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Study of electrical circuits using Runge Kutta method of order 4

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Abstract

In this paper, Runge Kutta method of order 4 is used to study the electrical circuits designs through past, intermediate and present voltages. When integrating differential equations with Runge Kutta method of order 4, a constant step size (h) is used until a testing procedure confirms that the discontinuity occurs in the present integration interval. This step size function calculations would take place at the end of the functional calculations, but before the dependent variables were updated. Runge Kutta methods along with convolution are given by array interpretation (Butcher matrix) representation, this leads to identify the equilibrium state. The input parameters indicate the voltage coefficient controlled by current sources and measures it a random periodic time. The output parameters provide stable independent values and calculated from past voltage and current values. Finally solutions are compared with exact values and RK method of order 4 along with Heun, Midpoint and Taylors's method with various h values.

Key Words: Runge Kutta method of order 4, step size, past, intermediate and present voltages, electrical circuits.

AMS Classification: 34B15, 34B16.

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1 Introduction

Numerous branches of economics, science, engineering and technology are commonly contain numerical methods and analysis. Runge Kutta Method of order 4 is reasonably simple and robust and is used to analyze the transient behaviors of RLC circuit taken into consideration of higher order accuracy in multi-stage. It involves slope calculations at multiple steps between the current and the consecutive discrete time values combined with an intelligent adaptive step-size routine [4]. RK method of order 4 preserves stability, provided that the step size h does not become too large [7, 11]. In the general observation, higher order RK methods allows us to increase / decrease the step size and obtaining good accuracy but the stability of the algorithms establishes limits to the value of h. This method avoids the higher order derivatives of the unknown function y and obtained the higher order accuracy [18] because of its computation potential, accuracy, flexibility and the possibility of changing the integration step. Large scale parallel processing system without redesigning the structure is processed by Runge Kutta method of order 4 with small h values [3, 14]. An efficient implementation and designing the circuits are evaluated and analyzed the scalability and throughput [5, 16].

The Runge Kutta technique is a one-step method with several stages whose order is determined by the number of stages [1, 17]. This technique and approach is used to solve differential equations of various types, including explicit, implicit, partial, and delay differential equations [2, 10]. This method played a significant role in the study of explicit and implicit iterative strategies for solving ODEs with temporal dissipation. Higher order stability preserving Runge Kutta discretizations were created for use with semi-discrete method of approximations of hyperbolic partial differential equations, and have been found to be useful in a diverse of other situations [6]. Because hyperbolic problems and its issues are often leads discontinuous solutions and the nonlinear stability features are almost critical [9, 15].

When utilizing a linearly stable, high-order approach that lacks the strong stability feature and numerical data showed that oscillations can arise [8, 13]. For a nonlinear ODE, any Runge Kutta technique of fifth order or above will have at least one negative coefficient. As a result, we recognize that negative coefficients must be addressed when using Runge Kutta methods to solve nonlinear ODEs [12].

This paper is organized as follows. Section II focused the convolution of Runge Kutta method of order 4. Equilibrium state and array interpretation (Butcher matrix) representation of RK method of order 4 along with the convolution of past, intermediate and present voltages are studied in this section. In section III, the input parameters indicate the voltage coefficient which is controlled by current sources and measures in a random periodical time. The output parameters provide stable independent values and calculated from past voltage and current values. Section IV presents the implementation of numerical integration for nonlinear capacitors. Comparing exact values with RK method of order 4, Heun, Midpoint and Taylors's method with various h values are calculated. Finally section V concludes the paper.

2 Convolution and Runge Kutta method of order 4

Consider the following first order linear differential equation,

$$y' = f(t, y)$$

Runge Kutta methods along with convolution are given by

$$Y_i = y_n + h \sum_{j=1}^{s} a_{ij} * f(t_n + c_j h\,, Y_j) \qquad (i = 0,1, \dots, s)$$

$$y_{n+1} = y_n + h \sum_{j=1}^{s} b_i * f(t_n + c_i h\,, Y_i) \tag{1}$$

(1) is given by the array interpretation (Butcher matrix) representation and this type arrangement is shown in table 1 and is explicit when upper triangular coefficients $a_{ij}(i \le j)$ are zero and pass through towards equilibrium state and implicit except these conditions (i.e., $a_{ij}(i \le j)$ are non zero and does not pass through equilibrium state.

c_1	a_{11}	a_{12}	$\cdots$	a_{1s}
c_2	a_{21}	a_{22}	$\cdots$	a_{2s}
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
c_s	a_{s1}	a_{s2}	$\cdots$	a_{ss}

