2456-8686, vii(i), 2023 : 120-130
https://doi.org/10.26524/cm167

ORIGINAL RESEARCH **Open Access**

A Study On Different Types Of Edge Sequence In Pseudo Regular Fuzzy Graphs

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Received: 25 April 2023/ Accepted: 25 May 2023/ Published online: 19 June 2023

Abstract

The concept of connectivity plays an important role in both theory and applications of fuzzy graphs. Depending on the strength of an edge, this paper classifies edge sequence of a fuzzy graph in to different types. We analyze the relation between different types of edge sequence in both pseudo regular and totally pseudo regular fuzzy graphs. Also we identify strong edge sequence in pseudo regular fuzzy graph.

Key words: Pseudo regular fuzzy graph, Totally pseudoregular fuzzy graph, α-edge sequence, β-edge sequence, δ-edge sequence, Strong edge sequence.

AMS classification: 46A45, 46B45.

1 Introduction

Euler in 1736 first introduced the concept of graph theory. Fuzzy graph theory is finding numerous application in the fields of information theory, neural network, expert systems, cluster analysis, medical diagnosis, control theory ect. Fuzzy set theory was first introduced by Zadeh in 1965. The first definition of fuzzy graph was introduced by Haufmann in 1973 based on Zadehs fuzzy relations in 1971. In 1975, A.Rosenfeld introduced the concept of fuzzy graphs. Sunil Mathew and Sunitha defined different types of arcs in fuzzy graphs and using them classified fuzzy graphs. Pathinathan and Jesintha Rosline defined relationship between different types of arcs in both regular and totally regular fuzzy graph. Santhi Maheswari and Sekar introduced on pseudo regular fuzzy graphs. Butani and Rosenfeld have introduced the concept of strong arcs. Kalaiarasi defined Optimization of fuzzy integrated vendor-buyer inventory models.

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In this article, the concept of pseudo regular fuzzy graph, totally pseudo regular fuzzy graph and different types of edge sequence are discussed. Also a comparative study is made between pseudo regular and totally pseudo regular fuzzy graphs with reference to different types of edge sequence.

2 Preliminaries

Definition 2.1 A graph $G^* = (V,E)$ is a pair of vertex set V and edge set E where $E : V \times V$ i.e E is a relation on V.

Definition 2.2 2-degree of v is defined as the sum of the degree of the vertices adjacent to v and it is denoted by $t(v)$.

Definition 2.3 Average degree of v is defined as $\frac{t(v)}{d(v)}$, where $t(v)$ is the 2-degree of v and $d(v)$ is the degree of v and it is denoted by $d_a(v)$.

Definition 2.4 A graph is called pseudo regular if every vertex of G has equal average degree.

Definition 2.5 A fuzzy graph G is a pair of function $G : (\sigma,\mu)$ where σ is a fuzzy subset of a non empty set V and μ is a symmetric fuzzy relation on σ. The underlying crisp graph of $G : (\sigma,\mu)$ is denoted by $G^* : (V,E)$ where $E \subseteq V \times V$.

Definition 2.6 The strength of connectedness between two nodes x and y is defined as the maximum of the strengths of all paths between x and y and is denoted by $CONN_G(x,y)$

Definition 2.7 Let $G : (\sigma,\mu)$ be a fuzzy graph on $G^* : (V,E)$. The degree of a vertex u is $d_G(u) = \sum \mu(u,v)$, for $uv \in E$ and $\mu(u,v) = 0$ for uv not in E, this is equivalent to $d_G(u) = \sum \mu(u,v), u \neq v$ and $uv \in E$

Definition 2.8 Let $G : (\sigma,\mu)$ be a fuzzy graph on $G^* : (V,E)$. If $d(v) = k$ for all $v \in V$, then G is said to be a regular fuzzy graph of degree k.

Definition 2.9 Let $G : (\sigma,\mu)$ be a fuzzy graph on $G^* : (V,E)$. A pseudo(average) degree of a vertex v in fuzzy graph G is denoted by $d_a(v)$ and is defined by $d_a(v) = \frac{t_G(v)}{d_G^*(v)}$, where $d_G^*(v)$ is the number of edges incident at v.

Definition 2.10 Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^* : (V, E)$. If $d_a(v) = k$, for all v in V then G is called k-pseudo regular fuzzy graph.

Definition 2.11 Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^* : (V, E)$. The total pseudo degree of a vertex v is G is denoted by $td_a(v)$ and is defined as $td_a(v) = d_a(v) + \sigma(v)$ for all $v \in V$

Definition 2.12 Let G be a fuzzy graph on $G^* : (V, E)$. If all the vertices of G have the same total pseudo degree k, then G is said to be a totally k-pseudo regular fuzzy graph.

