

Pentapartitioned Neutrosophic Generalized Semi-closed Sets

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Abstract

The aim of the paper is to introduce and investigate the concepts of Pentapartitioned neutrosophic generalized semi-closed sets (PNGSCS) and Pentapartitioned neutrosophic generalized semi-open sets (PNGSOS) in Pentapartitioned neutrosophic topological space.

Key words: Pentapartitioned Neutrosophic Set, Pentapartitioned Neutrosophic Generalized Semi-Closed Sets, Pentapartitioned Neutrosophic Generalized Semi-Open Sets.

AMS classification: 03E72, 03B52

1 Introduction

The fuzzy set was initiated by Zadeh [18] in 1965, where each element had a degree of membership. The neutrosophic set was introduced by Smarandache [16] and explained, neutrosophic set is a generalization of intuitionistic fuzzy set. They introduced neutrosophic topological space as a generalization of intuitionistic fuzzy topological space and a neutrosophic set besides the degree of membership, the degree of indeterminacy and therefore the degree of non-membership of each element. For the past few years, many researchers were going on in neutrosophic topological spaces and many concepts in intuitionistic fuzzy topology were extended to neutrosophic topology. Rama Malik and Surpati Pramanik [11] introduced Pentapartitioned neutrosophic set and its properties. Here indeterminacy is split into three parts as contradiction, ignorance and unknown membership function. Also we introduced the concept of Penta partitioned neutrosophic Pythagorean set [6] and establish a number of its properties in our previous work.

The concepts of pentapartitioned neutrosophic semi-open sets, pentaparti-

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tioned neutrosophic semi-closed sets, pentapartitioned neutrosophic semi-interior and pentapartitioned neutrosophic semi-closure in pentapartitioned neutrosophic topological spaces were introduced by R. Radha and A. Stanis Arul Mary[10]. In this paper, we introduce the definitions of pentapartitioned neutrosophic generalized semi-closed sets and pentapartitioned neutrosophic generalized semi-open sets in pentapartitioned neutrosophic topological spaces and We study some of their basic properties with examples.

2 Preliminaries

Definition 2.1 [11] Let X be a universe. A Pentapartitioned neutrosophic set A on X is an object of the form

$$A = \{ \langle x, T_A, C_A, I_A, U_A, F_A \rangle : x \in X \}$$

Where $T_A + F_A + I_A + C_A + U_A \leq 5$.

Here, $T_A(x)$ is the truth membership, $C_A(x)$ is contradiction membership, $U_A(x)$ is ignorance membership, $F_A(x)$ is the false membership and $I_A(x)$ is an unknown membership.

Definition 2.2 [6] A PN set A is contained in another PN set B (i.e) $A \subseteq B$ iff $T_A \leq T_B, C_A \leq C_B, I_A \geq I_B, U_A \geq U_B$ and $F_A \geq F_B$

Definition 2.3 [6] The complement of a PN set (H, A) on R is defined as $H^c(x) = \{ \langle x, F_A, U_A, 1 - I_A, C_A, T_A \rangle : x \in R \}$ and is denoted by $(H, A)^c$.

Definition 2.4 [6] Let R be a non-empty set, $A = \langle x, T_A, C_A, I_A, U_A, F_A \rangle$ and $B = \langle x, T_B, C_B, I_B, U_B, F_B \rangle$ are two PN sets. Then

$$A \cup B = \langle x, \max(T_A, T_B), \max(C_A, C_B), \min(I_A, I_B), \min(U_A, U_B), \min(F_A, F_B) \rangle$$

$$A \cap B = \langle x, \min(T_A, T_B), \min(C_A, C_B), \max(I_A, I_B), \max(U_A, U_B), \max(F_A, F_B) \rangle$$

Definition 2.5 [6] A PN set (F, A) over the universe X is said to be empty PN set 0_X with respect to the parameter A if

$T_{F(e)} = 0, C_{F(e)} = 0, I_{F(e)} = 1, U_{F(e)} = 1, F_{F(e)} = 1, \forall x \in X, \forall e \in A$. It is denoted by 0_X .



Definition 2.6 [6] A PN set (F, A) over the universe R is said to be universe Pentapartitioned neutrosophic set with respect to the parameter A if $T_{F(e)} = 1, C_{F(e)} = 1, I_{F(e)} = 0, U_{F(e)} = 0, F_{F(e)} = 0, \forall x \in X, \forall e \in A$. It is denoted by 1_R .

Definition 2.7 [10] Let A be PN set of a pentapartitioned neutrosophic topological space [PNTS] X . Then A is said to be PN semi-open set [PNSO set] of X if there exists a set PNO such that $PNO(A) \subseteq A \subseteq PNCl(PNO(A))$.

