Correlation Measure for Interval valued Pythagorean Fuzzy Sets

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Abstract

In this paper, we study the concept of Interval Valued Pythagorean set [IVPS] with special cases of membership and non-membership degrees. In this IVPS set membership and non-membership degrees are satisfying the condition \((u_A(x))^2 + (v_A(x))^2 \leq 1\) instead of \(u_A(x) + v_A(x) > 1\). Here the correlation measure of IVPS set is an extension of correlation measure of Interval valued Pythagorean fuzzy set.

Key words: Interval-Valued Pythagorean Fuzzy Set, Correlation Measures.

1 Introduction

Fuzzy sets are initiated by L.A.Zadeh. The extension of fuzzy set (IFS) describes the non-membership grade of an membership grades under a restriction that the sum of membership and non-membership grade of an imprecise does not exceed 1.

Based on the centric method, Hanafy introduced and studied the correlation and correlation coefficient. Correlation coefficients are beneficial tools used to determine the degree of similarity between objects. The correlation coefficients in fuzzy environments have many applications in the field of medicals diagnosis and clustering etc.

2 Preliminaries

Definition 2.1 Let X be a non empty set and I the unit interval [0,1]. A interval valued Pythagorean set A and B of the form \(A = \{x, u_A(x), v_A(x) : x \in X\}\) and
A valued Pythagorean fuzzy sets $A$ and $B$ are of the form

$$A$$

Definition 2.4 Let $X$ be a non-empty set and $I$ the unit interval $[0, 1]$. A Interval valued Pythagorean fuzzy set $S$ is an object having the form

$$S$$

Definition 2.5 Let $E$ be an universe set. An Intuitionistic fuzzy set $A$ on $E$ can be defined as follows:

$$A = \{ < x, u_A(x), v_A(x) >, x \in E \}, \text{ where } u_A : E \to [0, 1] \text{ and } v_A : E \to [0, 1]$$

such that $0 = u_A(x) + v_A \leq 1$ for any $x \in E$. $u_A(x)$ and $v_A(x)$ are the degree of membership and degree of non-membership of the element $x$ respectively.

Definition 2.2 Let $E$ be an universe set. An Intuitionistic fuzzy set $A$ on $E$ can be defined as follows:

$$A = \{ < x, u_A(x), v_A(x) >, x \in E \}, \text{ where } u_A : E \to [0, 1] \text{ and } v_A : E \to [0, 1]$$

such that $0 = u_A(x) + v_A \leq 1$ for any $x \in E$. $u_A(x)$ and $v_A(x)$ are the degree of membership and degree of non-membership of the element $x$ respectively.

Definition 2.3 Let $X$ is a non-empty set (universe). A Interval valued Pythagorean fuzzy set $A$ on $X$ is an object of the form $A = \{ (x, u_A(x), v_A(x)) : x \in X \}$, where $u_A(x), v_A(x) \in [0, 1], 0 \leq (u_A(x))^2 + v_A(x)^2 \leq 1$ for all $x$ in $X$. $u_A(x)$ is the degree of membership, and $v_A(x)$ is the degree of non-membership.

Definition 2.4 Let $X$ be a non-empty set and $I$ the unit interval $[0, 1]$. A Interval valued Pythagorean fuzzy sets $A$ and $B$ are of the form $A = \{ x, u_A(x), v_A(x) : x \in X \}$ and $B = \{ x, u_B(x), v_B(x) : x \in X \}$.

1) $A^c = \{ x, v_A(x), u_A(B(x)) : x \in X \}$
2) $A \cup B = \{ (x, max(u_A(x), u_B(x)), min(v_A(x), v_B(x)) ; x \in X \}$
3) $A \cap B = \{ (x, min(u_A(x), u_B(x)), max(v_A(x), v_B(x)) ; x \in X \}$

Definition 2.5 Let $X$ be a non-empty set and $I$ the unit interval $[0, 1]$. A Interval Valued Pythagorean fuzzy set $S$ is an object having the form

$$S$$

where the functions $u_A : X \to [0, 1]$ and $v_A : X \to [0, 1]$ denote respectively the degree of membership and degree of non-membership of each element $x \in X$ to the set $P$ and $0 \leq (u_A(x))^2 + (v_A(x))^2 \leq 1$. 

