Improved Correlation Coefficients of Quadripartitioned Neutrosophic Pythagorean sets for MADM
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Abstract

Quadripartitioned single valued neutrosophic pythagorean sets is a improvisation of Wang’s single valued neutrosophic sets. In this paper we have studied the improved correlation coefficients of quadripartitioned single valued neutrosophic pythagorean sets and investigate its properties. Further we have applied this concept in multiple attribute decision making methods with quadripartitioned single valued neutrosophic pythagorean environment. Finally we illustrated an example in the above proposed method to the multiple attribute decision making problems.

Key words: Quadripartitioned Neutrosophic Pythagorean Sets, Quadripartitioned Neutrosophic Sets, Improved Correlation Coefficient.

AMS classification: 60A86, 62A86

1 Introduction

Fuzzy sets were introduced by Zadeh [21] in 1965 which allows the membership function valued in the interval [0,1] and also it is an extension of classical set theory. Fuzzy set helps to deal the concept of uncertainty, vagueness and imprecision which is not possible in the cantorian set. As an extension of Zadeh’s fuzzy set theory intuitionistic fuzzy set (IFS) was introduced by Atanassov [1] in 1986, which consists of degree of membership and degree of non membership and lies in the interval of [0,1]. IFS theory widely used in the areas of logic programming, decision making problems, medical diagnosis etc.

Florentin Smarandache [13] introduced the concept of Neutrosophic set in 1995 which provides the knowledge of neutral thought by introducing the new factor
called indeterminacy in the set. Therefore neutrosophic set was framed and it includes
the components of truth membership function (T), indeterminacy membership
function (I), and falsity membership function (F) respectively. Neutrosophic sets deals
with non standard interval of $[-0, 1]$. Since neutrosophic set deals the indeterminacy
effectively it plays an vital role in many applications areas include information
technology, decision support system, relational database systems, medical diagnosis,
multicriteria decision making problems etc.,

To deal the real world problems, Wang [12](2010) introduced the concept
of single valued neutrosophic sets (SVNS) which is also known as an extension
of intuitionistic fuzzy sets and it became a very new hot research topic now. Rajashi Chatterjee, et al [12] proposed the concept of Quadripartitioned single valued
neutrosophic sets which is based on Belnap’s four valued logic and Smarandache’s
four numerical valued logic. In (QSVNS) indeterminacy is splitted into two functions
known as ‘Contradicition’ (both true and false) and ‘Unknown’ (neither true nor false)
so that QSVNS has four components $T, C, U, F$ which also lies in the non standard
unit interval $[-0, 1]$. Further, R. Radha and A. Stanis Arul Mary defined a new
hybrid model of Quadripartitioned Neutrosophic Pythagorean Sets in 2021.

Correlation coefficient is a effective mathematical tool to measure the
strength of the relationship between two variables. So many researchers pay the
attention to the concept of various correlation coefficients of the different sets like
fuzzy set, IFS, SVNS, QSVNS. In 1999 D.A Chiang and N.P.Lin [3] proposed
the correlation of fuzzy sets under fuzzy environment. Later D.H.Hong [4](2006)
defined fuzzy measures for a correlation coefficient of fuzzy numbers under Tw (the
weakest t-norm) based fuzzy arithmetic operations. Correlation coefficients plays an
important role in many real world problems like multiple attribute group decision
making, clustering analysis, pattern recognition, medical diagnosis etc., Jun Ye [20] defined the improved correlation coefficients of single valued neutrosophic sets and
interval neutrosophic sets for multiple attribute decision making to overcome the
drawbacks of the correlation coefficients of single valued neutrosophic sets (SVNSs)
which is defined in [16].