Definition 2.13 Let $G : (\sigma, \mu)$ be a pseudo regular fuzzy graph with $\sigma^* = \{v_1, v_2, \ldots, v_i\}$ in some order. Then a finite sequence $\alpha_{edge-seq}(G) = (n_1, n_2, \ldots, n_t)$ is called the α - edge sequence of G if $n_i =$ number of α-strong edges incident on v_i and equal to zero, if no α -strong edges are incident on v_i.

Example 2.1.

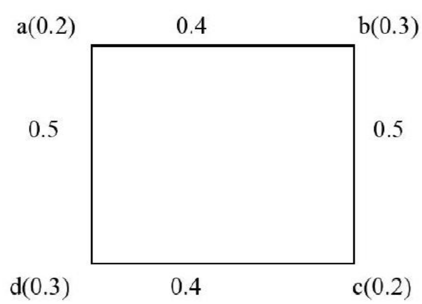

Figure 1: Pseudo regular fuzzy graph with α-edge sequence $\alpha_{edge-seq}(G) = (1, 1, 1, 1)$

Definition 2.14 Let $G : (\sigma, \mu)$ be a pseudo regular fuzzy graph with $\sigma^* = \{v_1, v_2, \ldots, v_t\}$ in some order. Then a finite sequence $\beta_{\text{edge-seq}}\,(G) = (n_1, n_2, \ldots, n_t)$

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is called the β - edge sequence of G if $n_i =$ number of β-strong edges incident on v_i and equal to zero, if no β -strong edges are incident on v_i.

Example 2.2.

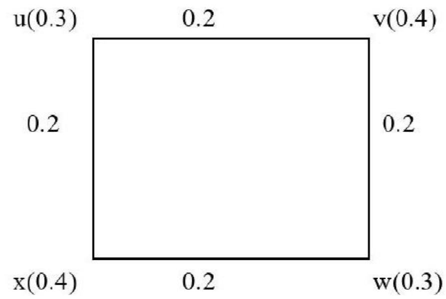

Figure 2: Pseudo regular fuzzy graph with β - edge sequence
$\beta_{edge-seq}(G) = (2,2,2,2)$

Definition 2.15 Let $G : (\sigma, \mu)$ be a pseudo regular fuzzy graph with $\sigma^* = \{v_1, v_2, \ldots, v_t\}$ in some order. Then a finite sequence $\delta_{\text{edge-seq}}\ (G) = (n_1, n_2, \ldots, n_t)$ is called the δ - edge sequence of G if $n_i =$ number of δ-edges incident on v_i and equal to zero, if no δ-edges are incident on v_i

Example 2.3.

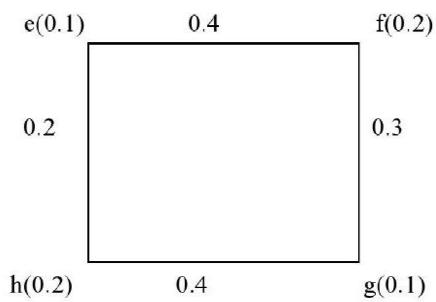

Figure 3: Pseudo regular fuzzy graph with δ - edge sequence
$\delta_{edge-seq}(G) = (1,0,0,1)$

Definition 2.16 Let $G : (\sigma, \mu)$ be a pseudo regular fuzzy graph with $\sigma^* = \{v_1, v_2, \ldots, v_t\}$ in some order. Then a finite sequence $S_{\text{edge-seq}}\ (G) = (n_1, n_2, \ldots, n_t)$ is called the strong edge sequence of G if $n_i =$ number of α or β strong edges incident on v_i and equal to zero, if no α and β strong edges are incident on v_i.

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Example 2.4.

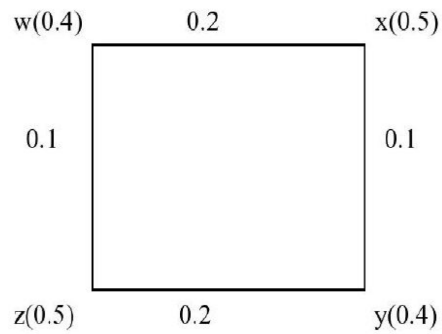

Figure 4: Pseudo regular fuzzy graph with strong edge sequence
$\alpha_{edge-seq}(G) = (1,1,1,1)$ $\beta_{edge-seq}(G) = (1,1,1,1) S_{edge-seq}(G) = (2,2,2,2)$

Example 2.5.