\[ \rho(A, B) = \frac{C(A, B)}{\sqrt{C(A, A) \cdot C(B, B)}} \]
3 Interval-valued Pythagorean fuzzy set

Definition 3.1 Let X be a non-empty set (universe). A set A on X is an object of the form \( A = \{ x, u_A(x), v_A(x) : x \in X \} \), where \( u_A(x), v_A(x) \in [0, 1], 0 = u_A(x) + v_A(x) \leq 1 \), for all \( x \) in \( X \). \( u_A(x) \) is the degree of membership, and \( v_A(x) \) is the degree of non-membership. Here \( u_A(x) \) and \( v_A(x) \) are dependent components.

Definition 3.2 Let \( X \) be a nonempty set and \( I \) be the unit interval \([0,1]\). Let object sets \( A = \{ x, u_A(x), v_A(x) : x \in X \} \) and \( B = \{ x, u_B(x), v_B(x) : x \in X \} \). Then,

1) \( A^c = \{ x, u_A(x), v_A(x) : x \in X \} \)
2) \( A \cup B = \{(x, \max(u_A(x), u_B(x)), \min(v_A(x), v_B(x))) ; x \in X \} \)
3) \( A \cap B = \{(x, \min(u_A(x), u_B(x)), \max(v_A(x), v_B(x))) ; x \in X \} \)

Proposition 3.3 The defined correlation measure between IVPS A and IVPS B satisfies the following properties

(i) \( 0 \rho(A, B) \leq 1 \)
(ii) \( \rho(A, B) = 1 \) if and only if \( A=B \)
(iii) \( \rho(A, B) = \rho(B, A) \).

Proof: (i) \( 0 \leq \rho(A, B) \leq 1 \)

As the membership, indeterminate and non-membership functions of the IVPS lies between 0 and 1, \( \rho(A, B) \), Also lies between 0 and 1. We will prove

\[
c(A, B) = \sum_{i=1}^{n} (u_A(x_i))^2 . (u_B(x_i))^2 + (v_A(x_i))^2 . (v_B(x_i))^2
\]

\[
= ((u_A(x_1))^2 . (u_B(x_1))^2 + (v_A(x_1))^2 . (v_B(x_1))^2 + ((u_A(x_2))^2 . (u_B(x_2))^2 + (v_A(x_2))^2 . (v_B(x_2))^2 + \cdots + ((u_A(x_n))^2 . (u_B(x_n))^2 + (v_A(x_n))^2 . (v_B(x_n))^2
\]

By Cauchy-Schwartz inequality

\[
(x_1y_1 + x_2y_2 + \cdots + x_ny_n)^2 = (x_1^2 + x_2^2 + \cdots + x_n^2)(y_1^2 + y_2^2 + \cdots + y_n^2),
\]

where \( (x_1 + x_2 + \cdots + x_n) \in \mathbb{R}^n \) and \((y_1 + y_2 + \cdots + y_n) \in \mathbb{R}^n \),
we get \( (c(A, B))^2 = ((u_A(x_1))^4 + (v_A(x_1))^4) + ((u_A(x_2))^4 + (v_A(x_2))^4)
+ \cdots + ((u_A(x_n))^4 + (v_A(x_n))^4) \times ((u_B(x_1))^4 + (v_B(x_1))^4)
+ ((u_B(x_2))^4 + (v_B(x_2))^4) + \cdots + ((u_B(x_n))^4 + (v_B(x_n))^4)
= (u_A(x_1))^2 . (u_A(x_1))^2 ((v_A(x_1))^2 . (v_A(x_1))^2) ((u_A(x_2))^2 . (u_A(x_2))^2
+ (v_A(x_2))^2 . (v_A(x_2))^2) + \cdots + ((u_A(x_n))^2 . (u_A(x_n))^2
+ (v_A(x_n))^2 . (v_A(x_n))^2)\)
\[ \rho(A, B) = \frac{C(A, B)}{\sqrt{C(A, A).C(B, B)}} = \frac{C(A, A)}{\sqrt{C(A, A).C(A, A)}} = 1. \]

