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Correlation Measure for Interval valued Pythagorean Fuzzy Sets

Saji Antonia S[1] and Mohana K[*,2]

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Abstract

In this paper, we study the concept of Interval Valued Pythagorean set [IVPS] with special cases of membership and non-membership degrees. In this IVPS set membership and non-membership degrees are satisfying the condition $(u_A(x))^2 + (\nu_A(x))^2 \leq 1$ instead of $u_A(x) + \nu_A(x) > 1$. Here the correlation measure of IVPS set is an extension of correlation measure of Interval valued Pythagorean fuzzy set.

Key words: Interval-Valued Pythagorean Fuzzy Set, Correlation Measures.

1 Introduction

Fuzzy sets are initiated by L.A.Zadeh. The extension of fuzzy set (IFS) describes the non-membership grade of an membership grades under a restriction that the sum of membership and non-membership grade of an imprecise does not exceed 1.

Based on the centric method, Hanafy introduced and studied the correlation and correlation coefficient. Correlation coefficients are beneficial tools used to determine the degree of similarity between objects. The correlation coefficients in fuzzy environments have many applications in the field of medicals diagnosis and clustering etc.

2 Preliminaries

Definition 2.1 Let X be a non empty set and I the unit interval [0,1]. A interval valued Pythagorean set A and B of the form $A = \{x, u_A(x), v_A(x) : x \in X\}$ and

[1,2]Department of Mathematics, Nirmala College for Women, Coimbatore.
Emails: [1]sajiantonia@gmail.com, [*,2]riyaraju116@gmail.com

$B = \{x, u_B(x), v_B(x) : x \in X\}$. Then the correlation coefficient of A and B is

$$\rho(A, B) = \frac{C(A, B)}{\sqrt{C(A, A).C(B, B)}} \tag{1}$$

$$C(A, B) = \sum_{i=1}^{n} ((u_A(x_i))^2.(u_B(x_i))^2 + (v_A(x_i))^2.(v_B(x_i))^2)$$

$$C(A, A) = \sum_{i=1}^{n} ((u_A(x_i))^2.(u_A(x_i))^2 + (v_A(x_i))^2.(v_A(x_i))^2)$$

$$C(B, B) = \sum_{i=1}^{n} ((u_B(x_i))^2.(u_B(x_i))^2 + (v_B(x_i))^2.(v_B(x_i))^2)$$

Definition 2.2 Let E be an universe set. An Intuitionistic fuzzy set A on E can be defined as follows:
$A = \{< x, u_A(x), v_A(x) >, x \in E\}$, where $u_A : E \to [0, 1]$ and $v_A : E \to [0, 1]$ such that $0 = u_A(x) + v_A \leq 1$ for any $x \in E$. $u_A(x)$ and $v_A(x)$ are the degree of membership and degree of non-membership of the element x respectively.

Definition 2.3 Let X is a non-empty set (universe). A Interval valued Pythagorean fuzzy set A on X is an object of the form $A = \{(x, u_A(x), v_A(x)) : x \in x\}$, where $u_A(x), v_A(x) \in [0, 1], 0 \leq (u_A(x))^2 + v_A(x))^2 \leq 1$ for all x in X. $u_A(x)$ is the degree of membership, and $v_A(x)$ is the degree of non-membership.

Definition 2.4 Let X be a non-empty set and I the unit interval [0, 1]. A Interval-valued Pythagorean fuzzy sets A and B are of the form $A = \{x, u_A(x), v_A(x) : x \in X\}$and $B = \{x, u_B(x), v_B(x) : x \in X\}$.
1) $A^c = \{x, v_A(x), u_A(B(x)) : x \in X\}$
2) $A \cup B = \{(x, max(u_A(x), u_B(x)), min(v_A(x), v_B(x))); x \in X\}$
3) $A \cap B = \{(x, min(u_A(x), u_B(x)), max(v_A(x), v_B(x))); x \in X\}$

Definition 2.5 Let X be a non-empty set and I the unit interval [0, 1]. A Interval Valued Pythagorean fuzzy set S is an object having the form
$A = \{x, u_A(x), v_A(x) : x \in X\}$, where the functions $u_A : X \to [0, 1]$ and
$v_A : X \to [0, 1]$ denote respectively the degree of membership and degree of non-membership of each element $x \in X$ to the set P and $0 \leq (u_A(x))^2 + (v_A(x))^2 \leq 1$.