Definition 2.8 [10] A subset A in a PNTS X is PNSO set if and only if $A \subseteq PNCl(PNInt(A))$.

Definition 2.9 [10] A PNPS A is called Pentapartitioned neutrosophic semi-closed set [PNSC set] if the complement of $C(A)$ is a PNSO set.

3 Pentapartitioned Neutrosophic Generalized Semi-closed Sets

Definition 3.1 A Pentapartitioned neutrosophic set [PNS] A of a Pentapartitioned neutrosophic topological space $PNTS(R, \tau)$ is called a Pentapartitioned neutrosophic generalized semi-closed set [PNGSC set] if $PNSCl(A) \subseteq U$, whenever $A \subseteq U$ and U is a Pentapartitioned neutrosophic open (PNO) set.

Example 3.2 Let $R = \{a\}$ with $\tau = \{0_R, A, B, C, D, E, F, 1_R\}$ where $A = \{0.1, 0.6, 0.5, 0.2, 0.6\}$, $B = \{0.4, 0.6, 0.1, 0.2, 0.4\}$, $C = \{0.4, 0.1, 0.5, 0.6, 0.4\}$, $D = \{0.1, 0.4, 0.7, 0.5, 0.6\}$, $E = \{0.4, 0.6, 0.5, 0.2, 0.4\}$ and $F = \{0.4, 0.4, 0.5, 0.5, 0.4\}$. Then the PNGSO sets are $0_R, A, B, C, D, E, F, C(A), C(D), C(F), 1_R$. Let us take $M = \{0.4, 0.2, 0.5, 0.1, 0.3\}$. Then M is PNGSC set.

Definition 3.3 A PNPS A in R is called PN generalized semi-open set [PNGSO set] in R if $C(A)$ is PNGSC set in R . That is, $U \subseteq PNSInt(A)$, whenever $A \subseteq U$ and U is a PNC set.

Definition 3.4 Let (R, τ) be a PNTS. Then a PN subset P of the PNTS R is said to be PN regular open if $P = PNInt(PNCl(P))$ and PN regular closed if $A = PNCl(PNInt(P))$

Theorem 3.5 Every PN closed set in PN topological space (R, τ) is a PN generalized semi-closed set.

Proof : Let A be a PNC set in PN topological space (R, τ) . Let $A \subseteq U$ and U be a PN open set in R . Then by Definition, $A = \text{PNCl}(A)$. We know that $\text{PNsCl}(A) \subseteq \text{PNCl}(A)$, then we get $\text{PNsCl}(A) \subseteq \text{PNCl}(A) = A \subseteq U$. Hence A is a PN generalized semi-closed set in R .

Example 3.6 From the above (3.2), M is PNGSC set. but not PNC set.

Theorem 3.7 Every PNSC set in the PNTS (R, τ) is a PNGSC set.

Proof: Let P be a PNSC set in the PNTS R . Let $P \subseteq U$ and U be a PN open set in R . Since P is PN semi-closed set, $\text{PNInt}(\text{PNCl}(P)) \subseteq P$. Therefore $\text{PNsCl}(P) \subseteq U$, $P \subseteq U$ and U is a PN open set. Hence P is a PNGSC set in R .

Example 3.8 From the above Example (3.2), M is PNGSC set but not PNSC set.

Theorem 3.9 If P and Q are PNGSC sets, then their intersection is also a PNGSC set.

Proof: Let P and Q are PNGSC sets. If $P \cap Q \subseteq U$ and U is PN open set, then $P \subseteq U$ and $Q \subseteq U$. Since P and Q are PNGSC sets, $\text{PNsCl}(P) \subseteq U$ and $\text{PNsCl}(Q) \subseteq U$. Hence $\text{PNsCl}(P) \cap \text{PNsCl}(Q) \subseteq U$. By theorem, $\text{PNCl}(P \cap Q) \subseteq \text{PNsCl}(P) \cap \text{PNsCl}(Q) \subseteq U$. Thus $P \cap Q$ is PNGSC set.

Remark 3.10 Union of any two PNGSC sets in (R, τ) need not be a PNGSC set, as seen from the following example.

Example 3.11 From Example (3.2), B^C and D are PNGSC sets but their union S is not PNGSC set.

Theorem 3.12 If P is a PNGSC set in R and $P \subseteq Q \subseteq \text{PNsCl}(P)$, then Q is a PNGSC set in R .

