

A note on weak fuzzy P -spaces

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Abstract

In this paper, the concepts of weak fuzzy P -spaces, are extensively established. The inter-relations of weak fuzzy P -spaces, fuzzy P -spaces, fuzzy almost P -spaces, fuzzy weakly Lindelöf spaces and fuzzy almost Lindelöf spaces are also investigated in this paper.

Key words: Fuzzy dense set, Fuzzy G_δ -set, Fuzzy Regular open set, Fuzzy P -space, Fuzzy almost P -space, weak Fuzzy P -space, Fuzzy weakly Lindelöf space, Fuzzy almost Lindelöf space.

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1 Introduction

The concept of fuzzy sets and fuzzy set operations were first introduced by L.A.Zadeh in his classical paper [16] in the year 1965. Thereafter the paper of C.L.Chang [5] in 1968 paved the way for the subsequent tremendous growth of the numerous fuzzy topological concepts. Since then much attention has been paid to generalize the basic concepts of General Topology in fuzzy setting and thus a modern theory of fuzzy topology has been developed. In recent years, fuzzy topology has been found to be very useful in solving many practical problems. In this paper, the concepts of weak fuzzy P -spaces, are extensively established. The inter-relations of weak fuzzy P -spaces, fuzzy P -spaces, fuzzy almost P -spaces, fuzzy weakly Lindelöf spaces and fuzzy almost Lindelöf spaces are also investigated in this paper.

2 Preliminaries

Now we give some basic notions and results used in the sequel. In this work by (X, T) or simply by X , we will denote a fuzzy topological space due to Chang (1968).

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Let X be a non-empty set and I , the unit interval $[0,1]$. A fuzzy set λ in X is a function from X into I . The null set 0 is the function from X into I which assumes only the value 0 and the whole fuzzy set 1 is the function from X into I which takes 1 only.

Definition 2.1 [5] Let λ and μ be fuzzy sets in X . Then for all $x \in X$,

1. $\lambda = \mu \Leftrightarrow \lambda(x) = \mu(x)$,
2. $\lambda \leq \mu \Leftrightarrow \lambda(x) \leq \mu(x)$,
3. $\psi = \lambda \vee \mu \Leftrightarrow \psi(x) = \max\{\lambda(x), \mu(x)\}$,
4. $\delta = \lambda \wedge \mu \Leftrightarrow \delta(x) = \min\{\lambda(x), \mu(x)\}$,
5. $\eta = \lambda^c \Leftrightarrow \eta(x) = 1 - \lambda(x)$.

For a family $\{\lambda_i/i \in I\}$ of fuzzy sets in X , the union $\psi = \bigvee_i \lambda_i$ and intersection $\delta = \bigwedge_i \lambda_i$ are defined by $\psi(x) = \sup_i\{\lambda_i(x), x \in X\}$, and $\delta(x) = \inf_i\{\lambda_i(x), x \in X\}$.

The fuzzy set 0_X is defined as $0_X(x) = 0$, for all $x \in X$ and the fuzzy set 1_X defined as $1_X(x) = 1$, for all $x \in X$.

Definition 2.2 [6] A fuzzy point x_α in X is a fuzzy set with membership function defined as:

$$x_\alpha(y) = \begin{cases} \alpha & \text{if } x = y; \\ 0 & \text{if } x \neq y. \end{cases}$$

Definition 2.3 [5] A fuzzy topology is a family ‘ T ’ of fuzzy sets in X which satisfies the following conditions:

- (1) $\Phi, X \in T$,
- (2) If $A, B \in T$, then $A \cap B \in T$,
- (3) If $A_i \in T$, for each $i \in I$, then $\bigcup_{i \in I} A_i \in T$.

T is called a fuzzy topology for X and the pair (X, T) is a fuzzy topological space or fts in short. Every member of T is called a T -open fuzzy set. A fuzzy set is T -closed if and only if its complement is T -open. When no confusion is likely to arise, we shall call a T -open (T -closed) fuzzy set simply an open (closed) fuzzy set.