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3 Interval- valued Pythagorean fuzzy set

Definition 3.1 Let X be a non-empty set (universe). A set A on X is an object of the form $A = \{x, u_A(x), v_A(x) : x \in X\}$, where $u_A(x), v_A(x) \in [0, 1], 0 = u_A(x) + v_A(x) \leq 1$, for all x in X. $u_A(x)$ is the degree of membership, and $v_A(x)$ is the degree of non-membership. Here $u_A(x)$ and $v_A(x)$ are dependent components.

Definition 3.2 Let X be a nonempty set and I be the unit interval [0,1]. Let object sets $A = \{x, u_A(x), v_A(x) : x \in X\}$ and $B = \{x, u_B(x), v_B(x) : x \in X\}$. Then,
1) $A^c = \{x, u_A(x), v_A(x) : x \in X\}$
2) $A \cup B = \{(x, max(u_A(x), u_A(x)), min(v_A(x), v_A(x))); x \in X\}$
3) $A \cap B = \{(x, min(u_A(x), u_A(x)), max(v_A(x), v_A(x))); x \in X\}$

Proposition 3.3 The defined correlation measure between IVPS A and IVPS B satisfies the following properties
(i) $0\rho(A, B) \leq 1$
(ii)$\rho(A, B) = 1$ if and only if A=B
(iii)$\rho(A, B) = \rho(B, A)$.
Proof: (i) $0 \leq \rho(A, B) \leq 1$
As the membership, indeterminate and non-membership functions of the IVPS lies between 0 and 1, $\rho(A, B)$, Also lies between 0 and 1. We will prove

$$c(A, B) = \sum_{i=1}^{n} (u_A(x_i))^2.(u_B(x_i))^2 + (v_A(x_i))^2.(v_B(x_i))^2$$

$$= ((u_A(x_1))^2.(u_B(x_1))^2 + (v_A(x_1))^2.(v_B(x_1))^2 + ((u_A(x_2))^2.(u_B(x_2))^2 + (v_A(x_2))^2.(v_B(x_2))^2 + \cdots + ((u_A(x_n))^2.(u_B(x_n))^2 + (v_A(x_n))^2.(v_B(x_n))^2)$$

By Cauchy-Schwartz inequality
$(x_1y_1 + x_2y_2 + \cdots + x_ny_n)^2 = (x_1^2 + x_2^2 + \cdots + x_n^2).(y_1^2 + y_2^2 + \cdots + y_n^2)$,
where $(x_1 + x_2 + \cdots x_n) \in R^n$and $(y_1 + y_2 + \cdots y_n) \in R^n$,
we get $(c(A, B))^2 = ((u_A(x_1))^4 + (v_A(x_1))^4) + (u_A(x_2))^4 + (v_A(x_2))^4)$

$$+ \cdots + ((u_A(x_1))^4 + (v_A(x_n))^2) \times ((u_B(x_1))^4 + (v_B(x_1))^4) + ((u_B(x_2))^4 + (v_B(x_n))^4) + \cdots + ((u_B(x_n))^4 + (v_B(x_n))^4)$$

$$= (u_A(x_1))^2.(u_A(x_1))^2 ((v_A(x_1))^2.(v_A(x_1))^2) ((u_A(x_2))^2.(u_A(x_2))^2 + (v_A(x_2))^2.(v_A(x_2))^2) + \cdots + ((u_A(x_n))^2.(u_A(x_n))^2$$

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$$+ (v_A(x_n))^2.(v_A(x_n))^2) \times ((u_B(x_1))^2.(u_B(x_1))^2 + (v_B(x_1))^2.(v_B(x_1))^2)$$
$$+ ((u_B(x_2))^2.(u_B(x_2))^2 + (v_B(x_2))^2.(v_B(x_2))^2$$
$$+ \cdots + ((u_B(x_n))^2.((u_B(x_n))^2 + (v_B(x_n))^2.(v_B(x_n))^2)$$
$$= C(A, A) \times C(B, B)$$

