n-Edge Magic Labeling of Splitting Graphs
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Abstract

In this article, we proposed a few Splitting graphs with accept n -EML. Also, we analyze the few graphs which accepts this n- EML.

Key words: Labeling, Magic labeling, n-Edge Magic Labeling [n-EML], Splitting Graph.

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1 Introduction

A graph labeling was first introduced by Rosa in 1967. Origin of the zero-edge magic labeling by Jayapriya J and Thirusangu K in 2012(5). One -edge magic labeling was first introduced in 2013(7) by Neelam Kumari and Seema Mehra, n-edge magic labeling was first developed in 2013(8) by Neelam Kumari and Seema Mehra, In this article on n-edge magic labelling of a few splitting graphs

Definition 1.1 For each node \(X\) about a graph \(G\), holding a recent node \(X'\), link \(X'\) be entire nodes \(G\) adjoining to \(X\). then The graph is splitting Graph \(G\) and is mean by \(S(G)\).

![Image of splitting graph](image)

Definition 1.2 Zero Edge Magic Labeling

A graph \(G\) along s nodes and t edges as a operation from the nodes of \(G\) to \([-1, 1]\)

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s.t every edge xy is determined by the label \( o(x) + o(y) = 0 \). the resulting is called zero edge magic labeling.

**Definition 1.3**: One Edge Magic Labeling

A graph G along s nodes and t edges as a operation from the nodes of G to \([-1, 2]\] s.t every edge xy is determined by the label \( o(x) + o(y) = 1 \). the resulting is called one edge magic labeling.

**Definition 1.4**: \( n \) Edge Magic Labeling

A graph G along s nodes and t edges as a operation from the nodes of G to \([-1, n+1]\] s.t every edge xy is determined by the label \( o(x) + o(y) = n \). the resulting is called \( n \) edge magic labeling.

### 2 Main Results

**Theorem 2.1** \( AS(P_z) \) graph accept \( n-EML \) for every \( z \).

**proof:**

\( AS(P_z) \) graph has \( 2z \) nodes and \( 3z-3 \) edges.

let \( X(G) = x_i : 1 \leq i \leq z \cup ss x'_i : 1 \leq i \leq z \) and

\[ E(G) = x'_ix_{i+1} : 1 \leq i \leq z - 1 \cup x_ix_{i+1} : 1 \leq i \leq z - 1 \cup x'_ix_{i-1} : 2 \leq i \leq z. \]

Let \( o : X \rightarrow [-1, n+1] \) s.t

1. \( o(x_i) = o(x'_i) = (-1)^i : 1 \leq i \leq z \) if \( i \) is odd
2. \( o(x_i) = o(x'_i) = n + 1 : 1 \leq i \leq z \) if \( i \) is even

The edge loads are estimated as displayed:

- For \( 1 \leq i \leq z - 1 \) : \( o(x_i) + o(x_{i+1}) = n \).
- For \( 2 \leq i \leq z \) : \( o(x'_i) + o(x_{i-1}) = n \).
- For \( 1 \leq i \leq z - 1 \) : \( o(x'_i) + o(x_{i+1}) = n \).

So edges gather cost \( n \).then the \( S(P_z) \) graph accept \( n-EML \).

n-EML of \( S(P_6) \) is displayed in the diagram.
Theorem 2.2 A $S(C_z)$ graph accept $n - EML$ only for $z$ is even.

proof:
A $S(C_z)$ graph has $2z$ nodes and $3z$ edges.

let $X(G) = x_i : 1 \leq i \leq z \cup x'_i : 1 \leq i \leq z$ and 
$E(G) = x'_{i+1} : 1 \leq i \leq z - 1 \cup x_i x_{i-1} : 2 \leq i \leq z \cup x_1 x_n, x'_n x_1, x'_1 x_n.$

Let $o : X \rightarrow [-1,n+1]$ s.t

1. $o(x_i) = o(x'_i) = (-1)^i : 1 \leq i \leq z$ if $i$ is odd
2. $o(x_i) = o(x'_i) = n + 1 : 1 \leq i \leq z$ if $i$ is even

The edge loads are estimated as displayed:

For $1 \leq i \leq z - 1 : o(x_i) + o(x_{i+1}) = n.$

For $2 \leq i \leq z : o(x'_i) + o(x_{i+1}) = n.$

For $1 \leq i \leq z - 1 : o(x'_i) + o(x_{i+1}) = n.$

Also

$\quad o(x_1) + o(x_n) = n.$

$\quad o(x'_n) + o(x_1) = n.$

$\quad o(x'_1) + o(x_n) = n.$

So edges gather cost $n$. Then the $S(C_z)$ graph accept $n$-EML only for $z$ is even. n-EML of $S(C_6)$ is displayed in the diagram.