Let the \( \rho(A, B) = 1 \). Then, the unite measure is possible only if

\[ \frac{C(A, B)}{\sqrt{C(A, A).C(B, B)}} = 1 . \] This refer that \( u_A(x_i) = u_B(x_i) \) and \( v_A(x_i) = v_B(x_i) \) for all \( i \) A=B

iii) If \( \rho(A, B) = \rho(B, A) \), it obvious that

\[ \frac{C(A, B)}{\sqrt{C(A, A).C(B, B)}} = \frac{C(A, B)}{\sqrt{C(A, A).C(B, B)}} = \rho(B, A) \]

\[ C(A, B) = \sum_{i=1}^{n}((u_A(x_i))^2.(u_B(x_i))^2 + (v_A(x_i))^2.(v_B(x_i))^2) \]

\[ = \sum_{i=1}^{n}((u_A(x_i))^2.(u_A(x_i))^2 + (v_B(x_i))^2.(v_B(x_i))^2) = C(B, A), \]

which completes the proof.
Definition 3.4 Let A and B be two IVPS. The correlation coefficient is defined as

\[ \rho' = \frac{C(A, B)}{\max\{C(A, A), C(B, B)\}} \] (2)

Theorem 3.5 The defined correlation measure between IVPS A and IVPS B satisfies the following properties

i) \(0 \leq \rho'(A, B) \leq 1\)

ii) \(\rho'(A, B) = 1\) if and only if \(A = B\)

iii) \(\rho'(A, B) = \rho'(B, A)\)

Proof: The property is straightforward, so omit here. Also \(\rho'(A, B) \geq 0\) is evident.

We now prove only \(\rho'(A, B) \leq 1\). Since \((C(A, B))^2 \leq C(A, A) \cdot C(B, B)\) we have \(C(A, B) \leq \max\{C(A, A), C(B, B)\}\) and thus \(\rho'(A, B) \leq 1\). However, in many practical situations, the different set may have taken different weights and weight \(\omega_i\) of the element \(x_i \in X(i = 1, 2, \cdots, n)\) should be taken into account. In the following, we develop a weighted correlation coefficient between IVPS.

Let \(\omega = \{\omega_1, \omega_2, \cdots, \omega_n\}\) be the weight vector of the elements \(x_i(i = 1, 2, \cdots, n)\) with \(\omega_i \geq 0\) and \(\sum_{i=1}^{n} \omega_i = 1\), then we have extended the above correlation coefficient \(\rho(A, B)\) and \(\rho'(A, B)\) to weighted correlation coefficient as follows:

\[ \rho'' = \frac{C_{\omega}(A, B)}{\sqrt{C_{\omega}(A, A) \cdot C_{\omega}(B, B)}} \] (3)

\[ C_{\omega}(A, B) = \sum_{i=1}^{n} \omega_i ((u_A(x_i))^2, (u_B(x_i))^2, (v_A(x_i))^2, (v_B(x_i))^2) \]

\[ C_{\omega}(A, A) = \sum_{i=1}^{n} \omega_i ((u_A(x_i))^2, (u_A(x_i))^2, (v_A(x_i))^2, (v_A(x_i))^2) \]

\[ C_{\omega}(B, B) = \sum_{i=1}^{n} \omega_i ((u_B(x_i))^2, (u_B(x_i))^2, (v_B(x_i))^2, (v_B(x_i))^2) \]

and

\[ \rho'' = \frac{C_{\omega}(A, B)}{\max\{C_{\omega}(A, A) \cdot C_{\omega}(B, B)\}} \] (4)
Let \( \omega = (\omega_1, \omega_2, \cdots, \omega_n)^T \) be the weight vector of \( x_i (i = 1, 2, \ldots, n) \) with \( \omega_i \geq 0 \) and \( \sum_{i=1}^{n} \omega_i = 1 \), then the weighted correlation coefficient between IVPS A and IVPS B satisfies:

i) \( 0 \leq \rho'' (A, B) \leq 1 \)

ii) \( \rho'' (A, B) = 1 \) if and only if \( A = B \)

iii) \( \rho'' (A, B) = \rho'' (B, A) \)