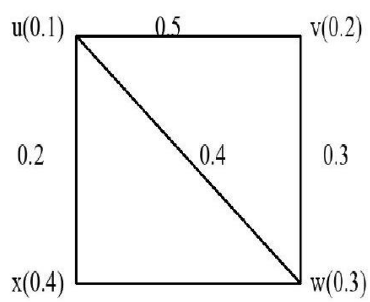

Figure 5: Pseudo regular fuzzy graph without strong edge sequence
$\alpha_{edge-seq}(G) = (1,2,2,1) \delta_{edge-seq}(G) = (1,0,0,1)$

The graph G is pseudo regular fuzzy graph. But $\beta_{edge-seq}(G) = (0,0,0,0)$. Hence G is not a strong edge sequence of fuzzy graph.

Remark 2.17 A strong edge sequence of fuzzy graph need not be a pseudo regular fuzzy graph.

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Example 2.6.

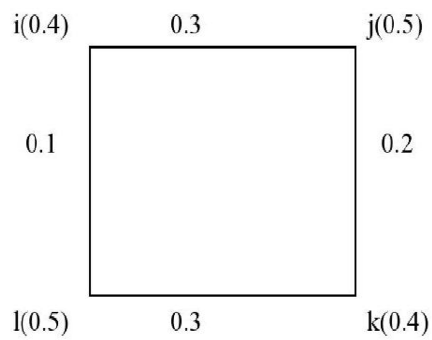

Figure 6: Strong edge sequence of fuzzy graph
$\alpha_{edge-seq}(G) = (2,1,1,0)\beta_{edge-seq}(G) = (1,0,1,2)S_{edge-seq}(G) = (3,1,2,2)$

The graph G is strong edge sequence of fuzzy graph. But $d_a(u) \neq d_a(w)$. Hence G is not a pseudo regular fuzzy graph.

3 Pseudo regular fuzzy graph with some special edge sequence

Theorem 3.1 "Let $G:(\sigma,\mu)$ be a fuzzy graph on $G^*:(V,E)$, a cycle of length n. If μ is a constant function, then G is a pseudo regular fuzzy graph".

Theorem 3.2 "Let $G:(\sigma,\mu)$ be a fuzzy graph on $G^*:(V,E)$, an even cycle of length n. If the alternative edges have same membership values, then G is a pseudo regular fuzzy graph".

Theorem 3.3 A pseudo regular fuzzy graph $G:(\sigma,\mu)$ whose a cycle is the crisp graph $G^*:(V,E)$ contains only β-edge sequence, but no α - edge sequence and δ - edge sequence.

Proof: If pseudo regular fuzzy graph $G:(\sigma,\mu)$ contains only β-edge sequence. This means G does not contain α - edge sequence and δ - edge sequence. Then by the definition, we have $\mu(u,v) = \text{CONN}_{G-(u,v)}(u,v)$. Thus all the edges in G will have same membership value. Then by the theorem, we get G is pseudo regular fuzzy

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graph.
Conversely, Let G be a pseudo regular fuzzy graph. Then by the theorem, membership value μ is a constant function. Thus the deletions of any edge in G will not affect the strength of connectivity of an $x-y$ path in G. That is $\mu(u,v) = CONN_{G-(u,v)}(u,v)$, $\forall(u,v) \in G$. Thus G contains only β-edge sequence.

Example 3.1
Consider a fuzzy graph on $G:(\sigma,\mu)$.

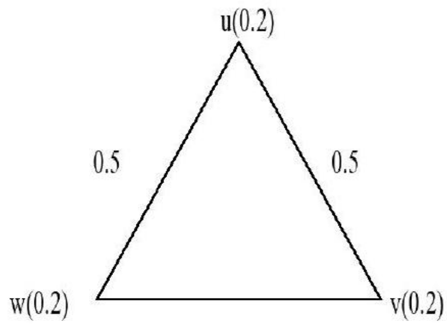

Figure 7: Totally pseudo regular fuzzy graph with β - edge sequence
$\beta_{\text{edge-seq}}\ (G) = (2,2,2)$

The graph is totally 1.2-pseudo regular fuzzy graph with β - edge sequence.

Remark 3.4 The above condition is also true for totally pseudo regular fuzzy graph.