Proof: Let U be a PNO set in R such that $Q \subseteq U$. Since $P \subseteq Q$, $P \subseteq U$. Again since P is a PNGSC set, $\text{PNsCl}(P) \subseteq U$. By hypothesis, $Q \subseteq \text{PNsCl}(P)$. We know that $\text{PNsCl}(Q) \subseteq \text{PNsCl}(\text{PNsCl}(P)) = \text{PNsCl}(P)$. That is



$PNSCl(Q) \subseteq PNSCl(P)$. This implies that $PNSCl(q) \subseteq U$. Hence Q is a PNGSC set in X .

Theorem 3.13 A PN set P of a PN topological space (R, τ) is a PNGSC set iff $PNSCl(P) \subseteq Q$ where Q is a PNO set and $P \subseteq Q$.

Proof : Assume that P is a PNGSC set in X . Let Q be a PNO set in R such that $P \subseteq Q$. Then $C(Q)$ is a PNC set in R such that $C(Q) \subseteq C(P)$. Since $C(P)$ is a PNGSO set, $C(Q) \subseteq PNSInt(C(P))$. By Proposition 6.2 (ii) [5], $PNSInt(C(P)) = C(PNSCl(P))$. Therefore $C(Q) \subseteq C(PNSCl(P))$ implies that $PNSCl(P) \subseteq Q$. Conversely, assume that $PNSCl(P) \subseteq Q$ where Q is a PNO set and $P \subseteq Q$. Then $C(Q) \subseteq C(PNSCl(P))$ where $C(Q)$ is a PNC set and $C(Q) \subseteq PNSInt(C(P))$. Therefore $C(P)$ is a PNGSO set. This implies that P is a PNGSC set.

Theorem 3.14 A PN set P of a PNTS (R, τ) is a PNGSC set iff $PNInt(PNCl(P) \subseteq Q$ whenever Q is a PNO set and $P \subseteq Q$.

Proof : Assume that P is a PNGSC set in R . Let Q be a PNO set in R and $P \subseteq Q$. Then $C(Q)$ is a PNC set in R such that $C(Q) \subseteq C(P)$. Since $C(P)$ is a PNGSO set, $C(Q) \subseteq PNSInt(C(P))$. Therefore $C(Q) \subseteq PNCl(PNInt(C(P)))$.

Hence $C(Q) \subseteq C(PNInt(PNCl(P)))$. This implies that $PNInt(PNCl(P)) \subseteq Q$. Conversely, let P be PN set of R and $PNInt(PNCl(P) \subseteq Q$ whenever Q is a PNO set and $P \subseteq Q$. Then $C(Q) \subseteq C(P)$ and $C(Q)$ is PNC set. By hypothesis $C(Q) \subseteq C(PNInt(PNCl(P)))$.

Hence $C(Q) \subseteq PNCl(PNInt(C(P)))$ implies that $C(Q) \subseteq PNSInt(C(P))$. So that $C(P)$ is PNGSO set. Hence P is PNGSC set of R .

4 Pentapartitioned Neutrosophic Generalized Semi-open Sets

In this section, we study the concepts of Pentapartitioned neutrosophic generalized semi-open sets(PNGSO) and some of their basic properties.

Definition 4.1 The family of all PNGSC set (resp,PNGSO set) of a PN topological space (R, τ) will be denoted by PNGSC (R) (resp. PNGSO (R)).

Example 4.2 From Example (3.2), Let $H = \{0.3,0.1,0.5,0.2,0.4\}$.Hence H is PN generalized semi-open set.



Theorem 4.3 Every PNO set in PNTS (R, τ) is a PNGSO set.

Proof: Let P be a PNO set in PNTS R . Then by Definition , $P = \text{PNInt}(P)$. We know that $\text{PNInt}(P) \subseteq \text{PNSInt}(P) \subseteq P$. Therefore $P = \text{PNSInt}(P)$. Hence P is a PNGSO set in R .

Example 4.4 From the above Example (3.2), H is PNGSO set but not PNO set.

Theorem 4.5 Every PNSO set in PNPTS (R, τ) is a PNGSO set.

Proof : Let P be a PNSO set in PNTS R . Let $U \subseteq P$ and U be a PNO set in R . Since P is PNSO set, $P \subseteq \text{PNCI}(\text{PNInt}(P))$. This implies that $P \subseteq \text{PNSInt}(P) = P \cap \text{PNCI}(\text{PNInt}(P))$. Since $U \subseteq P, U \subseteq \text{PNSInt}(P)$. Therefore $U \subseteq \text{PNSInt}(P), U \subseteq P$ and U is a PNC set. Hence P is a PNGSO set in R .

Example 4.6 From the Example (3.2), H is PNGSO set but not PNSO set.

Theorem 4.7 If P and Q are PNGSO sets, then $P \cup Q$ is also a PNGSO set.

