_\sigma}$ and

$$\|\lambda R(\lambda, A)\| \leq M, \qquad \lambda \in \mathbb{C} \backslash \Sigma_\sigma.$$

A closed and densely defined operator $(A, D(A))$ on a Banach space X is called sectorial of type $(M, w, \sigma) \in \mathbb{R}_+ \times \mathbb{R} \times (0, \pi)$ if $A + w$ is sectorial of type (M, σ).

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Let $H^\infty(\Sigma_\varphi)$ denote the space of all bounded holomorphic functions $f : \Sigma_\varphi \to \mathbb{C}$ and let $\|f\|_{H^\infty(\Sigma_\varphi)} = \sup_{z\in\Sigma_\varphi} |f(z)|$. Let $H_0^\infty(\Sigma_\varphi) \subseteq H^\infty(\Sigma_\varphi)$ be the set of all f for which there exists an $\varepsilon > 0$ and $C > 0$ such that $|f(z)| \leq C\frac{|z|^\varepsilon}{1+|z|^{2\varepsilon}}$.
For an operator A which is sectorial of type $(M, \sigma), \sigma < \nu < \varphi$, and $f \in H_0^\infty(\Sigma_\varphi)$ define

$$f(A) = \frac{1}{2\pi i}\int_{\partial\Sigma_\nu} f(\lambda)R(\lambda, A)d\lambda,$$

where the orientation is such that $\sigma(A)$ is on the right side of the integration path. The operator A is said to have a bounded H^∞-calculus of angle φ if there exists a constant C such that for all $f \in H_0^\infty(\Sigma_\varphi)$.

$$\|f(A)\| \leq C\|f\|_{H^\infty(\Sigma_\varphi)}.$$

For details on the H^∞-functional calculus we refer. A list of examples has been given in the introduction.
For an interpolation couple (X_0, X_1) let

$$X_\theta = [X_0, X_1]_\theta, \quad \text{and} \quad X_{\theta,p} = [X_0, X_1]_{\theta,p}$$

denote the complex and real interpolation spaces at $\theta \in (0, 1)$ and $p \in [1, \infty]$, respectively.

Definition 2.3 (Measure Space) A Pair $[[X, S, \mu]]$ is called a measure space if $[[X, S]]$ is a measurable space and μ is a measure on S.

Definition 2.4 (Hilbert Space) A Hilbert space is a complex Banach Space whose norm arises from an inner product, that is in which there is defined a complex function (x, y) of vectors x and y with

(i) $(\alpha x + \beta y, z) = \alpha(x, z) + \beta(y, z)$.
(ii) $(x, y) = (y, x)$
(iii) $(x, x) = ||x||^2$

Definition 2.5 (Banach Space) A normed linear space is a linear space N in which to each vector x there corresponds to a real number, denoted by $||x||$, called norm of x, such that

(i) $||x|| \geq 0$, and $||x|| = 0 \Rightarrow x = 0$.
(ii) $||x + y|| \leq ||x|| + ||y||$
(iii) $||\alpha x|| = |\alpha| ||x||$

Definition 2.6 (Holder inequility) Let the spaces $X = \{x_1, x_2, \cdots, x_n\}$, $Y = \{y_1, y_2, \cdots, y_n\} \in \ell$. Define $||x||_p = \left(\sum_1^n |x_i|^p \right)^{\frac{1}{p}}$, for $p > 1$, then $\sum_\ell |x_i y_i| \leq \left(\sum_{ell} |x_i|^p \right)^{\frac{1}{p}} \left(\sum_\ell |y_i|^q \right)^{\frac{1}{q}}$ that $||xy|| \leq ||x||_p ||y||_q$, where $\dfrac{1}{p} + \dfrac{1}{q} = 1$.

Definition 2.7 (Continuous Function) Let (S, d_s) and (T, d_T) be metric spaces and let $f : S \to T$ be a function from S to T. The function f is said to be continuous at a point P in S if for every $\epsilon > 0$ there is a $f > 0$ such that, $d_T(f(x), f(p)), \epsilon$, whenever $d_s(x, p), \epsilon$.

Definition 2.8 (Dense Set) A point x is a limit point of A if given $\epsilon < 0$, there exixts $Y \in A, Y \notin x$, with $\rho(y, x) < \epsilon$. We say that A is a dense set.