Theorem 2.3 : A $S(K_1,z)$ graph accept n-EML.

Proof: A $S(K_1,z)$ graph has $2z + 2$ nodes and $3z$ edges.

Let $X(G) = \{x_i : 1 \leq i \leq z + 1\} \cup \{x'_i : 1 \leq i \leq z + 1\}$ and
Let \( o : X \to [-1, n+1] \) s.t:

\[
\begin{align*}
(i) & \quad o(x_1) = o(x'_1) = -1 \\
(ii) & \quad o(x_1) = o(x'_i) = n + 1 : 2 \leq i \leq z + 1
\end{align*}
\]

The edge loads are estimated as displayed:

For \( 2 \leq i \leq z + 1 \) :
\[
\begin{align*}
o(x_1) + o(x_i) &= n. \\
o(x'_1) + o(x'_i) &= n if \quad i = 2 and i = z + 1
\end{align*}
\]

So edges gather cost \( n \). Then the \( S(K_{1,z}) \) graph accept \( n \)-EML. 

\( n \)-EML of \( S(K_{1,4}) \) is displayed in the diagram.
Theorem 2.4 A \((B_{yz})\) graph \(y,z \geq 2\) accept \(n-\text{EML}\).

Proof: A \(S(B_{yz})\) graph has \(2y + 2z + 4\) vertices and \(3y + 3z + 1\) edges.

Let \(X(G) = \{x_i, x'_i : 1 \leq i \leq z + 1\} \cup \{y_i, y'_i : 1 \leq i \leq y + 1\}\) and \(E(G) = \{x_i x'_i : 2 \leq i \leq z + 1\} \cup \{y_i y'_i : 2 \leq i \leq y + 1\} \cup \{x'_1 x_i : 2 \leq i \leq z + 1\} \cup \{y'_1 y_i : 2 \leq i \leq y + 1\}\). Let \(o : X \rightarrow [-1, n + 1]\) s.t

- (i) \(o(y_i) = o(y'_i) = -1\) : for \(2 \leq i \leq y + 1\)
- (ii) \(o(x_i) = o(x'_i) = n + 1\) : for \(2 \leq i \leq z + 1\)
- (iii) \(o(y'_1) = o(y_1) = n + 1\)
- (iv) \(o(x'_1) = o(x_1) = -1\)

The edge loads are estimated as displayed:

- For \(2 \leq i \leq z + 1\) : \(o(x_1) + o(x_i) = n\)
- For \(2 \leq i \leq z + 1\) : \(o(x'_i) + o(x_1) = n\)
- For \(2 \leq i \leq z + 1\) : \(o(x'_1) + o(x_i) = n\)
- For \(2 \leq i \leq y + 1\) : \(o(y_1) + o(y_i) = n\)
- For \(2 \leq i \leq y + 1\) : \(o(y'_1) + o(y'_i) = n\)
- For \(2 \leq i \leq y + 1\) : \(o(y'_i) + o(y_1) = n\)

Also

- \(o(y_1) + o(x_1) = n\)
- \(o(y_1) + o(x'_1) = n\)
- \(o(y'_1) + o(x_1) = n\)

So edges gather cost \(n\). Then the \(S(B_{yz})\) graph accept \(n\)-\(\text{EML}\).

\(n\)-\(\text{EML}\) of \(S(B_{3,4})\) is displayed in the diagram.

Theorem 2.5 Splitting graph of every tree is \(n\)-\(\text{EML}\).

Proof: Let \(y\) be a random vertex in the tree \(T\), such that \(\text{deg}(y) = k, k > 1\).

Let consider \(y\) has \(k\) successors \(x_1, x_2, \ldots, x_k\), location all \(x'_i\)'s are pendent then by sum a new vertex \(y'\) and joining we get \(y' x_1, y' x_2, \ldots, y' x_k\), which form \((k - 1)C_4\) cycle namely \(\{y x_1 y' x_2 y x_3 y' x_4 y, \ldots , y x_{k - 1} y' x_k y\}\).

After we include \(x'_1, x'_2, \ldots, x'_k\) and join each \(y'_i\) to the vertex \(y\).
we get k - new pendent edge $yx_1', yx_2', \ldots, yx_k'$.

Consider $o(y) = o(y') = n + 1$ (say),

then $o(x_i') = o(x_i) = -1$.

So edges loads gather n -EML of S(T) with random vertex is displayed in the diagram.

3 Conclusion

In this article reviewed a few Splitting graphs with accept n -EML. More analysis can be done to attain the educate on that a few graphs accept n -EML.

References