Definition 2.4 [5] Let (X, T) be any fuzzy topological space and λ be any fuzzy set in (X, T) . The closure and interior of a fuzzy set λ in a fuzzy topological space (X, T) are respectively denoted as $cl(\lambda)$ and $int(\lambda)$ are defined as

- (1) $cl(\lambda) = \wedge\{\mu/\lambda \leq \mu, 1 - \mu \in T\}$ and
- (2) $int(\lambda) = \vee\{\mu/\mu \leq \lambda, \mu \in T\}$.

Lemma 2.5 [15] For a fuzzy set λ of a fuzzy space X ,

- (a) $1 - cl(\lambda) = int(1 - \lambda)$ and
- (b) $1 - int(\lambda) = cl(1 - \lambda)$.

Lemma 2.6 [1] For a family $\mathcal{A} = \{\lambda_\alpha\}$ of fuzzy sets of a fuzzy space X , $\vee cl \lambda_\alpha \leq cl(\vee \lambda_\alpha)$. In case \mathcal{A} is a finite set, $\vee cl(\lambda_\alpha) = cl(\vee \lambda_\alpha)$. Also $\vee int \lambda_\alpha \leq int(\vee \lambda_\alpha)$.

Definition 2.7 [2] A fuzzy set λ in a fuzzy topological space (X, T) is called a fuzzy F_σ -set in (X, T) if $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where $1 - \lambda_i \in T$ for $i \in I$.

Definition 2.8 [2] A fuzzy set λ in a fuzzy topological space (X, T) is called a fuzzy G_δ -set in (X, T) if $\lambda = \wedge_{i=1}^{\infty}(\lambda_i)$, where $\lambda_i \in T$ for $i \in I$.

Definition 2.9 [13] A fuzzy set λ in a fuzzy topological space (X, T) is called a fuzzy dense set if there exists no fuzzy closed set μ in (X, T) such that $\lambda < \mu < 1$.

Definition 2.10 [8] A non-zero fuzzy set λ in a fuzzy topological space (X, T) is called a fuzzy somewhere dense set if $intcl(\lambda) \neq 0$ in (X, T) .

Definition 2.11 [14] Let (X, T) be a fuzzy topological space. A fuzzy set λ in (X, T) is called a fuzzy σ -nowhere dense set if λ is a fuzzy F_σ -set in (X, T) such that $int(\lambda) = 0$.

Definition 2.12 [11] A fuzzy topological space (X, T) is called a fuzzy P -space if countable intersection of fuzzy open sets in (X, T) is fuzzy open. That is, every non-zero fuzzy G_δ set in (X, T) is fuzzy open in (X, T) .

Definition 2.13 [12] A fuzzy topological space (X, T) is called a fuzzy almost Lindelöf space if every fuzzy open cover $\{\lambda_\alpha\}_{\alpha \in \Delta}$ of (X, T) admits a countable subcover $\{\lambda_n\}_{n \in \mathbb{N}}$ such that $\bigvee_{n \in \mathbb{N}} cl(\lambda_n) = 1$.

Definition 2.14 [12] A fuzzy topological space (X, T) is said to be fuzzy weakly Lindelöf if for every fuzzy open cover $\{\lambda_\alpha\}_{\alpha \in \Delta}$ of (X, T) there exists a countable subcover $\{\lambda_n\}_{n \in \mathbb{N}}$ such that $cl[\bigvee_{n \in \mathbb{N}}(\lambda_n)] = 1$.

Definition 2.15 [10] A fuzzy topological space (X, T) is called a fuzzy almost P -space if for every non-zero fuzzy G_δ -set λ in (X, T) , $int(\lambda) \neq 0$ in (X, T) .

Definition 2.16 [7] A fuzzy topological space (X, T) is called a fuzzy hyperconnected space if every fuzzy open set λ is fuzzy dense in (X, T) . That is, $cl(\lambda) = 1$ for all $\lambda(\neq 0) \in T$.

Definition 2.17 A fuzzy set λ in a fuzzy topological space X is called

- (1) fuzzy pre-open if $\lambda \leq intcl(\lambda)$ and fuzzy pre-closed if $clint(\lambda) \leq \lambda$ [4].
- (2) fuzzy semi-open if $\lambda \leq clint(\lambda)$ and fuzzy semi-closed if $intcl(\lambda) \leq \lambda$ [4].
- (3) fuzzy β -open if $\lambda \leq clintcl(\lambda)$ and fuzzy β -closed if $intclint(\lambda) \leq \lambda$ [3].
- (4) fuzzy regular open if $intcl(\lambda) = \lambda$ and fuzzy regular closed if $clint(\lambda) = \lambda$ [1].