Definition 2.9 (Function spaces) Let $S \subseteq \mathbb{R}^d$ be open. For a weight function $w : \mathbb{R}^d \to (0, \infty)$ which is integrable on bounded subset of $\mathbb{R}^d, p \in [1, \infty)$, and X a Banach space, we work with the Bochner spaces $L^p(S, w; X)$ with norm defined by

$$\|u\|^p_{L^p(S,w;X)} = \int\limits_S \|u(t)\|^p_X w(t) dt,$$

We also use the corresponding Sobolev spaces defined by

$$\|u\|^p_{W^{1,p}(S,w;X)} = \|u\|^p_{L^p(S,w;X)} + \|u'\|^p_{L^p(S,w;X)}$$

If $q < p$, and $w_\alpha(x) = |x|^\alpha$ with $\alpha/d < \frac{p}{q} - 1$, note that, by Hölder inequality $L^p(S, w_\alpha; X) \hookrightarrow L^q(S; X)$.

In several cases the class of weight we will consider is the class of A_p-weights $w : \mathbb{R}^d \to (0, \infty)$. Recall that $w \in A_p$ if and only if the Hardy-Littlewood maximal function is bounded on $L^p(\mathbb{R}^d, w)$.

For $p \in (1, \infty)$ and an A_p-weight w let the Bessel potential spaces $H^{s,p}(\mathbb{R}^d, w; X)$

be defined as the space of all $f \in \mathcal{S}'(\mathbb{R}^d; X) := \mathcal{L}(\mathcal{S}(\mathbb{R}^d), X)$ for which $\mathcal{F}^{-1}[(1 + |\cdot|^2)^{s/2}\widehat{f} \in L^p(\mathbb{R}^d, w; X)$. Here $\mathcal{F}$ denotes the Fourier transform. Then $H^{s,p}(\mathbb{R}^d, w; X)$ is a Banach space when equipped with the norm

$$\|f\|_{H^{s,p}(\mathbb{R}^d,w;X)} = \|\mathcal{F}^{-1}[(1 + |\cdot|^2)^{s/2}\widehat{f}]\|_{L^p(\mathbb{R}^d,w;X)}.$$

The following is a well known consequence of Fourier multiplier theory.

Lemma 2.10 Let X be a UMD Banach space, $p \in (1, \infty), s \in \mathbb{R}, r > 0$ and $k \in \mathbb{N}$. Then the following give equivalent norms on $H^{s,p}(\mathbb{R}^d; X)$:

$$\begin{gathered}\|(-\Delta)^{r/2}u\|_{H^{s-r,p}(\mathbb{R}^d;X)} + \|u\|_{H^{s-r,p}(\mathbb{R}^d;X)},\\ \sum_{|\alpha|=k} \|\partial^\alpha u\|_{H^{s-k,p}(\mathbb{R}^d;X)} + \|u\|_{H^{s-k,p}(\mathbb{R}^d;X)}.\end{gathered}$$

The spaces $H^{s,p}$ will also be needed on bounded open intervals I. For a $I \subseteq \mathbb{R}, p \in (1, \infty), w \in A_p, s \in \mathbb{R}$ the space $H^{s,p}(I, w; X)$ is defined as all restriction $f|_I$ where $f \in H^{s,p}(I, w; X)$. This is a Banach space when equipped with the norm

$$\|f\|_{H^{s,p}(I,w;X)} = \inf\{\|g\|_{H^{s,p}(\mathbb{R},w;X)} : g|_I = f\}.$$

Either by repeating the proof of Lemma (2.10) or by reducing to it by applying a bounded extension operator from $H^{\theta,p}(I, w; Y) \to H^{\theta,p}(\mathbb{R}, w; Y)$ and Fubini, we obtain the following norm equivalence.