In this paper , We have discussed some of its properties and decision
making method using the improved correlation coefficient with quadripartitioned
neutrosophic pythagorean environment. Additionally, an illustrative example is given
in above proposed correlation method particularly in multiple criteria decision making
problems.
2 Preliminaries

Definition 2.1 (13) Let X be a universe. A Neutrosophic set A on X can be defined as follows:

\[ A = \{ \langle x, T_A(x), I_A(x), F_A(x) \rangle : x \in X \} \]

Where \( T_A, I_A, F_A : U \rightarrow [0, 1] \) and \( 0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3 \)

Here, \( T_A(x) \) is the degree of membership, \( I_A(x) \) is the degree of indeterminacy and \( F_A(x) \) is the degree of non-membership.

Here, \( T_A(x) \) and \( F_A(x) \) are dependent neutrosophic components and \( I_A(x) \) is an independent component.

Definition 2.2 (10) Let X be a universe. A Quadripartitioned neutrosophic pythagorean set A with \( T, F, C \) and \( U \) as dependent neutrosophic components for A on X is an object of the form

\[ A = \{ \langle x, T_A, C_A, U_A, F_A \rangle : x \in X \} \]

Where \( T_A + F_A \leq 1, C_A + U_A \leq 1 \) and

\[ (T_A)^2 + (C_A)^2 + (U_A)^2 + (F_A)^2 \leq 2 \]

Here, \( T_A(x) \) is the truth membership, \( C_A(x) \) is contradiction membership, \( U_A(x) \) is ignorance membership, \( F_A(x) \) is the false membership.

Definition 2.3 (12) Let P be a non-empty set. A Quadripartitioned neutrosophic set A over P characterizes each element \( p \) in P a truth-membership function \( T_A \), a contradiction membership function \( C_A \), ignorance membership function \( U_A \) and a false membership function \( F_A \), such that for each \( p \) in P

\[ T_A + C_A + U_A + F_A \leq 4 \]

Definition 2.4 (10) The complement of a quadripartitioned neutrosophic pythagorean set \((F, A)\) on X Denoted by \((F, A)^c\) and is defined as

\[ F^c(x) = \{ \langle x, F_A(x), U_A(x), C_A(x), T_A(x) \rangle : x \in X \} \].

Definition 2.5 (10) Let \( A = \langle x, T_A(x), C_A(x), U_A(x), F_A(x) \rangle \) and
\[ B = \langle x, T_B(x), C_B(x), U_B(x), F_B(x) \rangle \] are quadripartitioned neutrosophic pythagorean sets. Then
\[ A \cup B = \langle x, \max(T_A(x), T_B(x)), \max(C_A(x), C_B(x)), \min(U_A(x), U_B(x)), \min(F_A(x), F_B(x)) \rangle \]
\[ A \cap B = \langle x, \min(T_A(x), T_B(x)), \min(C_A(x), C_B(x)), \max(U_A(x), U_B(x)), \max(F_A(x), F_B(x)) \rangle. \]

### 3 Improved Correlation Coefficients

**Definition 3.1** Let \( P \) and \( Q \) be any two QNPS in the universe of discourse \( R = \{r_1, r_2, r_3, \cdots, r_n\} \), then the improved correlation coefficient between \( P \) and \( Q \) is defined as follows

\[
K(P, Q) = \frac{1}{4n} \sum_{k=1}^{n} [\mu_k(1 - \Delta T_k) + \varphi_k(1 - \Delta C_k) + \gamma_k(1 - \Delta U_k) + v_k(1 - \Delta F_k)] \tag{1}
\]

