Skolem Mean Labeling of Four Star Graphs

$K_{1,a_1} \cup K_{1,a_2} \cup K_{1,a_3} \cup K_{1,b}$ where $|b - (a_1 + a_2 + a_3)| = 4$

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Abstract

In this paper, we prove that four star graph $G = K_{1,a_1} \cup K_{1,a_2} \cup K_{1,a_3} \cup K_{1,b}$ where $a_1 \leq a_2 \leq a_3$ is a skolem mean graph if $|b - (a_1 + a_2 + a_3)| = 4$.

Key words: Skolem mean graph, skolem mean labeling, star graphs.

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1. Introduction

In this paper all graphs are finite, simple and undirected. Terms and notations are used in the sense of Harary [3]. Much work is done by many researchers on skolem mean labelling [1], [2] and [3]. In [5], [6] and [7] some results are proved in four star graph $G = K_{1,a_1} \cup K_{1,a_2} \cup K_{1,a_3} \cup K_{1,b}$ on skolem mean labelling. In this paper, we prove that four star graph $G = K_{1,a_1} \cup K_{1,a_2} \cup K_{1,a_3} \cup K_{1,b}$ where $a_1 \leq a_2 \leq a_3$ is a skolem mean graph if $|b - (a_1 + a_2 + a_3)| = 4$. That is when $b = (a_1 + a_2 + a_3) + 4$ and $b = (a_1 + a_2 + a_3) - 4$.

Definition 1.1 A graph $G = (V, E)$ with $p$ vertices and $q$ edges is said to be a skolem mean graph if there exists a function $f$ from the vertex set of $G$ to $1, 2, \cdots, p$ such that the induced map $f^*$ from the edge set of $G$ to $2, 3, \cdots, p$ defined by

$$f^*(e = uv) = \begin{cases} \frac{f(u) + f(v)}{2} & \text{if } f(u) + f(v) \text{ is even} \\ \frac{f(u) + f(v) + 1}{2} & \text{if } f(u) + f(v) \text{ is odd} \end{cases}$$

the resulting edges get unique labels from the set $2, 3, \cdots, p$.

Theorem 1.2 The four star $G = K_{1,a_1} \cup K_{1,a_2} \cup K_{1,a_3} \cup K_{1,b}$ where $a_1 \leq a_2 \leq a_3$
is a skolem mean graph if \(|b - (a_1 + a_2 + a_3)| = 4\). Proof: Let \(A_i = \sum_{k=1}^{i} a_k\). That is, \(A_1 = a_1; A_2 = a_1 + a_2\) and \(A_3 = a_1 + a_2 + a_3\). Consider the graph \(G = K_{1,a_1} \cup K_{1,a_2} \cup K_{1,a_3} \cup K_{1,b}\). Let \(V = \bigcup_{k=1}^{4} V_k\) be the vertex set of \(G\) where

\[V_k = \{v_{k,i} : 0 \leq i \leq a_k\}\] for \(1 \leq k \leq 3\) and \(V_4 = \{v_{4,i} : 0 \leq i \leq b\}\). Let \(E = \bigcup_{k=1}^{4} E_k\) be the edge set of \(G\) where \(E_k = \{v_{k,0}v_{k,i} : 0 \leq i \leq a_k\}\) for \(1 \leq k \leq 3\) and \(E_4 = \{v_{4,0}v_{4,i} : 0 \leq i \leq b\}\).

The condition \(|b - (a_1 + a_2 + a_3)| = 4\) \(\Rightarrow\) \(b = A_3 - 4\) or \(b = A_3 + 4\).

That is, there are two cases viz. \(b = A_3 - 4\) and \(b = A_3 + 4\).

Let us prove in each of the two cases the graph \(G\) is a skolem mean graph.

**Case 1:** Let \(b = A_3 + 4\)

\(G\) has \(A_3 + b + 4 = 2A_3 + 8\) vertices and \(A_4 + b = 2A_3 + 4\) edges.