Lemma 2.11 Let X be a UMD space, $p \in (1, \infty), s \in \mathbb{R}, r > 0, k \in \mathbb{N}$, and let $I \subseteq \mathbb{R}$ be an open interval. Let $\theta \in (0, 1)$ and $w \in A_p$. Then the following two norms give equivalent norms on $H^{\theta,p}(I; H^{s,p}(\mathbb{R}^d; X))$:

$$\begin{gathered}\|(-\Delta)^{r/2}u\|_{H^{\theta,p}(I,w;H^{s-r,p}(\mathbb{R}^d;X))} + \|u\|_{H^{\theta,p}(I;H^{s-r,p}(\mathbb{R}^d;X))},\\ \sum_{|\beta|=k} \|\partial^\beta u\|_{H^{\theta,p}(I;H^{s-k,p}(\mathbb{R}^d;X))} + \|u\|_{H^{\theta,p}(I;H^{s-k,p}(\mathbb{R}^d;X))}.\end{gathered}$$

Stochastic integration

Let $L^p_{\mathcal{F}}(\Omega; L^q(I; X))$ denote the space of progressively measurable processes in $L^p(\Omega; L^q(I; X))$.

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The Itô integral of an $\mathcal{F}$-adapted finite rank step process in $\gamma(H,X)$, with respect to an $\mathcal{F}$-cylindrical Brownian motion W_H, is defined by

$$\int_{\mathbb{R}_+} \sum_{k=1}^{N}\sum_{j=1}^{M} \mathbf{1}_{(t_k,t_{k+1}]\times F_k} \otimes (h_j \otimes x_k)\, dW_H := \sum_{k=1}^{N}\sum_{j=1}^{M} \mathbf{1}_{F_k}\Big[W_H(t_{k+1})h_j - W_H(t_k)h_j\Big] \otimes x_k,$$

for $N \in \mathbb{N}, 0 \le t_1 < t_2 < \cdots < t_{N+1}$, and for all $k = 1,\dots,N, F_k \in \mathcal{F}_{t_k}, h_k \in H, x_k \in X$. The following version of Itô's isomorphism holds for such processes:

Theorem 2.12 Let X be a UMD Banach space and let G be an $\mathcal{F}$-adapted finite rank step process in $\gamma(H,X)$. For all $p \in (1,\infty)$ one has the two-sided estimate

$$\mathbb{E}\sup_{t\ge 0}\Big\|\int_0^t G(s)\, dW_H(s)\Big\|^p \eqsim_p \mathbb{E}\|G\|^p_{\gamma(L^2(\mathbb{R}_+;H),X))}, \tag{2}$$

with implicit constants depending only on p and (the UMD constant of) X.

The class of UMD Banach spaces includes all Hilbert spaces, and all $L^q(\mathcal{O};G)$ spaces for $q \in (1,\infty)$, and G another UMD space. It is stable under isomorphism of Banach spaces, and included in the class of reflexive Banach spaces. Closed subspaces, quotients, and duals of UMD spaces are UMD.

Theorem (2.12) allows one to extend the stochastic integral, by density, to the closed linear span in $L^p(\Omega;\gamma(L^2(\mathbb{R}_+;H),X))$ of all $\mathcal{F}$-adapted finite rank step processes in $\gamma(H,X)$). We denote this closed linear span by $L^p_{\mathcal{F}}(\Omega;\gamma(L^2(\mathbb{R}_+;H),X))$. Moreover, this set coincides with the progressively measurable processes in $L^p(\Omega;\gamma(L^2(\mathbb{R}_+;H),X))$.

If the UMD Banach space X has type 2 (and thus martingale type 2), then one has a continuous embedding $L^2(\mathbb{R}_+;\gamma(H,X)) \hookrightarrow \gamma(L^2(\mathbb{R}_+;H),X)$.

In such a Banach space, (2) implies that

$$\mathbb{E}\sup_{t\ge 0}\Big\|\int_0^t G(s)\, dW_H(s)\Big\|^p \le C^p\mathbb{E}\|G\|^p_{L^2(\mathbb{R}_+;\gamma(H,X))}, \tag{3}$$

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where C depends on X and p. The stochastic integral thus uniquely extends to $L^p_{\mathcal{F}}(\Omega; L^2(\mathbb{R}_+; \gamma(H, X)))$.

Note, however, that the sharp version of Itô's isomorphism given in Theorem (2.12) is critical to prove stochastic maximal regularity, even in time-independent situations. The weaker estimate (3) (where the right hand side would typically be $L^2(\mathbb{R}_+; L^p(\mathbb{R}^d))$ instead of $L^p(\mathbb{R}^d; L^2(\mathbb{R}_+)))$ does not suffice for this purpose.