**Proof:** The property (i) and (ii) are straightforward. Also \( \rho'' (A, B) \geq 0 \) is evident so we need to show \( \rho'' (A, B) \leq 1 \). Since,

\[
C_{\omega} (A, B) = \sum_{i=1}^{n} \omega_i ((u_A(x_i))^2(u_B(x_i))^2 + (v_A(x_i))^2(v_B(x_i))^2)
\]

\[
= \omega_1 ((u_A(x_1))^2(u_B(x_1))^2 + (v_A(x_1))^2(v_B(x_1))^2 + \omega_2 (u_A(x_2))^2(u_B(x_2))^2 + (v_A(x_2))^2(v_B(x_2))^2 + \cdots + \omega_n (u_A(x_n))^2(u_B(x_n))^2 + (v_A(x_n))^2(v_B(x_n))^2
\]

\[
= (\sqrt{\omega_1 (u_A(x_1))^2} \sqrt{\omega_1 (u_B(x_1))^2} + \sqrt{\omega_1 (v_A(x_1))^2} \sqrt{\omega_1 (v_B(x_1))^2})
\]

\[
+ (\sqrt{\omega_2 (u_A(x_2))^2} \sqrt{\omega_2 (u_B(x_2))^2} + \sqrt{\omega_2 (v_A(x_2))^2} \sqrt{\omega_2 (v_B(x_2))^2})
\]

\[
+ \cdots + (\sqrt{\omega_n (u_A(x_n))^2} \sqrt{\omega_n (u_B(x_n))^2} + \sqrt{\omega_n (v_A(x_n))^2} \sqrt{\omega_n (v_B(x_n))^2})
\]

By using Cauchy-Schwarz inequality, we get

\[
C_{\omega} (A, B)^2 = (w_1 (u_A(x_1))^2(u_A(x_1))^2 + (v_A(x_1))^2(v_A(x_1))^2)
\]

\[
+ (w_2 (u_A(x_2))^2(u_A(x_2))^2 + (v_A(x_2))^2(v_A(x_2))^2)
\]

\[
+ \cdots + (w_n (u_A(x_n))^2(u_A(x_n))^2
\]

\[
+(v_A(x_n))^2(v_A(x_n))^2) \times (w_1 (u_B(x_1))^2(u_B(x_1))^2 + (v_B(x_1))^2(v_B(x_1))^2)
\]

\[
+(v_B(x_1))^2(v_B(x_1))^2) + (w_2 (u_B(x_2))^2(u_B(x_2))^2 + (v_B(x_2))^2(v_B(x_2))^2)
\]

\[
+(v_B(x_2))^2(v_B(x_2))^2) + \cdots + (w_n (u_B(x_n))^2(u_B(x_n))^2 + (v_B(x_n))^2(v_B(x_n))^2)
\]

\[
\leq 1.
\]
\[ + \cdots + (w_n(u_B(x_n))^2(u_B(x_n))^2 + (v_B(x_n))^2(v_B(x_n))^2) \]
\[ = \sum_{i=1}^{n} w_i((u_A(x_i))^2(u_A(x_i))^2 + (v_A(x_i))^2(v_A(x_i))^2) \]
\[ \times \sum_{i=1}^{n} w_i((u_B(x_i))^2(u_B(x_i))^2 + (v_B(x_i))^2(v_B(x_i))^2) \]
\[ = C_\omega(A, A) \times C_\omega(B, B) \]

Therefore \[ C_\omega(A, B) \leq \sqrt{C_\omega(A, A) \times C_\omega(B, B)} \] and hence \[ 0 \leq \rho''(A, B) \leq 1. \]

**Theorem 3.7** The correlation coefficient of IVPS A and IVPS B defined in \[ \rho''(A, B) \] satisfies the properties as given in Theorem 3.6.