The vertex labeling

\[f : V \rightarrow \{1, 2, \cdots, A_3 + b + 4 = 2A_3 + 8\}\]

is defined as follows:

\[
\begin{align*}
    f(v_{1,0}) & = 1; \quad f(v_{2,0}) = 2; \quad f(v_{3,0}) = 4; \\
    f(v_{4,0}) & = A_3 + b + 3 = 2A_3 + 7 \\
    f(v_{1,i}) & = 2i + 4 \quad 1 \leq i \leq a_1 \\
    f(v_{2,i}) & = 2A_1 + 2i + 4 \quad 1 \leq i \leq a_2 \\
    f(v_{3,i}) & = 2A_2 + 2i + 4 \quad 1 \leq i \leq a_3 \\
    f(v_{4,i}) & = 2i + 11 \leq i \leq b - 2 = A_3 + 2 \\
    f(v_{4,b-1}) & = A_3 + b + 2 = 2A_3 + 6 \\
    f(v_{4,b}) & = A_3 + b + 4 = 2A_3 + 8
\end{align*}
\]

The corresponding edge labels are as follows:

The edge label of \(v_{1,0}v_{1,i}\) is \(3 + i\) for \(1 \leq i \leq a_1\) (edge labels are \(4, 5, \cdots, A_1 + 3 = A_1 + 3\)), \(v_{2,0}v_{2,i}\) is \(A_1 + 3 + i\) for \(1 \leq i \leq a_2\) (edge labels are \(A_1 + 4, A_1 + 5, \cdots, A_2 + 3\)), \(v_{3,0}v_{3,i}\) is \(A_2 + 4 + i\) for \(1 \leq i \leq a_3\) (edge labels are \(A_2 + 5, A_2 + 6, \cdots, A_3 + 4\)), \(v_{4,0}v_{4,i}\) is \(A_3 + 4 + i\) for \(1 \leq i \leq b - 2 = A_3 + 2\) (edge labels are \(A_3 + 5, A_3 + 6, \cdots, 2A_3 + 6\)), \(v_{4,0}v_{b-1}\) is \(2A_3 + 7\) and \(v_{4,0}v_{4,b}\) is \(2A_3 + 8\).

These induced edge labels of graph \(G\) are unique.
Hence $G$ is a skolem mean graph.

![Figure 1: $K_{1,5} \cup K_{1,6} \cup K_{1,7} \cup K_{1,22}$](image)

**Case 2:** Let $b = A_3 - 4$

$G$ has $A_3 + b + 4 = 2A_3$ vertices and $A_3 + b = 2A_3 - 4$ edges.

The vertex labeling $f : V \rightarrow \{1, 2, \cdots, A_3 + b + 4 = 2A_3\}$ is defined as follows:

$$
\begin{align*}
&f(v_{1,0}) = 2; \quad f(v_{2,0}) = 4; \quad f(v_{3,0}) = 6; \\
&f(v_{4,0}) = A_3 + b + 4 = 2A_3 \\
&f(v_{1,i}) = 2i - 1 \quad \quad 1 \leq i \leq a_1 \\
&f(v_{2,i}) = 2A_1 + 2i - 1 \quad 1 \leq i \leq a_2 \\
&f(v_{3,i}) = 2A_2 + 2i - 1 \quad 1 \leq i \leq a_3 \\
&f(v_{4,i}) = 2i + 6 \quad 1 \leq i \leq b
\end{align*}
$$

The corresponding edge labels are as follows:

The edge label of $v_{1,0}v_{1,i}$ is $1 + i$ for $1 \leq i \leq a_1$ (edge labels are $2, 3, \cdots, a_1 + 1 = A_1 + 1$), $v_{2,0}v_{2,i}$ is $A_1 + 2 + i$ for $1 \leq i \leq a_2$ (edge labels are $1, 2, \cdots, a_2 + 1 = A_2 + 1$).
$A_1 + 3, A_1 + 4, \cdots , A_2 + 2)$, $v_{3,0}v_{3,i}$ is $A_2 + 3 + i$ for $1 \leq i \leq a_3$ (edge labels are $A_2 + 4, A_2 + 5, \ldots , A_2 + a_3 + 3 = A_3 + 3$), $v_{4,0}v_{4,i}$ is $A_3 + 3 + i$ for $1 \leq i \leq b = A_3 - 4$ (edge labels are $A_3 + 4, A_3 + 5, \cdots , A_3 + 3 + b = A_3 + 3 + A_3 - 4 = 2A_3 - 1$). These induced edge labels of graph $G$ are unique. Hence $G$ is a skolem mean graph.

**Example 1.3**

![Diagram](image)

**Figure 2: $K_{1,6} \cup K_{1,7} \cup K_{1,1} \cup K_{1,20}$**

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