Where

\[
\begin{align*}
\mu_k &= \frac{2 - \Delta T_k - \Delta T_{\max}}{2 - \Delta T_{\min} - \Delta T_{\max}}, \\
\varphi_k &= \frac{2 - \Delta C_k - \Delta C_{\max}}{2 - \Delta C_{\min} - \Delta C_{\max}}, \\
\gamma_k &= \frac{2 - \Delta U_k - \Delta U_{\max}}{2 - \Delta U_{\min} - \Delta U_{\max}}, \\
v_k &= \frac{2 - \Delta F_k - \Delta F_{\max}}{2 - \Delta F_{\min} - \Delta F_{\max}}, \\
\Delta T_K &= |T^2_P(r_k) - T^2_Q(r_k)|, \\
\Delta C_K &= |C^2_P(r_k) - C^2_Q(r_k)|, \\
\Delta U_K &= |U^2_P(r_k) - U^2_Q(r_k)|, \\
\Delta F_K &= |F^2_P(r_k) - F^2_Q(r_k)|, \\
\Delta T_{\min} &= \min_k |T^2_P(r_k) - T^2_Q(r_k)|, \\
\Delta C_{\min} &= \min_k |C^2_P(r_k) - C^2_Q(r_k)|, \\
\Delta U_{\min} &= \min_k |U^2_P(r_k) - U^2_Q(r_k)|,
\end{align*}
\]
\[ \Delta F_{\min} = \min_k |F_P^2(r_k) - F_Q^2(r_k)|, \]
\[ \Delta T_{\max} = \max_k |T_P^2(r_k) - T_Q^2(r_k)|, \]
\[ \Delta C_{\max} = \max_k |C_P^2(r_k) - C_Q^2(r_k)|, \]
\[ \Delta U_{\max} = \max_k |U_P^2(r_k) - U_Q^2(r_k)|, \]
\[ \Delta F_{\max} = \max_k |F_P^2(r_k) - F_Q^2(r_k)|, \]

For any \( r_k \in R \) and \( k = 1, 2, 3, \cdots, n \).

**Theorem 3.2** For any two QNPSs \( P \) and \( Q \) in the universe of discourse \( R = \{r_1, r_2, r_3, \cdots, r_n\} \), the improved correlation coefficient \( K(P, Q) \) satisfies the following properties.

(i) \( K(P, Q) = K(Q, P) \);
(ii) \( 0 \leq K(P, Q) \leq 1 \);
(iii) \( K(P, Q) = 1 \) iff \( P = Q \).

**Proof:**

(i) It is obvious and straightforward.
(ii) Here, \( 0 \leq \mu_k \leq 1, \ 0 \leq \varphi_k \leq 1, \ 0 \leq \gamma_k \leq 1, \ 0 \leq v_k \leq 1, \)
\[ 0 \leq 1 - \Delta T_k \leq 1, \ 0 \leq 1 - \Delta C_k \leq 1, \ 0 \leq 1 - \Delta U_k \leq 1, \ 0 \leq 1 - \Delta F_k \leq 1, \]
Therefore the following inequation satisfies
\[ 0 \leq \mu_k (1 - \Delta T_k) + \varphi_k (1 - \Delta C_k) + \gamma_k (1 - \Delta U_k) + v_k (1 - \Delta F_k) \leq 5. \]
Hence we have \( 0 \leq K(P, Q) \leq 1 \).
(iii) If \( K(P, Q) = 1 \), then we get
\[ \mu_k (1 - \Delta T_k) + \varphi_k (1 - \Delta C_k) + \gamma_k (1 - \Delta U_k) + v_k (1 - \Delta F_k) = 4. \]
Since \( 0 \leq \mu_k (1 - \Delta T_k) \leq 1, 0 \leq \varphi_k (1 - \Delta C_k) \leq 1, 0 \leq \gamma_k (1 - \Delta U_k) \leq 1 \) and \( 0 \leq v_k (1 - \Delta F_k) \leq 1 \), there are \( \mu_k (1 - \Delta T_k) = 1, \varphi_k (1 - \Delta C_k) = 1, \gamma_k (1 - \Delta U_k) = 1 \) and \( v_k (1 - \Delta F_k) = 1 \).
And also since \( 0 \leq \mu_k \leq 1, 0 \leq \varphi_k \leq 1, 0 \leq \gamma_k \leq 1 \) and \( 0 \leq v_k \leq 1 \),
\[ 0 \leq 1 - \Delta T_k \leq 1, 0 \leq 1 - \Delta C_k \leq 1, 0 \leq 1 - \Delta U_k \leq 1, 0 \leq 1 - \Delta F_k \leq 1. \]
We get \( \mu_k = \varphi_k = \gamma_k = v_k = 1 \) and
\[ 1 - \Delta T_k = 1 - \Delta C_k = 1 - \Delta U_k = 1 - \Delta F_k = 1. \]
This implies, \( \Delta T_k = \Delta T_{\min} = \Delta T_{\max} = 0, \)
\[ \Delta C_k = \Delta C_{\min} = \Delta C_{\max} = 0, \Delta U_k = \Delta U_{\min} = \Delta U_{\max} = 0, \]
\[ \Delta F_k = \Delta F_{\min} = \Delta F_{\max} = 0. \] Hence \( T_P(r_k) = T_Q(r_k), \) 
\( C_P(r_k) = C_Q(r_k), U_P(r_k) = U_Q(r_k) \) and \( F_P(r_k) = F_Q(r_k) \) for any \( r_k \in R \) and \( k = 1, 2, 3, \ldots, n \). Hence \( P = Q \).