Theorem 2.18 [1] In a fuzzy topological space (X, T) ,

- (a). The closure of a fuzzy open set is a fuzzy regular closed set
- (b). The interior of a fuzzy closed set is a fuzzy regular open set.

Theorem 2.19 [9] A fuzzy topological space (X, T) is a weak fuzzy P -space if and only if $\bigvee_{i=1}^{\infty}(\mu_i)$, where (μ_i) 's are fuzzy regular closed sets in (X, T) , is fuzzy regular closed in (X, T) .

Theorem 2.20 [9] If a fuzzy topological space (X, T) is a weak fuzzy P -space, then $cl(\bigvee_{i=1}^{\infty}(\lambda_i)) = \bigvee_{i=1}^{\infty}(cl(\lambda_i))$, where (λ_i) 's are non-zero fuzzy open sets in (X, T) .

3 Weak fuzzy P-spaces

Definition 3.1 [9] A fuzzy topological space (X, T) is called a weak fuzzy P -space if the countable intersection of fuzzy regular open sets in (X, T) is a fuzzy regular open set in (X, T) . That is., $\bigwedge_{i=1}^{\infty}(\lambda_i)$ is fuzzy regular open in (X, T) , where (λ_i) 's are fuzzy regular open sets in (X, T) .

4 Characterizations of weak fuzzy P-spaces

Proposition 4.1 If $\lambda = \bigvee_{i=1}^{\infty} (cl(\lambda_i))$, where (λ_i) 's are fuzzy open sets in a weak fuzzy P -space (X, T) and $\lambda \neq 1$, then λ is not a fuzzy dense set in (X, T) .

Proof: Suppose that $\lambda = \bigvee_{i=1}^{\infty} [cl(\lambda_i)]$, where (λ_i) 's are fuzzy open sets in a weak fuzzy P -space (X, T) and $\lambda \neq 1$. Now, (λ_i) 's are fuzzy open sets in (X, T) , implies that $(cl(\lambda_i))$'s are fuzzy regular closed sets in (X, T) . Since (X, T) is a weak fuzzy P -space, by theorem (2.19), $\bigvee_{i=1}^{\infty} [cl(\lambda_i)]$ is a fuzzy regular closed set in (X, T) and hence λ is a fuzzy regular closed set in (X, T) . Then, $clint(\lambda) = \lambda$, in (X, T) . It has to be proved that $cl(\lambda) \neq 1$, in (X, T) . Assume the contrary. Suppose that $cl(\lambda) = 1$, in (X, T) . Then, $cl[clint(\lambda)] = cl(\lambda) = 1$, in (X, T) and hence $clint(\lambda) = 1$, in (X, T) . Then, $clint(\lambda) = 1$ and $clint(\lambda) = \lambda$, implies that $\lambda = 1$, contradiction. Hence, the assumption that $cl(\lambda) = 1$, does not hold in (X, T) and therefore, $cl(\lambda) \neq 1$, in (X, T) . Thus, λ is not a fuzzy dense set in (X, T) .

Proposition 4.2 If $\lambda = \bigvee_{i=1}^{\infty} (cl(\lambda_i))$, where (λ_i) 's are fuzzy open sets in a weak fuzzy P -space (X, T) and $\lambda \neq 1$, then $1 - \lambda$ is a fuzzy somewhere dense set in (X, T) .

Proof: Suppose that $\lambda = \bigvee_{i=1}^{\infty} [cl(\lambda_i)]$, where $(\lambda_i) \in T$. Now, (λ_i) 's are fuzzy open sets in (X, T) , implies that $(cl(\lambda_i))$'s are fuzzy regular closed sets in (X, T) . Since (X, T) is a weak fuzzy P -space, by theorem (2.19), $\bigvee_{i=1}^{\infty} [cl(\lambda_i)]$ is a fuzzy regular closed set in (X, T) and hence λ is a fuzzy regular closed set in (X, T) . Then, $clint(\lambda) = \lambda$, in (X, T) . This implies that $1 - clint(\lambda) = 1 - \lambda \neq 0$, in (X, T) and thus $intcl(1 - \lambda) = 1 - \lambda \neq 0$, in (X, T) . Therefore, $1 - \lambda$ is a fuzzy somewhere dense set in (X, T) .