**Proof:** \[ \rho(A, B) \] and \[ \rho'(A, B) \] to weighted correlation coefficient as follows:

\[ \rho'' = \frac{C_\omega(A, B)}{\sqrt{C_\omega(A, A) \cdot C_\omega(B, B)}} \] (5)

\[ C_\omega(A, B) = \sum_{i=1}^{n} \omega_i((u_A(x_i))^2(u_B(x_i))^2 + (v_A(x_i))^2(v_B(x_i))^2) \]
\[ C_\omega(A, A) = \sum_{i=1}^{n} \omega_i((u_A(x_i))^2(u_A(x_i))^2 + (v_A(x_i))^2(v_A(x_i))^2) \]
\[ C_\omega(B, B) = \sum_{i=1}^{n} \omega_i((u_B(x_i))^2(u_B(x_i))^2 + (v_B(x_i))^2(v_B(x_i))^2) \]

And

\[ \rho'' = \frac{C_\omega(A, B)}{\max\{C_\omega(A, A), C_\omega(B, B)\}} \] (6)

\[ = \frac{\sum_{i=1}^{n} \omega_i((u_A(x_i))^2(u_B(x_i))^2 + (v_A(x_i))^2(v_B(x_i))^2)}{\max\{\sum_{i=1}^{n} \omega_i((u_A(x_i))^2(u_A(x_i))^2 + (v_A(x_i))^2(v_A(x_i))^2), \sum_{i=1}^{n} \omega_i((u_B(x_i))^2(u_B(x_i))^2 + (v_B(x_i))^2(v_B(x_i))^2)\}} \]

It is easy to verify that if \[ \omega = (\frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n})^T \], then equation (6) and (5) reduces (1) and (2) respectively.

**Example 3.8** Let \[ P = \{P_1, P_2, P_3\} \] be a set of patients,
\[ D = \{\text{Viral Fever, Malaria, Typhoid, Dengue}\} \] be a set of diseases and
\[ S = \{\text{Temperature, Headache, Cough, Joint Pain}\} \] be a set of symptoms
**Table 1:** M (the relation between Patient and Symptoms)

<table>
<thead>
<tr>
<th>M</th>
<th>Temperature</th>
<th>Headache</th>
<th>Cough</th>
<th>Joint Pain</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_1$</td>
<td>(0.7,0.6);(0.5,0.7)</td>
<td>(0.4,0.6);(0.7,0.7)</td>
<td>(0.5,0.7);(0.3,0.5)</td>
<td>(0.2,0.4);(0.1,0.3)</td>
</tr>
<tr>
<td>$P_2$</td>
<td>(0.1,0.3);(0.8,0.9)</td>
<td>(0.4,0.6);(0.7,0.7)</td>
<td>(0.3,0.5);(0.2,0.4)</td>
<td>(0.1,0.2);(0.8,0.9)</td>
</tr>
<tr>
<td>$P_3$</td>
<td>(0.2,0.4);(0.4,0.6)</td>
<td>(0.7,0.8);(0.5,0.5)</td>
<td>(0.3,0.5);(0.8,0.8)</td>
<td>(0.4,0.6);(0.1,0.3)</td>
</tr>
</tbody>
</table>

**Table 2:** N (the relation between Symptoms and Diseases)

<table>
<thead>
<tr>
<th>N</th>
<th>Viral Fever</th>
<th>Malaria</th>
<th>Typhoid</th>
<th>Dengu</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature</td>
<td>(0.8,0.7);(0.3,0.5)</td>
<td>(0.4,0.6);(0.5,0.7)</td>
<td>(0.7,0.8);(0.3,0.5)</td>
<td>(0.1,0.3);(0.4,0.6)</td>
</tr>
<tr>
<td>Headache</td>
<td>(0.1,0.2);(0.2,0.4)</td>
<td>(0.4,0.6);(0.6,0.8)</td>
<td>(0.3,0.4);(0.8,0.8)</td>
<td>(0.8,0.9);(0.2,0.4)</td>
</tr>
<tr>
<td>Cough</td>
<td>(0.2,0.4);(0.7,0.9)</td>
<td>(0.8,0.8);(0.3,0.5)</td>
<td>(0.1,0.2);(0.8,0.9)</td>
<td>(0.4,0.5);(0.7,0.7)</td>
</tr>
<tr>
<td>Joint Pain</td>
<td>(0.6,0.8);(0.4,0.5)</td>
<td>(0.7,0.8);(0.5,0.6)</td>
<td>(0.4,0.6);(0.5,0.7)</td>
<td>(0.1,0.2);(0.7,0.9)</td>
</tr>
</tbody>
</table>