Therefore $(C(A, B))^2 \leq C(A, A) \times C(B, B)$ and thus $\rho(A, B) \leq 1$. Hence we obtain the following property $0 \leq \rho(A, B) \leq 1$

(ii)$\rho(A, B)$=1 if and only if A=B

Let the two IVPS A and IVPS B be equal (i.e. A=B). Hence for any $u_A(x_i) = u_B(x_i)$ and $v_A(x_i) = v_B(x_i)$ Then

$C(A, A) = C(B, B) = \sum_{i=1}^{n} ((u_A(x_i))^2.(u_A(x_i))^2 + (v_A(x_i))^2.(v_A(x_i))^2)$ and

$C(A, B) = \sum_{i=1}^{n} ((u_A(x_i))^2.(u_B(x_i))^2 + (v_A(x_i))^2.(v_B(x_i))^2$

$$= \sum_{i=1}^{n} (u_A(x_i))^2.(u_A(x_i))^2 + (v_A(x_i))^2.\ (v_A(x_i))^2 = C(A, A)$$

Hence, $\rho(A, B) = \dfrac{C(A, B)}{\sqrt{C(A, A).C(B, B)}} = \dfrac{C(A, A)}{\sqrt{C(A, A)}.C(A, A)} = 1.$

Let the $\rho\,(A, B) = 1$. Then, the unite measure is possible only if

$\dfrac{C(A, B)}{\sqrt{C(A, A).C(B, B)}} = 1$ .This refer that $u_A(x_i) = u_B(x_i)$ and $v_A(x_i) = v_B(x_i)$ for all i A=B

iii) If$\rho(A, B) = \rho(B, A)$, it obvious that

$$\frac{C(A, B)}{\sqrt{C\,(A, A)\,.C(B, B)}} = \frac{C(A, B)}{\sqrt{C\,(A, A)\,.C(B, B)}} = \rho(B, A)$$

$$C\,(A, B) = \sum_{i=1}^{n} ((u_A\,(x_i))^2.(u_B\,(x_i))^2 + (v_A\,(x_i))^2.(v_A\,(x_i))^2)$$

$$= \sum_{i=1}^{n} ((u_A\,(x_i))^2.(u_A\,(x_i))^2 + (v_B\,(x_i))^2.\left(v_A\,(x_i))^2\right) = C(B, A),$$

which completes the proof.

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Definition 3.4 Let A and B be two IVPS. The correlation coefficient is defined as

$$\rho^{'} = \frac{C(A,B)}{max\{C\,(A,A)\,.C(B,B)\}} \tag{2}$$

Theorem 3.5 The defined correlation measure between IVPS A and IVPS B satisfies the following properties

i)$0 \leq \rho^{'}(A,B) \leq 1$

ii) $\rho^{'}(A,B)$=1 if and only if A=B

iii)$\rho^{'}\,(A,B) = \rho^{'}\,(B,A)$

Proof: The property is straight forward, so omit here. Also $\rho^{'}\,(A,B) \geq 0$ is evident. We now prove only $\rho^{'}\,(A,B) \leq 1$. Since $((C\,(A,B))^2 \leq C\,(A,A)\,.C(B,B)$ we have $C\,(A,B) \leq max\{C\,(A,A)\,, C\,(B,B)\}$ and thus $\rho^{'}\,(A,B) \leq 1$. However, in many practical situations, the different set may have taken different weights and weight ω_i of the element $x_i \in X(i = 1, 2, \cdots n)$ should be taken into account. In the following, we develop a weighted correlation coefficient between IVPS.