3 Maximal Regularity for Stochastic Evolution Equations

In this section we consider the semilinear stochastic evolution equation

$$\begin{cases} dU(t) + A(t)U(t)dt = F(t, U(t))dt + \big(B(t)U(t) + G(t, U(t))\big)dW_H(t), \\ \qquad U(0) = u_0. \end{cases} \tag{4}$$

Here $A(t)$ and $B(t)$ are linear operators which are (t, ω)-dependent. The functions F and G are nonlinear perturbations.

The deterministic case

Consider the following hypotheses.

Assumption 3.1 Let X_0 and X_1 be Banach spaces such that $X_1 \hookrightarrow X_0$ is dense. Let $X_\theta = [X_0, X_1]_\theta$ and $X_{\theta,p} = (X_0, X_1)_{\theta,p}$ denote the complex and real interpolation spaces at $\theta \in (0, 1)$ and $p \in [1, \infty]$, respectively.

For $f \in L^1(I; X_0)$ with $I = (0, T)$ and $T \in (0, \infty]$ we consider:

$$\begin{cases} u'(t) + A(t)u(t) = f(t), \quad t \in I \\ \qquad u(0) = 0. \end{cases} \tag{5}$$

We say that u is a strong solution of (5) if for any finite interval $J \subseteq I$ we have $u \in L^1(J; X_1)$ and

$$u(t) + \int_0^t A(s)u(s)ds = \int_0^t f(s)ds, \quad t \in \overline{J}, \tag{6}$$

Note that this identity yields that $u \in W^{1,1}(J; X_0)$ and $u \in C(\overline{J}; X_0)$ for bounded $J \subseteq I$.

Definition 3.1 (Deterministic maximal regularity) Let Assumption (3.1) be satisfied and assume that $A : [s, \infty) \to \mathcal{L}(X_1, X_0)$ is strongly measurable and $\sup_{t\in\mathbb{R}} \|A(t)\|_{\mathcal{L}(X_1,X_0)} < \infty$. Let $p \in (1, \infty), \alpha \in (-1, p-1), T \in (0, \infty]$, and set $I = (0, T)$. We say that $A \in \mathrm{DMR}(p, \alpha, T)$ if for all $f \in L^p(I, w_\alpha; X_0)$, there exists a strong solution

$$u \in W^{1,p}(I, w_\alpha; X_0) \cap L^p(I, w_\alpha; X_1)$$

of (5) and

$$\|u\|_{W^{1,p}(I,w_\alpha;X_0)} + \|u\|_{L^p(I,w_\alpha;X_1)} \leq C\|f\|_{L^p(I,w_\alpha;X_0)}. \tag{7}$$

In (6) we use the continuous version of $u : \overline{I} \to X_0$. By Proposition **??** for $\alpha \in [0, p-1)$ we have

$$u \in C_{ub}(\overline{I}; X_{1-\frac{1+\alpha}{p},p}) \text{ and } u \in C_{ub}([\varepsilon, T]; X_{1-\frac{1}{p},p}), \quad \varepsilon \in (0, T).$$

If $\alpha \in (-1, 0)$ the first assertion does not hold, but the second one holds on $[0, T] if T < \infty$.

Remark 3.1 Although we do allow $T = \infty$ in the above definition, most result will be formulated for $T \in (0, \infty)$ as this is often simpler and enough for applications to PDEs.

Note that $A \in \mathrm{DMR}\,(p, \alpha, T)$ implies that the solution u is unique (use (7)). Furthermore, it implies unique solvability of (5) on subintervals $J = (a, b) \subseteq I$. In particular, $\mathrm{DMR}\,(p, \alpha, T)$ implies $\mathrm{DMR}(p, \alpha, t)$ for all $t \in (0, T]$.

Hypothesis on A and B and the definition of SMR

Consider the following hypotheses.