Proposition 4.3 If $\lambda = \bigvee_{i=1}^{\infty} (cl(\lambda_i))$, where (λ_i) 's are fuzzy open sets in a weak fuzzy P -space (X, T) and $\lambda \neq 1$, then $int(\bigvee_{i=1}^{\infty} (cl(\lambda_i))) \neq 0$, in (X, T) .

Proof: Suppose that $\lambda = \bigvee_{i=1}^{\infty} [cl(\lambda_i)]$, where $(\lambda_i) \in T$ and $\lambda \neq 1$. Now, (λ_i) 's are fuzzy open sets in (X, T) , implies that $(cl(\lambda_i))$'s are fuzzy regular closed sets in (X, T) . Since (X, T) is a weak fuzzy P -space, by theorem (2.19), $\bigvee_{i=1}^{\infty} [cl(\lambda_i)]$ is a fuzzy regular closed set in (X, T) and hence λ is a fuzzy regular closed set in (X, T) . Then, $clint(\lambda) = \lambda$, in (X, T) . This implies that $int(\lambda) \neq 0$, in (X, T) . [For, if $int(\lambda) = 0$ in (X, T) will imply that $clint(\lambda) = cl(0) = 0 \neq \lambda$ in (X, T) , a contradiction]. Thus, $int(\bigvee_{i=1}^{\infty} (cl(\lambda_i))) \neq 0$, where (λ_i) 's are fuzzy open sets in (X, T) .

Proposition 4.4 If $\lambda = \bigvee_{i=1}^{\infty} (cl(\lambda_i))$, where (λ_i) 's are fuzzy open sets in a weak fuzzy P -space (X, T) and $\lambda \neq 1$, then λ is not a fuzzy σ -nowhere dense set in (X, T) .

Proof: Suppose that $\lambda = \bigvee_{i=1}^{\infty} [cl(\lambda_i)]$, where $(\lambda_i) \in T$. Now, (λ_i) 's are fuzzy open sets in (X, T) , implies that $(cl(\lambda_i))$'s are fuzzy regular closed sets in (X, T) . Also, since fuzzy regular closed sets are fuzzy closed sets in a fuzzy topological space, $\lambda = \bigvee_{i=1}^{\infty} [cl(\lambda_i)]$, implies that λ is a fuzzy F_{σ} -set in (X, T) . Since (X, T) is a weak fuzzy P -space, by theorem (2.19), $\bigvee_{i=1}^{\infty} [cl(\lambda_i)]$ is a fuzzy regular closed set in (X, T) and hence λ is a fuzzy regular closed set in (X, T) . Then, $clint(\lambda) = \lambda$, in (X, T) . This implies that $int(\lambda) \neq 0$, in (X, T) . Thus, λ is a fuzzy F_{σ} -set in (X, T) such that $int(\lambda) \neq 0$ in (X, T) , implies that λ is not a fuzzy σ -nowhere dense set in (X, T) .

Proposition 4.5 If $\mu = \bigwedge_{i=1}^{\infty} (int(\mu_i))$, where (μ_i) 's are fuzzy closed sets in a weak fuzzy P -space (X, T) , then μ is not a fuzzy dense set in (X, T) and $cl(\bigwedge_{i=1}^{\infty} (int(\mu_i))) \neq 0$, in (X, T) .