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	b_1	b_2	$\cdots$	b_s

Table 1: Array interpretation (Butcher matrix) representation of equation 1

Assume that $m_{ij} = a_{ij}, m_{10} = a_{20} = 0, \ldots, m_{ij} = a_{(i-1)(j-1)}$.

$$Y(t_a) - y(t_n) - hm_{10}f(t_n) - hm_{11}f(t_a) - hm_{12}f(t_{n+1}) = 0$$

$$y(t_{n+1}) - y(t_n) - hm_{20} * f(t_n) - hm_{21} * f(t_a) - hm_{22} * f(t_{n+1}) = 0 \quad (2)$$

where t_n, t_a and t_{n+1} indicates past, intermediate and present time domain.

3 Equivalent models for a linear element -Linear device

Applying (2) to the following equation for a linear capacitor (C) and voltage v_c:

$$i(t) = C\frac{dv_c}{dt}$$

And we get

$$i_a = \frac{(m_{12} - m_{22})v_n + m_{22}v_a - m_{22}v_{n+1} - \frac{h}{c}(m_{10}m_{22} - m_{12}m_{20})i_n}{\frac{h}{c}(m;_{11}m_{22} - m_{12}m_{21})}$$

$$i_{n+1} = \frac{(-m_{11} + m_{21})v_n - m_{21}v_a + m_{11}v_{n+1} + \frac{h}{c}(m_{10}m_{22} - m_{11}m_{20})i_n}{\frac{h}{c}(m_{11}m_{22} - m_{12}m_{21})} \quad (3)$$

where v_n, v_a and v_{n+1} indicate a past, intermediate and present voltages. i_n represent the final current value at time t. The equation for the simulation is given by

$$\begin{bmatrix} B & -B & A & -A \\ -B & B & -A & A \\ E & -E & D & -D \\ -E & E & -D & D \end{bmatrix} \begin{bmatrix} v_{1,n+1} \\ v_{2,n+1} \\ v_{1,a} \\ v_{2,a} \end{bmatrix} = \begin{bmatrix} -P \\ P \\ -F \\ F \end{bmatrix} \quad (4)$$

where $v_{1,a}$ and $v_{2,a}$ are the voltages in the intermediate time t_a . $v_{1,n+1}$ and $v_{2,n+1}$ are the voltages at a present time t_{n+1} and t_{n+1}. The input parameters A and E indicates the

coefficients of the voltage controlled by current sources. In particular A is controlled by v_{ca} and E is controlled by v_{cn+1}. Further A and E are defined like:

$$A = \frac{-m_{21}}{\frac{h}{c}(m_{11}m_{22} - m_{12}m_{21})}, \quad E = \frac{-m_{12}}{\frac{h}{c}(m_{11}m_{22} - m_{12}m_{21})}$$

The input parameters B and D indicates the coefficients of the voltage in a particular random periodic time. Further A and E are defined like:

$$B = \frac{m_{11}}{\frac{h}{c}(m_{11}m_{22} - m_{12}m_{21})}, \quad D = \frac{m_{22}}{\frac{h}{c}(m_{11}m_{22} - m_{12}m_{21})}$$

The output parameters P and F are stable independent values and calculated from past voltage and current values. Further P and F are defined like:

$$P = \frac{(-m_{11} + m_{21})v_{c_n} + \frac{h}{c}(m_{10}m_{20} - m_{12}m_{21})i_{c_n}}{\frac{h}{c}(m_{11}m_{22} - m_{12}m_{21})}$$

$$F = \frac{(m_{12} - m_{22})v_{c_n} + \frac{h}{c}(m_{10}m_{22} - m_{12}m_{20})i_{c_n}}{\frac{h}{c}(m_{11}m_{22} - m_{12}m_{21})}$$

Here i_{c_n} represents the current that pass through the capacitor. The similar equivalent circuits can be compared with the Runge Kutta method of order 4 and, moreover, to ensure the stability conditions of the variables are verified at random periodical time.