Let $\omega = \{\omega_1, \omega_2, \cdots \omega_n\}$ be the weight vector of the elements $x_i(i = 1, 2, .., n)$ with $\omega_i \geq 0$ and $\sum\limits_{i=1}^{n} \omega_i = 1$, then we have extended the above correlation coefficient $\rho\,(A,B)$ and $\rho^{'}\,(A,B)$ to weighted correlation coefficient as follows:

$$\rho^{''} = \frac{C_\omega(A,B)}{\sqrt{C_\omega\,(A,A)\,.C_\omega(B,B)}} \tag{3}$$

$$C_\omega\,(A,B) = \sum_{i=1}^{n} \omega_i((u_A\,(x_i))^2.(u_B\,(x_i))^2 + (v_A\,(x_i))^2.(v_B\,(x_i))^2$$

$$C_\omega\,(A,A) = \sum_{i=1}^{n} \omega_i((u_A\,(x_i))^2.(u_A\,(x_i))^2 + (v_A\,(x_i))^2.(v_A\,(x_i))^2$$

$$C_\omega\,(B,B) = \sum_{i=1}^{n} \omega_i((u_B\,(x_i))^2.(u_B\,(x_i))^2 + (v_B\,(x_i))^2.(v_B\,(x_i))^2$$

and

$$\rho^{''} = \frac{C_\omega(A,B)}{\max\{C_\omega\,(A,A)\,.C_\omega\,(B,B)\}} \tag{4}$$

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$$= \frac{\sum_{i=1}^{n} \omega_i((u_A(x_i))^2.(u_B(x_i))^2 + (v_A(x_i))^2.(v_B(x_i))^2)}{\max\{\begin{matrix} \sum_{i=1}^{n} \omega_i((u_A(x_i))^2.(u_A(x_i))^2+(v_A(x_i))^2.(v_A(x_i))^2 \\ \sum_{i=1}^{n} \omega_i((u_B(x_i))^2.(u_B(x_i))^2+(v_B(x_i))^2.(v_B(x_i))^2 \end{matrix}\}}$$

It is easy to verify that if $\omega = (\frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n})^T$, then equations (3) and (4) reduces to (1) and (2) respectively.

Theorem 3.6 Let $\omega = (\omega_1, \omega_2, \cdots, \omega_n)^T$ be the weight vector of $x_i (i = 1, 2..n)$ with $\omega_i \geq 0$ and $\sum_{i=1}^{n} \omega_i = 1$,then the weighted correlation coefficient between IVPS A and IVPS B satisfies:

i) $0 \leq \rho''(A, B) \leq 1$

ii)$\rho''(A, B) = 1$ if and only if A=B

iii)$\rho''(A, B) = \rho''(B, A)$

Proof: The property (i) and (ii) are straight forward .Also $\rho''(A, B) \geq 0$ is evident so we need to show $\rho''(A, B) \leq 1$ .Since,

$$C_\omega(A, B) = \sum_{i=1}^{n} \omega_i((u_A(x_i))^2.(u_B(x_i))^2 + (v_A(x_i))^2.(v_B(x_i))^2)$$

$$= \omega_1((u_A(x_1))^2.(u_B(x_1))^2 + (v_A(x_1))^2.(v_B(x_1) + \omega_2(u_A(x_2))^2.(u_B(x_2))^2$$

$$+ (v_A(x_2))^2.(v_B(x_2))^2 + \cdots + \omega_n((u_A(x_n))^2.(u_B(x_n))^2 + (v_A(x_n))^2.(v_B(x_n))^2$$

$$= (\sqrt{\omega_1(}u_A(x_1))^2.\sqrt{\omega_1(}u_B(x_1))^2 + \sqrt{\omega_1(}v_A(x_1))^2.\sqrt{\omega_1}(v_B(x_1))^2)$$

$$+ (\sqrt{\omega_2(}u_A(x_2))^2.\sqrt{\omega_2(}u_B(x_2))^2(\sqrt{\omega_2(}v_A(x_2))^2.\sqrt{\omega_2}(v_B(x_2))^2)$$

$$+.. + (\sqrt{\omega_n(}u_A(x_n))^2.\sqrt{\omega_n(}u_B(x_n))^2 + \sqrt{\omega_n(}v_A(x_n))^2.\sqrt{\omega_n}(v_B(x_n))^2)$$