Proof : Let P and Q are PNGSO sets. If $U \subseteq P \subseteq Q$ and U is a PNC set, then $U \subseteq P$ and $U \subseteq Q$. Since P and Q are PNGSO sets, $U \subseteq \text{PNSInt}(P)$ and $U \subseteq \text{PNSInt}(Q)$. Hence $U \subseteq \text{PNSInt}(P) \cup \text{PNSInt}(Q)$. We know that $\text{PNSInt}(P \cup Q) \supseteq \text{PNSInt}(P) \cup \text{PNSInt}(Q) \supseteq U$. This implies that $U \subseteq \text{PNSInt}(P \cup Q)$. Therefore $U \subseteq \text{PNSInt}(P \cup Q), U \subseteq P \cup Q$ and U is PNC set. Thus $P \cup Q$ is PNGSO set.

Remark 4.8 Intersection of any two PNGSO sets in (R, τ) need not be a PNGSO set, as seen from the following example.

Example 4.9 From Example (3.2), we consider the two PNGSO sets $C(A)$ and $C(F)$. Their union is PNGSO set but their intersection is not PNGSO set.

Theorem 4.10 If P is a PNGSO in R and if $\text{PNSInt}(P) \subseteq Q \subseteq P$, then Q is a PNGSO set in R .

Proof : Let P be a PNGSO set in R . Since $\text{PNSInt}(P) \subseteq Q \subseteq P$ and we have $C(P) \subseteq C(Q) \subseteq C(\text{PNSInt}(P)) = \text{PNSCI}(C(P))$. Again since $C(P)$ is a PNGSC



set and by theorem we get $C(Q)$ is a PNGSC set in R . Hence Q is a PNGSO set in R .

Theorem 4.11 A PN set P of a PN topological space (R, τ) is a PNGSO set if and only if $Q \subseteq PNSInt(P)$ where Q is a PN closed set and $Q \subseteq P$.

Proof : Assume that P is a PNGSO set in R . Let Q be a PN closed set in R such that $Q \subseteq P$. Then $C(Q)$ is a PNO set in R such that $C(P) \subseteq C(Q)$. Since $C(P)$ is a PNGSC set, $PNSCl(C(P)) \subseteq C(Q)$.

Then $PNSCl(C(P)) = C(PNSInt(P))$. Therefore $C(PNSInt(P)) \subseteq C(Q)$ implies that $Q \subseteq PNSInt(P)$. Conversely, assume that $Q \subseteq PNSInt(P)$ where Q is a PNC set and $Q \subseteq P$. Then $C(PNSInt(P)) \subseteq C(Q)$ where $C(Q)$ is a PNO set and $PNSCl(C(P)) \subseteq C(Q)$. Therefore $C(P)$ is a PNGSC set. This implies that P is a PNGSO set.

Theorem 4.12 A PN set P of a PNTS (R, τ) is a PNGSO set if and only if $Q \subseteq PNCl(PNInt(P))$ whenever Q is a PNC set and $Q \subseteq P$.

Proof : Assume that P is a PNGSO set in R . Let Q be a PNC set in R and $Q \subseteq P$. Then $C(Q)$ is a PNO set in R such that $C(P) \subseteq C(Q)$.

Since $C(P)$ is a PNGSC set, $PNSCl(C(P)) \subseteq C(Q)$. Therefore $PNInt(PNCl(C(P))) \subseteq C(Q)$. Hence $C(PNCl(PNInt(P))) \subseteq C(Q)$. This implies that $Q \subseteq PNCl(PNInt(P))$. Conversely, let P be PN set of R and $Q \subseteq PNCl(PNInt(P))$ whenever Q be PNC set and $Q \subseteq P$. Then $C(P) \subseteq C(Q)$ and $C(Q)$ is PNO set. By hypothesis $C(PNCl(PNInt(P))) \subseteq C(Q)$. Hence $PNInt(PNCl(C(P))) \subseteq C(Q)$ implies that $PNSCl(C(P)) \subseteq C(Q)$. So that $C(P)$ is PNGSC set. Hence P is PNGSO set of R .

5 Conclusions

The concepts of pentapartitioned neutrosophic generalized semi-closed sets, pentapartitioned neutrosophic generalized semi-open sets, and their properties in pentapartitioned neutrosophic topological spaces were studied in this paper. In the future, we will extend these pentapartitioned neutrosophic topology concepts to include pentapartitioned neutrosophic generalized semi-continuous, pentapartitioned neutrosophic semi-generalized continuous, and pentapartitioned neutrosophic semi-generalized continuous in PNTS. We also contribute nets, filters, and borders to this pentapartitioned neutrosophic concept.



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