Proof: Suppose that $\mu = \bigwedge_{i=1}^{\infty} (int(\mu_i))$, where (μ_i) 's are fuzzy closed sets in a weak fuzzy P -space (X, T) . Now, (μ_i) 's are fuzzy closed sets in (X, T) , implies that $(int(\mu_i))$'s are fuzzy regular open sets in (X, T) and hence $[1 - (int(\mu_i))]$'s are fuzzy regular closed sets in (X, T) . Since (X, T) is a weak fuzzy P -space, by theorem (2.19), $\bigvee_{i=1}^{\infty} [1 - (int(\mu_i))]$ is a fuzzy regular closed set in (X, T) and hence $1 - [\bigwedge_{i=1}^{\infty} (int(\mu_i))]$ is a fuzzy regular closed set in (X, T) . Thus, $\bigwedge_{i=1}^{\infty} (int(\mu_i))$ is a fuzzy regular open set in (X, T) and hence μ is a fuzzy regular open set in (X, T) . This implies that $intcl(\mu) = \mu$, in (X, T) . Hence $cl(\mu) \neq 1$, in (X, T) . [For, if $cl(\mu) = 1$ in (X, T) , then $intcl(\mu) = int(1) = 1 \neq \mu$ in (X, T) , a contradiction]. Therefore, μ is not a fuzzy dense set in (X, T) . Also, $cl(\mu) \neq 0$, in (X, T) . [For, if $cl(\mu) = 0$ in (X, T) , then $intcl(\mu) = int(0) = 0 \neq \mu$ in (X, T) , a contradiction]. Thus, $cl(\bigwedge_{i=1}^{\infty} (int(\mu_i))) \neq 0$, in (X, T) .

5 Weak fuzzy P -spaces and other fuzzy topological spaces

Proposition 5.1 If each fuzzy open set is a fuzzy regular open set in a weak fuzzy P -space (X, T) , then (X, T) is a fuzzy P -space.

Proof: Let λ be a fuzzy G_δ -set in (X, T) . Then, $\lambda = \bigwedge_{i=1}^{\infty} (\lambda_i)$, where (λ_i) 's are fuzzy open sets in (X, T) . By hypothesis, each fuzzy open set is a fuzzy regular open set in (X, T) . Then, (λ_i) 's are fuzzy regular open sets in (X, T) . Since (X, T) is a weak fuzzy P -space, $\bigwedge_{i=1}^{\infty} (\lambda_i)$ is a fuzzy regular open set in (X, T) and hence λ is a fuzzy regular open set in (X, T) . Now, each fuzzy regular open set is a fuzzy open set in a fuzzy topological space (X, T) , λ is a fuzzy open set in (X, T) . Hence, each fuzzy G_δ -set in (X, T) is fuzzy open in (X, T) , implies that (X, T) is a fuzzy P -space.

Proposition 5.2 If each fuzzy closed set is a fuzzy regular closed set in a weak fuzzy P -space (X, T) , then (X, T) is a fuzzy P -space.

Proof: Let λ be a fuzzy closed set in (X, T) such that $clint(\lambda) = \lambda$, in (X, T) . Then, $1 - \lambda$ is a fuzzy open set in (X, T) such that $1 - clint(\lambda) = 1 - \lambda$, in (X, T) and hence $intcl(1 - \lambda) = 1 - \lambda$, in (X, T) . Thus, the fuzzy open set $1 - \lambda$ is a fuzzy regular open set in the weak fuzzy P -space. Hence, by proposition (5.1), (X, T) is a fuzzy P -space.

Remark 5.3 Obviously every fuzzy almost Lindelöf space is a fuzzy weakly Lindelöf space. For, $\bigvee_{n \in N} cl(\lambda_n) \leq cl[\bigvee_{n \in N} (\lambda_n)]$ and $\bigvee_{n \in N} cl(\lambda_n) = 1$, implies that $cl[\bigvee_{n \in N} (\lambda_n)] = 1$.

Proposition 5.4 If the fuzzy topological space (X, T) is a weak fuzzy P -space, then every fuzzy weakly Lindelöf space is a fuzzy almost Lindelöf space.

Proof: Let (X, T) be a fuzzy weakly Lindelöf space and $\{\lambda_\alpha\}_{\alpha \in \Delta}$ be a fuzzy open cover of (X, T) . Then there exists a countable fuzzy subcover $\{\lambda_n\}_{n \in N}$ such that $cl[\bigvee_{n \in N} (\lambda_n)] = 1$, in (X, T) . Since (X, T) is a weak fuzzy P -space, by theorem (2.20), $cl[\bigvee_{n \in N} (\lambda_n)] = \bigvee_{n \in N} cl(\lambda_n)$ where (λ_n) 's are non-zero fuzzy open sets in (X, T) . Hence, for the fuzzy open cover $\{\lambda_\alpha\}_{\alpha \in \Delta}$ of (X, T) , there exists a countable subcover $\{\lambda_n\}_{n \in N}$ such that $\bigvee_{n \in N} cl(\lambda_n) = 1$. Therefore (X, T) is a fuzzy almost Lindelöf space.