Assumption 3.2 Let H be a separable Hilbert space. Assume X_0 and X_1 are UMD spaces with type 2. Let $A : \mathbb{R}_+ \times \Omega \to \mathcal{L}(X_1, X_0)$ be strongly progressively measurable and $C_A := \sup_{t\in\mathbb{R},\omega\in\Omega} \|A(t, \omega)\|_{\mathcal{L}(X_1,X_0)} < \infty$.
Let $B : \mathbb{R}_+ \times \Omega \to \mathcal{L}(X_1, \mathcal{L}(H, X_{\frac{1}{2}}))$ be such that for all $x \in X_1$ and $h \in H, (Bx)h$ is strongly progressively measurable and assume there is a constant C such that

$$C_B := \sup_{t\in\mathbb{R},\omega\in\Omega} \|B(t, \omega)\|_{\mathcal{L}(X_1,\mathcal{L}(H,X_{\frac{1}{2}}))} < \infty.$$

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For $f \in L^1(I; X_0)$ and $g \in L^2(I; \gamma(H, X_{\frac{1}{2}}))$ with $I = (0,T)$ and $T \in (0,\infty]$ we consider:

$$\begin{cases} dU(t) + A(t)U(t)dt = f(t)dt + \big(B(t)U(t) + g(t)\big)dW_H(t), \\ \qquad\qquad U(0) = 0. \end{cases} \tag{8}$$

We say that U is a strong solution of (4) if for any finite interval $J \subseteq I$ we have $U \in L^0_{\mathcal{F}}(\Omega; L^2(J; \gamma(H, X_1)))$ and almost surely for all $t \in I$,

$$U(t) + \int_0^t A(s)U(s)ds = \int_0^t f(s)ds + \int_0^t \Big(g(s) + B(s)U(s)\Big)dW_H(s), \tag{9}$$

The above stochastic integrals are well-defined by (3). Identity (9) yields that U has paths in $C(\overline{J}; X_0)$ for bounded $J \subseteq I$.

Definition 3.2 (Stochastic maximal regularity) Suppose Assumptions 3.1 and 3.2 hold. Let $p \in [2,\infty), \alpha \in (-1, \frac{p}{2} - 1) (\alpha = 0$ is included if $p = 2), T \in (0,\infty]$, and set $I = (0,T)$. We say that $(A, B) \in \mathrm{SMR}(p, \alpha, T)$ if for all $f \in L^p_{\mathcal{F}}(\Omega \times I, w_\alpha; X_0)$ all $g \in L^p_{\mathcal{F}}(\Omega \times I, w_\alpha; \gamma(H, X_{\frac{1}{2}}))$, there exists a strong solution

$$U \in \bigcap_{\theta \in [0, \frac{1}{2})} L^p(\Omega; H^{\theta,p}(I, w_\alpha; X_{1-\theta}))$$

of (8) and for each $\theta \in [0, \frac{1}{2})$ there is a constant C_θ such that

$$\begin{aligned} &\|U\|_{L^p(\Omega; H^{\theta,p}(I, w_\alpha; X_{1-\theta}))} \\ &\qquad \leq C_\theta \|f\|_{L^p(\Omega \times I, w_\alpha; X_0)} + C_\theta \|g\|_{L^p(\Omega \times I, w_\alpha; \gamma(H, X_{\frac{1}{2}}))}. \end{aligned} \tag{10}$$

In the case $B = 0$ we write $A \in \mathrm{SMR}(p, \alpha, T)$ instead of $(A, 0) \in \mathrm{SMR}\,(p, \alpha, T)$

In the above we use a pathwise continuous version of $U : \Omega \times \overline{I} \to X_0$. By Proposition 2.5 if $\alpha \in [0, \frac{p}{2} - 1)$ we even have

$$U \in L^p(\Omega; C(\overline{I}; X_{1-\frac{\alpha+1}{p},p})) \;\text{ and }\; U \in L^p(\Omega; C([\varepsilon, T]; X_{1-\frac{\alpha+1}{p},p})).$$

If $\alpha \in (-1, 0)$ the first assertion does not hold, but the second one holds on $[0, T] if T < \infty$. A variant of Remark holds for SMR. In particular, any of the estimates (10) implies uniqueness.

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4 Conclusion

In this paper, we discussed about Measurability, Functional calculus, Function spaces, Stochastic integration. Also we discussed about Maximal regularity for stochastic evolution equations and their Hypothesis.

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