4 Implementation of numerical integration for nonlinear capacitors

This implementation consists of resistors, diodes and voltages. A diode model consists of a contact resistance R_s, a nonlinear conductor $g(v)$ and a nonlinear capacitor $q(v)$. The relationship between the nonlinear capacitor $q(v)$ with the voltage be like:

$$q(v) = T_t I_s \left(exp\left(\frac{v}{NV_t}\right) - 1 \right) \tag{5}$$

where T_t, I_s and N indicates run time, a saturation current and coefficient of emission. Applying higher iterations to i_{c_n},

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$$i_{cn}^{(j+1)} = \frac{m_{22}}{h\alpha}\left\{q\left(v_a^{(j)}\right) + C_a^{(j)}\left(v_a^{(j+1)} - v_a^{(j)}\right)\right\} - \frac{m_{12}}{h\alpha}\left\{q\left(v_{n+1}^{(j)}\right) + C_{n+1}^{(j)}\left(v_{n+1}^{(j+1)} - v_{n+1}^{(j)}\right)\right\} + \frac{1}{h\alpha}\left\{(m_{12} - m_{22})q\left(v_{(n)}\right) - h(m_{10}m_{22} - m_{12}m_{20})C_n\right\}$$

$$i_{cn+1}^{(j+1)} = \frac{m_{22}}{h\alpha}\left\{q\left(v_{n+1}^{(j)}\right) + C_{n+1}^{(j)}\left(v_{n+1}^{(j+1)} - v_{n+1}^{(j)}\right)\right\} - \frac{m_{12}}{h\alpha}\left\{q\left(v_a^{(j)}\right) + C_a^{(j)}\left(v_a^{(j+1)} - v_a^{(j)}\right)\right\} + \frac{1}{h\alpha}\left\{(m_{21} - m_{11})q\left(v_{(n)}\right) + h(m_{10}m_{21} - m_{11}m_{20})C_n\right\} \quad (6)$$

where j represents the number of iterations. Parameters $C_a^{(j)}, C_{n+1}^{(j)}, C_n$ and α in (6) are given by

$$C_a^{(j)} = \left.\frac{\partial q(v)}{\partial v}\right|_{v=v_a^{(j)}} \qquad C_{n+1}^{(j)} = \left.\frac{\partial q(v)}{\partial v}\right|_{v=v_{n+1}^{(j)}}$$

$$C_n = \left.\frac{\partial q(v)}{\partial v}\right|_{v=v_n} \qquad \alpha = m_{11}m_{22} - m_{12}m_{21}$$

Equation (6) is rewritten as a simplified form as:

$$i_{ca}^{(j+1)} = G_{ca}^{(j)} v_a^{(j+1)} + g_{cn+1}^{(j)} v_{n+1}^{(j+1)} + I_{ca}^{(j)}$$

$$i_{cn+1}^{(j+1)} = G_{cn+1}^{(j)} v_{n+1}^{(j+1)} + g_{ca}^{(j)} v_a^{(j+1)} + I_{cn+1}^{(j)} \quad (7)$$

where $G_{ca}^{(j)}$ and $G_{cn+1}^{(j)}$ are conductance's and are defined by

$$G_{ca}^{(j)} = \frac{m_{22}}{h\alpha} C_a^{(j)} \qquad G_{cn+1}^{(j)} = \frac{m_{11}}{h\alpha} C_{n+1}^{(j)}$$

$g_{cn+1}^{(j)}$ and $g_{ca}^{(j)}$ in (7) shows the coefficients of voltage. Simultaneously the coefficient of $g_{cn+1}^{(j)}$ is connected to the equivalent circuit at t_a and controlled by voltage at t_{n+1}. Parameter $g_{ca}^{(j)}$ is connected to the equivalent circuit at t_{n+1} and controlled by voltage at t_a and defined as

$$g_{cn+1}^{(j)} = -\frac{m_{12}}{h\alpha} C_{n+1}^{(j)} \qquad g_{ca}^{(j)} = -\frac{m_{21}}{h\alpha} C_a^{(j)}$$

I_{ca}^{k} and I_{cn+1}^{k} in (7) are defined like

$$I_{ca}^{(j)} = I_{c1}^{(j)} + i_{c1}^{(j)} + I_{c1n}$$

$$I_{cn+1}^{(j)} = I_{c2}^{(j)} + i_{c2}^{(j)} + I_{c2n}$$

where each term is calculated by

$$I_{c1}^{(j)} = \frac{m_{22}}{h\alpha}\left[q\left(v_a^{(j)}\right) + T_t I_s - C_a^{(j)} v_a^{(j)}\right]$$

$$I_{c2}^{(j)} = \frac{m_{11}}{h\alpha}\left[q\left(v_{n+1}^{(j)}\right) + T_t I_s - C_{n+1}^{(j)} v_{n+1}^{(j)}\right]$$