Conversely, assume that \( P = Q \), this implies \( T_P(r_k) = T_Q(r_k), \) 
\( C_P(r_k) = C_Q(r_k), U_P(r_k) = U_Q(r_k) \) and \( F_P(r_k) = F_Q(r_k) \) for any \( r_k \in R \) and \( k = 1, 2, 3, \ldots, n \). Thus \( \Delta T_k = \Delta T_{\min} = \Delta T_{\max} = 0, \) 
\( \Delta C_k = \Delta C_{\min} = \Delta C_{\max} = 0, \Delta U_k = \Delta U_{\min} = \Delta U_{\max} = 0, \) 
\( \Delta F_k = \Delta F_{\min} = \Delta F_{\max} = 0. \) Hence we get \( K(P, Q) = 1. \)

The improved correlation coefficient formula which is defined is correct and also satisfies these properties in the above theorem. When we use any constant \( \varepsilon > 2 \) in the following expressions

\[ \mu_k = \frac{\varepsilon - \Delta T_k - \Delta T_{\max}}{\varepsilon - \Delta T_{\min} - \Delta T_{\max}}, \]
\[ \varphi_k = \frac{\varepsilon - \Delta C_k - \Delta C_{\max}}{\varepsilon - \Delta C_{\min} - \Delta C_{\max}}, \]
\[ \delta_k = \frac{\varepsilon - \Delta G_k - \Delta G_{\max}}{\varepsilon - \Delta G_{\min} - \Delta G_{\max}}, \]
\[ \gamma_k = \frac{\varepsilon - \Delta U_k - \Delta U_{\max}}{\varepsilon - \Delta U_{\min} - \Delta U_{\max}}, \]
\[ \upsilon_k = \frac{\varepsilon - \Delta F_k - \Delta F_{\max}}{\varepsilon - \Delta F_{\min} - \Delta F_{\max}}. \]

**Example 3.3** Let \( A = \{ r, 0, 0, 0, 0 \} \) and \( B = \{ r, 0.4, 0.2, 0.1, 0.2 \} \) be any two PNPS in \( R \). Therefore by equation (1) we get \( K(A, B) = 0.893305. \) It shows that the above defined improved correlation coefficient overcome the disadvantages of the correlation coefficient.

In the following, we define a weighted correlation coefficient between PNPS since the differences in the elements are considered into an account. Let \( w_k \) be the weight of each element \( r_k(k = 1, 2, \ldots, n), w_k \in [0, 1] \) and \( \sum_{k=1}^{n} w_k = 1 \), then the weighted correlation coefficient between the PNPS’s \( A \) and \( B \).
\[ K_w(A, B) = \frac{1}{n} \sum_{k=1}^{n} w_k \mu_k(1 - \Delta T_k) + \varphi_k(1 - \Delta C_k) + \gamma_k(1 - \Delta U_k) + \nu_k(1 - \Delta F_k) \quad (2) \]

If \( w = (1/n, 1/n, 1/n, \ldots, 1/n)^T \), then equation [4] reduces to equation [3]. \( K_w(A, B) \) also satisfies the three properties in the above theorem.