By using Cauchy-Schwarz inequality, we get

$$C_w(A, B))^2 = \left(w_1(u_A(x_1))^2.(u_A(x_1))^2 + (v_A(x_1))^2.(v_A(x_1))^2\right)$$

$$+ \left(w_2(u_A(x_2))^2.(u_A(x_2))^2 + (v_A(x_2))^2.(v_A(x_2))^2\right)$$

$$+ \cdots + \left(w_n(u_A(x_n))^2.(u_A(x_n))^2\right.$$

$$\left.+(v_A(x_n))^2.(v_A(x_n))^2\right) \times \left(w_1(u_B(x_1))^2.(u_B(x_1))^2\right.$$

$$\left.+(v_B(x_1))^2.(v_B(x_1))^2\right) + \left(w_2(u_B(x_2))^2.(u_B(x_2))^2 + (v_B(x_2))^2.(v_B(x_2))^2\right)$$

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$$+\cdots+\left(w_n(u_B(x_n))^2.(u_B(x_n))^2+(v_B(x_n))^2.(v_B(x_n))^2\right)$$

$$=\sum_{i=1}^{n} w_i((u_A(x_i))^2.(u_A(x_i))^2+(v_A(x_i))^2(v_A(x_i))^2$$

$$\times\sum_{i=1}^{n} w_i((u_B(x_i))^2.(u_B(x_i))^2+(v_B(x_i))^2(v_B(x_i))^2$$

$$=C_\omega(A,A)\times C_\omega(B,B)$$

Therefore $C_\omega\,(A,B)\leq\sqrt{C_\omega(A,A)\times C_\omega(B,B)}$ and hence $0\leq\,\rho^{''}(A,B)\leq 1$.

Theorem 3.7 The correlation coefficient of IVPS A and IVPS B defined in $\rho^{''}\,(A,B)$ satisfies the properties as given in Theorem 3.6.
Proof: $\rho\,(A,B)$ and $\rho^{'}\,(A,B)$ to weighted correlation coefficient as follows:

$$\rho^{''}=\frac{C_\omega(A,B)}{\sqrt{C_\omega\,(A,A)\,.C_\omega(B,B)}}\tag{5}$$

$$C_\omega\,(A,B)=\sum_{i=1}^{n}\omega_i((u_A\,(x_i))^2.(u_B\,(x_i))^2+(v_A\,(x_i))^2.(v_B\,(x_i))^2$$

$$C_\omega\,(A,A)=\sum_{i=1}^{n}\omega_i((u_A\,(x_i))^2.(u_A\,(x_i))^2+(v_A\,(x_i))^2.(v_A\,(x_i))^2$$

$$C_\omega\,(B,B)=\sum_{i=1}^{n}\omega_i((u_B\,(x_i))^2.(u_B\,(x_i))^2+(v_B\,(x_i))^2.(v_B\,(x_i))^2$$

And

$$\rho^{''}=\frac{C_\omega(A,B)}{\max\{C_\omega\,(A,A)\,.C_\omega\,(B,B)\}}\tag{6}$$

$$=\frac{\sum_{i=1}^{n}\omega_i((u_A\,(x_i))^2.(u_B\,(x_i))^2+(v_A\,(x_i))^2.(v_B\,(x_i))^2)}{\max\{\frac{\sum_{i=1}^{n}\omega_i((u_A(x_i))^2.(u_A(x_i))^2+(v_A(x_i))^2.(v_A(x_i))^2}{\sum_{i=1}^{n}\omega_i((u_B(x_i))^2.(u_B(x_i))^2+(v_B(x_i))^2.(v_B(x_i))^2}\}}$$

It is easy to verify that if $\omega=(\frac{1}{n},\frac{1}{n},\cdots,\frac{1}{n})^T$, then equation (6) and (5) reduces (1) and (2) respectively.