$$i_{c1}^{(j)} = \frac{m_{12}}{h\alpha}\left[C_{n+1}^{(j)} v_{n+1}^{(j)} - q\left(v_{n+1}^{(j)}\right) - T_t I_s\right]$$

$$i_{c2}^{(j)} = \frac{m_{21}}{h\alpha}\left[C_a^{(j)} v_a^{(j)} - q\left(v_a^{(j)}\right) - T_t I_s\right]$$

$$I_{c1n} = \frac{T_t I_s (m_{12} - m_{22})}{h\alpha} + \frac{T_t I_s}{h\alpha}\left[\{(m_{12} - m_{22}) - h(m_{10}m_{22} - m_{12}m_{20})\}q(v_n) + T_t I_s - 1\right]$$

$$I_{c2n} = \frac{T_t I_s (m_{21} - m_{11})}{h\alpha} + \frac{T_t I_s}{h\alpha}\left[\{(m_{21} - m_{11}) - h(m_{10}m_{21} - m_{11}m_{20})\}q(v_n) + T_t I_s - 1\right]$$

The nonlinear physical phenomenon is linearized by applying Newton's methodology on t_a and t_{n+1} both are associated intermediately. Therefore, the equivalent nonlinear electrical model consists of $G_{ga}^{(j)}$ and $G_{gn+1}^{(j)}$ (conductance) and the parallel current sources $I_{ga}^{(j)}$ and $I_{gn+1}^{(j)}$ which are defined as:

$$G_{ga}^{(j)} = \left.\frac{\partial g(v)}{\partial v}\right|_{v=v_a^{(j)}} \qquad G_{n+1}^{(j)} = \left.\frac{\partial g(v)}{\partial v}\right|_{v=v_{n+1}^{(j)}}$$

$$I_{ga}^{(j)} = g\left(v_a^{(j)}\right) - G_{ga}^{(j)} v_a^{(j)} \qquad I_{gn+1}^{(j)} = g\left(v_{n+1}^{(j)}\right) - G_{gn+1}^{(j)} v_{n+1}^{(j)}$$

Homogenous circuit in $(j+1)^{th}$ Newton's iteration and $(n+1)^{th}$ time domain are written like,

$$G_{d1}^{(j)} = G_{ga}^{(j)} + G_{ca}^{(j)} \qquad G_{d2}^{(j)} = G_{gn+1}^{(j)} + G_{cn+1}^{(j)}$$

$$I_{d1}^{(j)} = I_{ga}^{(j)} + I_{ca}^{(j)} \qquad I_{d2}^{(j)} = I_{gn+1}^{(j)} + I_{cn+1}^{(j)}$$

The dimension of equivalent circuits becomes Runge Kutta method of order 4. The voltage controlled by the current i_{CE} is controlled by v_{BC} and v_{BE} and is given by

$$i_{CE} = \frac{I_S}{Q_B}\left(exp\left\{\frac{v_{BE}}{N_F.V_T}\right\} - exp\left\{\frac{v_{BC}}{N_R.V_T}\right\}\right)$$

where Q_B is charge density, N_F and N_R are forward and reverse current emission coefficients. V_T is termed as thermal voltage and is given by kT/q. Forward that $i_{CE} = f_{CE}(v_{BE}, v_{BC})$, and apply Newton's methodology to voltages and current within the intervals. Then $i_{CE(n+1)}, i_{CE(a)}$ are given by

$$i_{CE(n+1)}^{(j+1)} = i_{0(n+1)}^{(j)} + g_{BC(n+1)}^{(j)} v_{BC(n+1)}^{(j+1)} + g_{BE(n+1)}^{(j)} v_{BE(n+1)}^{(j+1)} \quad (8)$$

$$i_{CE(a)}^{(j+1)} = i_{0(a)}^{(j)} + g_{BC(a)}^{(j)} v_{BC(a)}^{(j+1)} + g_{BE(a)}^{(j)} v_{BE(A)}^{(j+1)} \quad (9)$$