Using Equations (1), we get the value of (A,B)

**Table 3:** M and N (Correlation Measure)

<table>
<thead>
<tr>
<th>M</th>
<th>Viral Fever</th>
<th>Malaria</th>
<th>Typhoid</th>
<th>Dengu</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_1$</td>
<td>0.5089</td>
<td><strong>0.78106</strong></td>
<td><strong>0.8256</strong></td>
<td>0.5363</td>
</tr>
<tr>
<td>$P_2$</td>
<td>0.3745</td>
<td>0.6540</td>
<td>0.6112</td>
<td><strong>0.7383</strong></td>
</tr>
<tr>
<td>$P_3$</td>
<td><strong>0.67540</strong></td>
<td>0.6582</td>
<td>0.8023</td>
<td>0.7721</td>
</tr>
</tbody>
</table>

Using Equations (2), we get the value of $\rho(A,B)$

**Table 4:** M and N (Correlation Measure)
On the other hand, if we assign weights 0.10, 0.20, 0.30 and 0.40 respectively, then by applying correlation coefficient given in equations (3) and (4), we can give the following values of the correlation coefficient:

Using Equations (3), we get the value of \( \rho''(A,B) \).

**Table 5: M and N (Correlation Measure)**

<table>
<thead>
<tr>
<th>M</th>
<th>Viral Fever</th>
<th>Malaria</th>
<th>Typhoid</th>
<th>Dengu</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_1 )</td>
<td>0.4396</td>
<td>0.78106</td>
<td>0.8256</td>
<td>0.5363</td>
</tr>
<tr>
<td>( P_2 )</td>
<td>0.3745</td>
<td>0.6540</td>
<td>0.6112</td>
<td>0.7383</td>
</tr>
<tr>
<td>( P_3 )</td>
<td>0.67540</td>
<td>0.6582</td>
<td>0.8023</td>
<td>0.7721</td>
</tr>
</tbody>
</table>

Using Equations (4), we get the value of \( \rho''(A,B) \).

**Table 6: M and N (Correlation Measure)**

<table>
<thead>
<tr>
<th>M</th>
<th>Viral Fever</th>
<th>Malaria</th>
<th>Typhoid</th>
<th>Dengu</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_1 )</td>
<td>0.55205</td>
<td>0.5826</td>
<td>0.6760</td>
<td>0.5180</td>
</tr>
<tr>
<td>( P_2 )</td>
<td>0.3169</td>
<td>0.6222</td>
<td>0.35476</td>
<td>0.7977</td>
</tr>
<tr>
<td>( P_3 )</td>
<td>0.7550</td>
<td>0.704508</td>
<td>0.72632</td>
<td>0.78140</td>
</tr>
</tbody>
</table>

The highest correlation measure from the Tables 3, 4, 5 and 6 give the proper medical diagnosis. Therefore, patient \( P_1 \) suffers from Viral Fever, patient \( P_2 \) suffers from Malaria and patient \( P_3 \) suffers from Dengu. Hence, we can see from the above four kinds of correlation coefficient indices that the results are same.
4 Conclusion

In this paper, we found the correlation measure of Interval valued Pythagorean set [IVPS] and proved some of their basic properties. Based on that the present paper, we have extended the theory of correlation coefficient to the Pythagorean in which the constraint condition of sum of membership, non-membership and indeterminacy be less. Illustrate examples have handle the situation where the existing correlation coefficient in NS environment fails. Also to deal with the situations where the elements in a set are correlative, a weighted correlation coefficients has been defined. We studied an application of correlation measure of Interval valued Pythagorean set.

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