**Theorem 3.4** Let \( w_k \) be the weight for each element \( r_k(k = 1, 2, \ldots, n) \), \( w_k \in [0, 1] \) and \( \sum_{k=1}^{n} w_k = 1 \), then the weighted correlation coefficient between the QNPS s \( A \) and \( B \) which is denoted by \( K_w(A, B) \) defined in equation ( ) satisfies the following properties.

1) \( K_w(A, B) = K_w(B, A) \);
2) \( 0 \leq K_w(A, B) \leq 1 \);
3) \( K_w(A, B) = 1 \) iff \( A = B \).

It is similar to prove the properties in theorem (3.1).

4 Decision Making using the improved correlation coefficient of QNPSs

Multiple criteria decision making (MCDM) problems refers to make decisions when several attributes are involved in real-life problem. For example one may buy a vehicle by analysing the attributes which is given in tems of price, style, safety, comfort etc.,

Here we consider a multiple attribute decision making problem with quadripartitioned neutrosophic pythagorean information and the characteristic of an alternative \( A_i(i = 1, 2, \ldots, m) \) on an attribute \( C_j(j = 1, 2, \ldots, n) \) is represented by the following PNPS s:

\[ A_i = \{ C_j, T_{A_i}(C_j), C_{A_i}(C_j), U_{A_i}(C_j), F_{A_i}(C_j) / C_j \in C, j = 1, 2, \ldots, n \} \]

Where \( T_{A_i}(C_j), C_{A_i}(C_j), U_{A_i}(C_j), F_{A_i}(C_j) \in [0, 1] \) and

\[ 0 \leq T_{A_i}^2(C_j) + C_{A_i}^2(C_j) + U_{A_i}^2(C_j) + F_{A_i}^2(C_j) \leq 2 \]

for \( C_j \in C, j = 1, 2, \ldots, n \) and \( i = 1, 2, \ldots, m \).

To make it convenient, we are considering the following five functions \( T_{A_i}(C_j), C_{A_i}(C_j), U_{A_i}(C_j), F_{A_i}(C_j) \) in terms of pentapartitioned neutrosophic
pythagorean value (PNPV)

\[ d_{ij} = (t_{ij}, c_{ij}, u_{ij}, f_{ij}) \quad (i = 1, 2, \cdots m; j = 1, 2 \cdots n). \]

Here the values of \( d_{ij} \) are usually derived from the evaluation of an alternative \( A_i \) with respect to a criteria \( C_j \) by the expert or decision maker. Therefore we got a pentapartitioned neutrosophic pythagorean decision matrix \( D = (d_{ij})_{m \times n} \).

In the case of ideal alternative \( A^* \) an ideal PNPV can be defined by

\[ d^*_{j} = t^*_j, c^*_j, u^*_j, f^*_j = (1, 1, 0, 0)(j = 1, 2, \cdots, n) \] in the decision making method, Hence the weighted correlation coefficient between an alternative \( A_i (i = 1, 2 \cdots, m) \) and the ideal alternative \( A^* \) is given by,

\[
K_w(A_i, A^*) = \frac{1}{4} \sum_{j=1}^{n} w_j [\mu_{ij} (1 - \triangle t_{ij}) + \varphi_{ij} (1 - \triangle c_{ij}) + \gamma_{ij} (1 - \triangle u_{ij}) + \psi_{ij} (1 - \triangle f_{ij})]
\]

(3)