Example 5.5 Let $X = \{a, b, c\}$. The fuzzy sets λ , μ and ν are defined on X as follows:

$\lambda : X \rightarrow [0, 1]$ is defined as $\lambda(a) = 0.5$; $\lambda(b) = 0.6$; $\lambda(c) = 0.4$

$\mu : X \rightarrow [0, 1]$ is defined as $\mu(a) = 0.4$; $\mu(b) = 0.7$; $\mu(c) = 0.5$

$\nu : X \rightarrow [0, 1]$ is defined as $\nu(a) = 0.5$; $\nu(b) = 0.8$; $\nu(c) = 0.6$.

Let $T = \{0, \lambda, \mu, \nu, \lambda \vee \mu, \lambda \wedge \mu, 1\}$ is a fuzzy topology on X . Now for the fuzzy G_δ -sets $\lambda \wedge \mu \wedge \nu$ and $\{(\lambda \vee \mu) \wedge (\lambda \wedge \mu)\}$ in (X, T) , $\text{int}(\lambda \wedge \mu \wedge \nu) = \lambda \wedge \mu \neq 0$ and $\text{int}[(\lambda \vee \mu) \wedge (\lambda \wedge \mu)] = \lambda \wedge \mu \neq 0$, in (X, T) . Hence (X, T) is a fuzzy almost P -space.

Remark 5.6 (1) Clearly every fuzzy P -space is a fuzzy almost P -space, since for every non-zero fuzzy G_δ -set δ in (X, T) , $\text{int}(\delta) = \delta \neq 0$. But the converse need not be true. For, in example (5.5), for every non-zero fuzzy G_δ -set δ in (X, T) , $\text{int}(\delta) \neq 0$ in (X, T) . Hence (X, T) is a fuzzy almost P -space, but (X, T) is not a fuzzy P -space, since the fuzzy G_δ -set $\{(\lambda \vee \mu) \wedge (\lambda \vee \nu) \wedge (\mu \vee \nu)\}$ is not a fuzzy open set in (X, T) .

(2) A fuzzy almost P -space need not be a weak fuzzy P -space. For, consider the following example:

Example 5.7 Let $X = \{a, b, c\}$. The fuzzy sets λ , μ and ν are defined on X as follows:

$\lambda : X \rightarrow [0, 1]$ is defined as $\lambda(a) = 0.8$; $\lambda(b) = 0.6$; $\lambda(c) = 0.7$

$\mu : X \rightarrow [0, 1]$ is defined as $\mu(a) = 0.6$; $\mu(b) = 0.9$; $\mu(c) = 0.8$

$\nu : X \rightarrow [0, 1]$ is defined as $\nu(a) = 0.7$; $\nu(b) = 0.5$; $\nu(c) = 0.9$.

Let $T = \{0, \lambda, \mu, \nu, \lambda \vee \mu, \lambda \vee \nu, \mu \vee \nu, \lambda \wedge \mu, \lambda \wedge \nu, \mu \wedge \nu, \lambda \wedge (\mu \vee \nu), \lambda \vee (\mu \wedge \nu), \mu \wedge (\lambda \vee \nu), \mu \vee (\lambda \wedge \nu), \nu \wedge (\lambda \vee \mu), \nu \vee (\lambda \wedge \mu), \lambda \wedge \mu \wedge \nu, \lambda \vee \mu \vee \nu, 1\}$ is a fuzzy topology on X . In (X, T) , for every non-zero fuzzy G_δ -set δ , $\text{int}(\delta) \neq 0$ in (X, T) . Hence (X, T) is a fuzzy almost P -space, but (X, T) is not a weak fuzzy P -space.