Example 3.8 Let P=$\{P_1,P_2,P_3\}$ *be* a set of patients,
D={Viral Fever,Malaria,Typoid,Dengu} be a set of diseases and
S={Tempreaure,Headache,Cough,Joint Pain}be a set of symptoms

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Table 1:M (the relation between Patient and Symptoms)

M	Temperature	Headache	Cough	Joint Pain
P_1	(0.7,0.6);(0.5,0.7)	(0.4,0.6);(0.7,0.7)	(0.5,0.7);(0.3,0.5)	(0.2,0.4);(0.1,0.3)
P_2	(0.1,0.3);(0.8,0.9)	(0.4,0.6);(0.7,0.7)	(0.3,0.5);(0.2,0.4)	(0.1,0.2);(0.8,0.9)
P_3	(0.2,0.4);(0.4,0.6)	(0.7,0.8);(0.5,0.5)	(0.3,0.5);(0.8,0.8)	(0.4,0.6);(0.1,0.3)

Table 2:N(the relation between Symptoms and Diseases)

N	Viral Fever	Malaria	Typhoid	Dengu
Temperature	(0.8,0.7);(0.3,0.5)	(0.4,0.6);(0.5,0.7)	(0.7,0.8);(0.3,0.5)	(0.1,0.3);(0.4,0.6)
Headache	(0.1,0.2);(0.2,0.4)	(0.4,0.6);(0.6,0.8)	(0.3,0.4);(0.8,0.8)	(0.8,0.9);(0.2,0.4)
Cough	(0.2,0.4);(0.7,0.9)	(0.8,0.8);(0.3,0.5)	(0.1,0.2);(0.8,0.9)	(0.4,0.5);(0.7,0.7)
Joint Paint	(0.6,0.8);(0.4,0.5)	(07,0.8);(0.5,0.6)	(0.4,0.6);(0.5,0.7)	(0.1,0.2);(0.7,0.9)

Using Equations (1), we get the value of (A,B)

Table 3: M and N (Correlation Measure)

M	Viral Fever	Malaria	Typhoid	Dengu
P_1	0.5089	***0.78106***	***0.8256***	0.5363
P_2	0.3745	0.6540	0.6112	***0.7383***
P_3	***0.67540***	0.6582	0.8023	0.7721

Using Equations (2), we get the value of $\rho'(A,B)$

Table 4: M and N (Correlation Measure)

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M	Viral Fever	Malaria	Typhoid	Dengu
P_1	***0.4396***	0.78106	0.8256	0.5363
P_2	0.3745	***0.6540***	0.6112	0.7383
P_3	0.67540	0.6582	0.8023	***0.7721***

On the other hand, if we assign weights 0.10, 0.20, 0.30 and 0.40 respectively, then by applying correlation coefficient given in equations (3) and (4), we can give the following values of the correlation coefficient:

Using Equations (3), we get the value of $\rho''(A,B)$.

Table 5: M and N (Correlation Measure)

M	Viral Fever	Malaria	Typhoid	Dengu
P_1	0.55205	05826	0.6760	0.5180
P_2	0.3169	***0.6222***	0.35476	***0.7977***
P_3	***0.7550***	0.704508	***0.72632***	0.78140

Using Equations (4), we get the value of $\rho''(A,B)$.

Table 6: M and N (Correlation Measure)

M	Viral Fever	Malaria	Typhoid	Dengu
P_1	0.4014	0.4405	0.4427	0.33477
P_2	0.2967	***0.5826***	0.3544	0.7581
P_3	0.71321	0.55140	0.70868	***0.73417***

The highest correlation measure from the Tables 3, 4, 5 and 6 give the proper medical diagnosis. Therefore, patient P_1 suffers from Viral Fever, patient P_2 suffers from Malaria and patient P_3 suffers from Dengu. Hence, we can see from the above four kinds of correlation coefficient indices that the results are same.

4 Conclusion

In this paper, we found the correlation measure of Interval valued Pythagorean set [IVPS] and proved some of their basic properties. Based on that the present paper, we have extended the theory of correlation coefficient to the Pythagorean in which the constraint condition of sum of membership, non-membership and indeterminacy be less. Illustrate examples have handle the situation where the existing correlation coefficient in NS environment fails. Also to deal with the situations where the elements in a set are correlative, a weighted correlation coefficients has been defined. We studied an application of correlation measure of Interval valued Pythagorean set.

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