Parameters $g_{BE(n+1)}^{(j)}$, $g_{BE(a)}^{(j)}$, $g_{BC(n+1)}^{(j)}$ and $g_{BC(a)}^{(j)}$ indicate the coefficients of the current sources. These are invariably controlled by the v_{BE}. The parameters $i_{0(n+1)}^{(j)}$ and $i_{0(a)}^{(j)}$ are referred as current sources. The results of every parameter are shown as follows:

$$g_{BC(n+1)}^{(j)} = \frac{I_S}{Q_{B(n+1)}.N_F.V_T}\ exp\left\{\frac{v_{BE(n+1)}^{(j)}}{N_F.V_T}\right\}$$

$$g_{BC(a)}^{(j)} = \frac{I_S}{Q_{B(a)}.N_F.V_T}\ exp\left\{\frac{v_{BE(a)}^{(j)}}{N_F.V_T}\right\}$$

$$g_{BE(n+1)}^{(j)} = \frac{I_S}{Q_{B(n+1)}.N_R.V_T}\ exp\left\{\frac{v_{BC(n+1)}^{(j)}}{N_R.V_T}\right\}$$

$$g_{BE(a)}^{(j)} = \frac{I_S}{Q_{B(a)}.N_R.V_T}\ exp\left\{\frac{v_{BC(a)}^{(j)}}{N_R.V_T}\right\}$$

$$i_{0(n+1)}^{(j)} = \frac{I_S}{Q_{B(n+1)}}\left(1 - \frac{v_{BE(n+1)}^{(j)}}{N_F.V_T}\right) exp\left\{\frac{v_{BC(n+1)}^{(j)}}{N_F.V_T}\right\} + \left(-1 + \frac{v_{BC(n+1)}^{(j)}}{N_R.V_T}\right) exp\left\{\frac{v_{BC(n+1)}^{(j)}}{N_R.V_T}\right\}$$

$$i_{0(a)}^{(j)} = \frac{I_S}{Q_{B(a)}}\left(1 - \frac{v_{BE(a)}^{(j)}}{N_F.V_T}\right) exp\left\{\frac{v_{BC(a)}^{(j)}}{N_F.V_T}\right\} + \left(-1 + \frac{v_{BC(a)}^{(j)}}{N_R.V_T}\right) exp\left\{\frac{v_{BC(a)}^{(j)}}{N_R.V_T}\right\}$$

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Hence the computations of RK method of order 4, yield the exact solutions as well as how to diminish or control the error. The error in interpolation can be expressed as

$$f^{(x)}(t) = 2^x x! * O\left(x^{(2+n)}\right), \qquad n \to \infty$$

approximate values are: $t > h$. Figures 1 to 3 presents the comparison of exact values with RK method of order 4, Heun, Midpoint and Taylors's method with various h values.

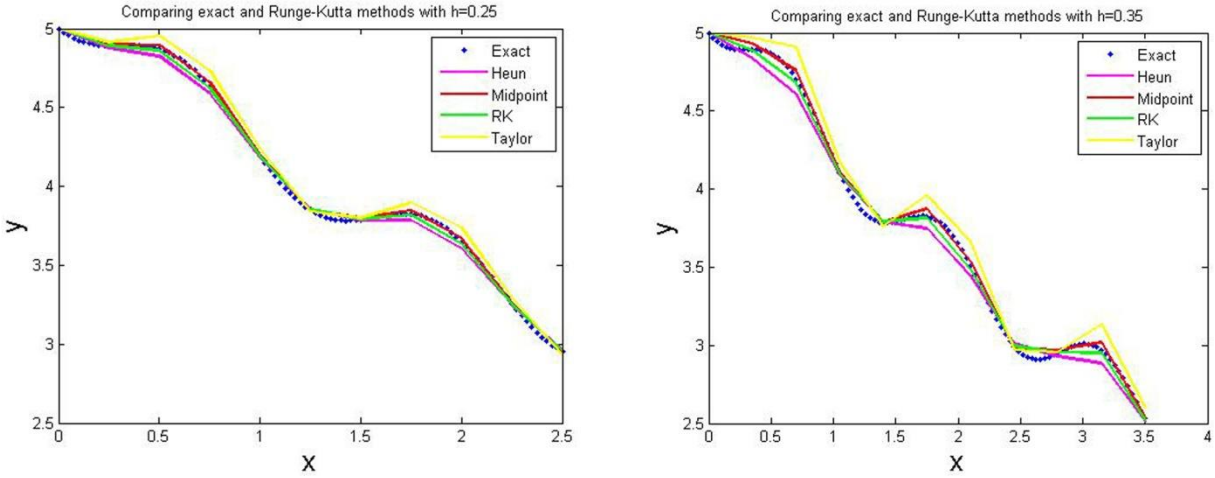

Figure 1: Comparing exact values with RK method of order 4, Heun, Midpoint and Taylors's method when $h = 0.25$ and $h = 0.35$