Theorem 3.5 Let $G:(\sigma,\mu)$ be a pseudo regular fuzzy graph on $G^*:(V,E)$, an even cycle. If μ is a alternative edges have same membership values, then G contains only strong edge sequence, but no δ - edge sequence.
Proof: Let $G:(\sigma,\mu)$ be a pseudo regular fuzzy graph on $G^*:(V,E)$, an even cycle. We want to prove that G contains a strong edge sequence. That is we need to prove that G contains only α-edge sequence and β-edge sequence. If G contains only strong edge sequence. That is G has no δ - edge sequence. Then by the definitions, we have $\mu(u,v) \geq CONN_{G-(u,v)}(u,v)$. This means alternative edges have same membership value. Then by the theorem, we get G as a pseudo regular fuzzy graph.

Conversely, Let G be a pseudo regular fuzzy graph. Then by the theorem, the alternative edges have same membership value. That is $\mu(u,v) \geq CONN_{G-(u,v)}(u,v)$. Thus G contains α-edge sequence and β-edge sequence. This means G contains only strong edge sequence.

Example 3.2.
Consider a fuzzy graph on $G:(\sigma,\mu)$.

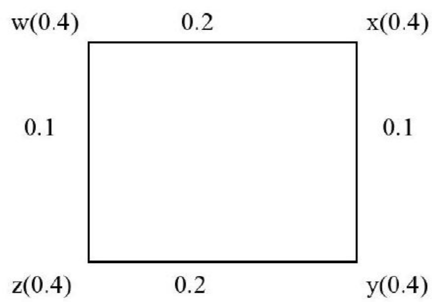

Figure 8: Totally pseudo regular fuzzy graph with strong edge sequence $\alpha_{edge-seq}(G) = (1,1,1,1)\beta_{edge-seq}(G) = (1,1,1,1)S_{edge-seq}(G) = (2,2,2,2)$

The graph is totally 0.7-pseudo regular fuzzy graph with strong edge sequence.

Remark 3.6 The above condition is also true for totally pseudo regular fuzzy graph.

Theorem 3.7 A pseudo regular fuzzy graph $G:(\sigma,\mu)$ with its crisp graph $G^*:(V,E)$ as even cycle is both pseudo regular and totally pseudo regular if G contains either β-edge sequence or strong edge sequence.
Proof: Let $G:(\sigma,\mu)$ be a pseudo regular fuzzy graph. Then its crisp graph $G^*:(V,E)$ as even cycle and G be both pseudo regular and totally pseudo regular fuzzy graph. There are two cases arise.

Case (i) Let G be both pseudo regular and totally pseudo regular fuzzy graph with constant values in σ and μ. Then by the definition G contains only β-edge sequence.
Example 3.3. Consider a fuzzy graph on $G:(\sigma,\mu)$.

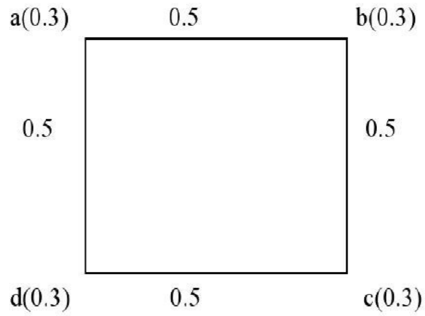

Figure 9: Pseudo regular and totally pseudo regular fuzzy graph
$\beta_{edge-seq}(G) = (2,2,2,2)$

The graph is pseudo regular and totally pseudo regular fuzzy graph with β-edge sequence.

Case (ii) Let G be both pseudo regular and totally pseudo regular fuzzy graph with constant values in σ and with same alternative values in μ. Then by the definitions, G contains α-edge sequence and β-edge sequence. Thus G contains a strong edge sequence.

Example 3.4.
Consider a fuzzy graph on $G : (\sigma, \mu)$.

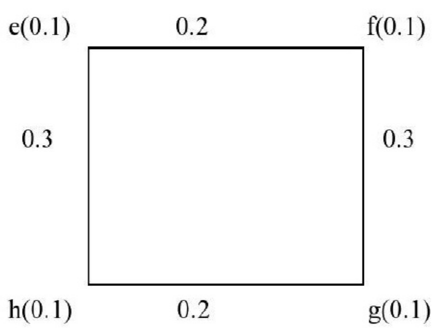

Figure 10: Pseudo regular and totally pseudo regular fuzzy graph with strong edge sequence $\alpha_{\text{edge-seq}}(G) = (1,1,1,1)\beta_{\text{edge-seq}}(G) = (1,1,1,1)S_{\text{edge-seq}}(G) = (2,2,2,2)$

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The graph G is pseudo regular and totally pseudo regular fuzzy graph with strong edge sequence.

4 Conclusion

In this paper we have discussed about fuzzy graphs and Pseudo regular fuzzy graph and defined some definitions, theorems also we have given some examples. We discussed about the different types of edge sequence in Pseudo regular fuzzy graphs.

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