(3) A weak fuzzy P -space need not be a fuzzy almost P -space. For, Consider the following example:

Example 5.8 Let $X = \{a, b, c\}$. The fuzzy sets λ , μ and ν are defined on X as follows:

$\lambda : X \rightarrow [0, 1]$ is defined as $\lambda(a) = 0$; $\lambda(b) = 0.4$; $\lambda(c) = 0.5$

$\mu : X \rightarrow [0, 1]$ is defined as $\mu(a) = 0.6$; $\mu(b) = 0$; $\mu(c) = 0.4$
 $\nu : X \rightarrow [0, 1]$ is defined as $\nu(a) = 0.5$; $\nu(b) = 0.6$; $\nu(c) = 0$.
 Let $T = \{0, \lambda, \mu, \nu, \lambda \vee \mu, \lambda \vee \nu, \mu \vee \nu, \lambda \wedge \mu, \lambda \wedge \nu, \mu \wedge \nu, \lambda \wedge (\mu \vee \nu), \lambda \vee (\mu \wedge \nu), \mu \wedge (\lambda \vee \nu), \mu \vee (\lambda \wedge \nu), \nu \wedge (\lambda \vee \mu), \nu \vee (\lambda \wedge \mu), \lambda \wedge \mu \wedge \nu, \lambda \vee \mu \vee \nu, 1\}$ is a fuzzy topology on X . Now $\lambda \vee \mu$, $\lambda \vee (\mu \wedge \nu)$ and $\lambda \vee \mu \vee \nu$ are fuzzy regular open sets in (X, T) and $\{[\lambda \vee \mu] \wedge [\lambda \vee (\mu \wedge \nu)] \wedge [\lambda \vee \mu \vee \nu] = [\lambda \vee (\mu \wedge \nu)]\}$ is a fuzzy regular open set (X, T) and hence (X, T) is a weak fuzzy P -space. But, (X, T) is not a fuzzy almost P -space, since for the non-zero fuzzy G_δ -set $\{[\lambda \wedge (\mu \vee \nu)] \wedge [\mu \wedge (\lambda \vee \nu)] \wedge [\nu \wedge (\lambda \vee \mu)]\}$, $\text{int}([\lambda \wedge (\mu \vee \nu)] \wedge [\mu \wedge (\lambda \vee \nu)] \wedge [\nu \wedge (\lambda \vee \mu)]) = 0$, in (X, T) .

Proposition 5.9 If a fuzzy topological space (X, T) is a weak fuzzy P -space, then (X, T) is not a fuzzy hyperconnected space.

Proof: Let $\mu = \bigwedge_{i=1}^{\infty} (\mu_i)$, where (μ_i) 's ($\neq 1$) are fuzzy regular open sets in (X, T) . Since (X, T) is a weak fuzzy P -space, $\mu = \bigwedge_{i=1}^{\infty} (\mu_i)$ is a fuzzy regular open set in (X, T) . Since every fuzzy regular open set is a fuzzy open set in a fuzzy topological space (X, T) , μ is a fuzzy open set in (X, T) . It has to be proved that $\text{cl}(\mu) \neq 1$ in (X, T) . Assume the contrary. Suppose that $\text{cl}(\mu) = 1$ in (X, T) . Then, $\text{cl}(\bigwedge_{i=1}^{\infty} (\mu_i)) = 1$ in (X, T) . Since (X, T) is a weak fuzzy P -space, by theorem (2.20), $\text{cl}(\bigwedge_{i=1}^{\infty} (\mu_i)) = \bigwedge_{i=1}^{\infty} (\text{cl}(\mu_i))$ in (X, T) . Then, $\bigwedge_{i=1}^{\infty} (\text{cl}(\mu_i)) = 1$ and $\text{cl}(\mu_i) = 1$, for all i . But, μ_i is a fuzzy regular open set in (X, T) , implies that $\text{intcl}(\mu_i) = \mu_i$ in (X, T) and thus $\text{int}(1) = \mu_i$ in (X, T) and hence it will be $\mu_i = 1$, a contradiction. Hence the assumption $\text{cl}(\mu) = 1$ does not hold in (X, T) . Thus, $\text{cl}(\mu) \neq 1$ in (X, T) , for the fuzzy open set μ in (X, T) , implies that (X, T) is not a fuzzy hyperconnected space.

6 Conclusions

In this paper, the concepts of weak fuzzy P -spaces, were extensively established and also inter-relations between weak fuzzy P -spaces and other fuzzy topological spaces, were investigated.

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