Where,

\[
\mu_{ij} = \frac{2 - \triangle t_{ij} - \triangle t_{imax}}{2 - \triangle t_{imin} - \triangle t_{imax}},
\]

\[
\varphi_{ij} = \frac{2 - \triangle c_{ij} - \triangle c_{imax}}{2 - \triangle c_{imin} - \triangle c_{imax}},
\]

\[
\gamma_{ij} = \frac{2 - \triangle u_{ij} - \triangle u_{imax}}{2 - \triangle u_{imin} - \triangle u_{imax}},
\]

\[
\psi_{ij} = \frac{2 - \triangle f_{ij} - \triangle f_{imax}}{2 - \triangle f_{imin} - \triangle f_{imax}},
\]

\[
\triangle t_{ij} = |t^2_{ij} - t^*_j|,
\]

\[
\triangle c_{ij} = |c^2_{ij} - c^*_j|,
\]

\[
\triangle u_{ij} = |u^2_{ij} - u^*_j|,
\]

\[
\triangle t_{imin} = min_j |t^2_{ij} - t^*_j|,
\]

\[
\triangle c_{imin} = min_j |c^2_{ij} - c^*_j|,
\]

\[
\triangle u_{imin} = min_j |u^2_{ij} - u^*_j|,
\]

\[
\triangle f_{imin} = min_j |f^2_{ij} - f^*_j|,
\]
\[ \Delta t_{\text{imax}} = \max_j |t_{ij}^2 - t_j^*|, \]
\[ \Delta c_{\text{imax}} = \max_j |c_{ij}^2 - c_j^*|, \]
\[ \Delta u_{\text{imax}} = \max_j |u_{ij}^2 - u_j^*|, \]
\[ \Delta f_{\text{imax}} = \max_j |f_{ij}^2 - f_j^*|. \]

For \( i = 1, 2 \ldots, m \) and \( j = 1, 2, \ldots, n \).

By using the above weighted correlation coefficient we can derive the ranking order of all alternatives and we can choose the best one among those.

**Example 4.1** This section deals the example for the multiple attribute decision making problem with the given alternatives corresponds to the criteria allotted under pentapartitioned neutrosophic pythagorean environment. For this example which we will discuss here is about the best laptop among all available alternatives based on various criteria. The alternatives \( A_1, A_2, A_3 \) respectively denotes the Lenovo, HP, Dell. The customer must take a decision according to the following four attributes that is (1) \( C_1 \) is the cost (2) \( C_2 \) is the storage space (3) \( C_3 \) is the camera quality (1) \( C_4 \) is the looks. According to this attributes we will derive the ranking order of all alternatives and based on this ranking order customer will select the best one.

The weight vector of the above attributes is given by \( w = (0.2, 0.35, 0.25, 0.20)^T \). Here the alternatives are to be evaluated under the above four attributes by the form of QNPSs. In general the evaluation of an alternative \( A_i \) with respect to an attribute \( C_j (i = 1, 2, 3; j = 1, 2, 3, 4) \) will be done by the questionnaire of a domain expert. In particularly, while asking the opinion about an alternative \( A_1 \) with respect to an attribute \( C_1 \), the possibility he (or) she say that the statement true is 0.3 the statement both true and false is 0.4, the statement neither true nor false is 0.3 and the statement false is 0.4. It can be denoted in neutrosophic notation as \( d_{11} = (0.3, 0.4, 0.3, 0.4) \). Continuing this procedure for all three alternatives with respect to four attributes we will get the following quadripartitioned single valued neutrosophic pythagorean decision value table.
Then by using the proposed method we will obtain the most desirable alternative. We can get the values of the correlation coefficient $M_w(A_i, A^*)(i = 1, 2, 3)$ by using Equation (3).

Hence $K_w(A_1, A^*) = 0.51535, K_w(A_2, A^*) = 0.53517, K_w(A_3, A^*) = 0.5708$. Therefore the ranking order is, $A_3 > A_2 > A_1$. The alternative $A_3$ (Dell) Laptop is the best choice among all the three alternatives.

5 Conclusion

In this paper we have defined the improved correlation coefficient of QNPSs. Decision making is a process which plays a vital role in real life problems. The main process in decision making is recognizing the problem (or) opportunity and deciding to address it. Here we have discussed the decision making method using the improved correlation coefficient of QNPSs and in particularly an illustrative example is given in multiple attribute decision making problems which involves the several alternatives based on various criteria. Hence our proposed improved correlation coefficient of QNPSs helps to identify the most suitable alternative to the customer based on the given criteria.

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