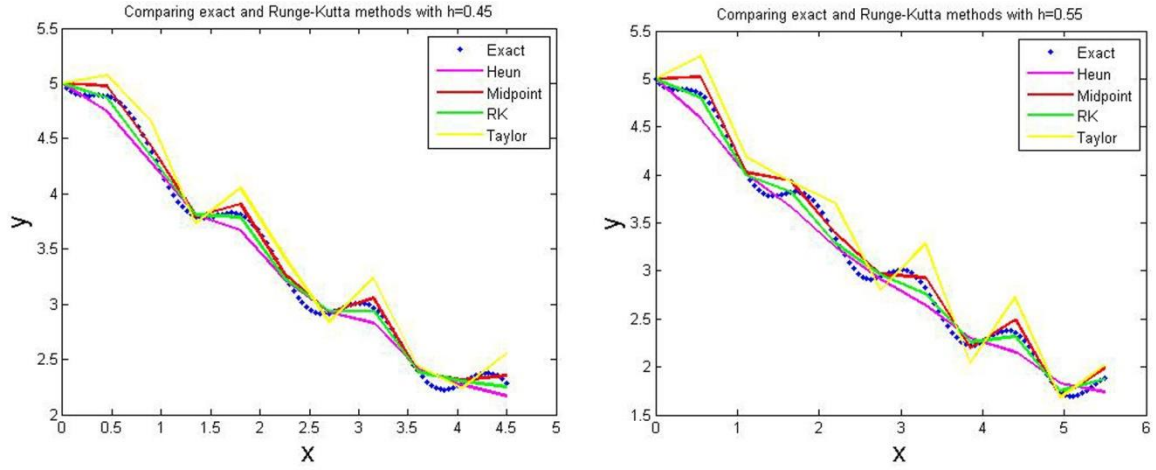

Figure 2: Comparing exact values with RK method of order 4, Heun, Midpoint and Taylors's method when $h = 0.45$ and $h = 0.55$

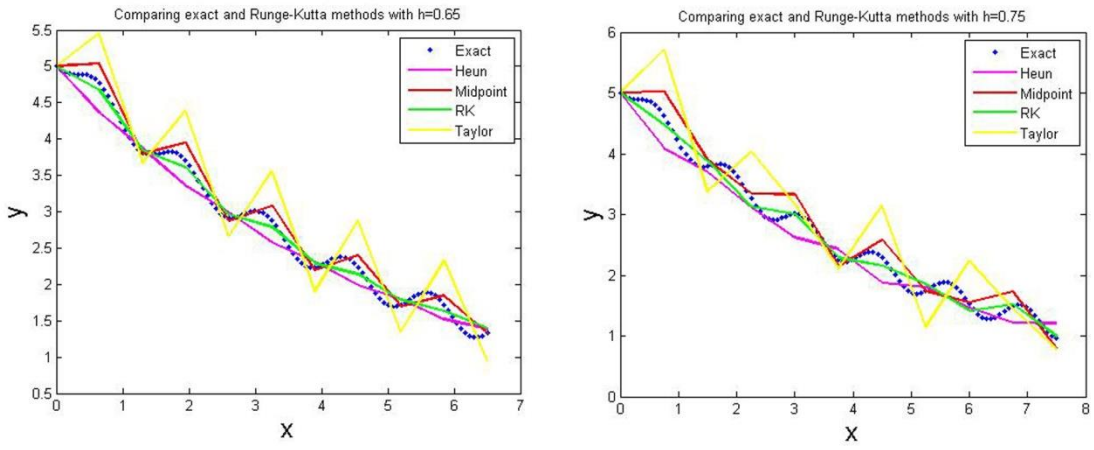

Figure 3: Comparing exact values with RK method of order 4, Heun, Midpoint and Taylors's method when $h = 0.65$ and $h = 0.75$

4 Conclusion

Equilibrium state and array interpretation (Butcher matrix) representation of RK method of order 4 along with the convolution of past, intermediate and present voltages are studied. Constant step size (h) is used until a testing procedure confirms that the discontinuity occurs in the present integration interval. This step size function calculations would take place at the end of the functional calculations, but before the dependent variables were updated. The input parameters indicate the voltage coefficient which is controlled by current sources and measures in a random periodic time. The output parameters provide stable independent values and calculated from past voltage and current values. Implementations of numerical integration for nonlinear capacitors are presented. Finally we compared the exact values with RK method of order 4, Heun, Midpoint and Taylors's method with various h values.

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