COEFFICIENT OF RANGE LABELING OF SOME GRAPHS

Senthamizh Selvan J¹, Jahir Hussain R²

Received: 20 September 2023/ Accepted: 25 September 2023/ Published online: 16 December 2023

©Sacred Heart Research Publications 2017

Abstract

The coefficient of range labeling is a new type of labeling since it was introduced in 2022 by Senthamizh Selvan and Jahir Hussain. In this paper, we obtained a coefficient of range labeling for the path graph, cycle graph, sun graph, and double star graph is coefficient of range graph. We can also determine the coefficient of range value of the path graph, cycle graph, sun graph, and double star graph.

Key Words: Labeling, coefficient of Range labeling, double star graph, sun graph.

AMS Classification: 05C78,05C90

1 Introduction

In the present study, the graphs here are only undirected, simple, finite graphs are taken into consideration. Vertex set V(G) and edge set E(G) are present in the graph G (V, E). Graph Labelling was first made available. Rosa's [7] research from 1967 is referenced in a lot of graph labeling studies. Rosa developed the function α, which assigns the separate labels | (u)-(v)| to each edge uv in a graph by starting with a set of vertices in the graph G and ending with the set of numbers {0, 1, 2...} Rosa label referred to this as labeling with "β-valuation." In a separate study, Golomb [2] identified the same kind of labeling and dubbed it graceful labeling. [1] has several very intriguing applications of graph labeling. Coefficient of Range labeling is a new type of labeling since it was introduced in 2022 by Senthamizh Selvan and Jahir Hussain [5]. The coefficient of range labeling conditions satisfied graphs; we say the graph is a coefficient of range graph. In this paper coefficient of range labeling applies to some graphs and we can also find the coefficient of range value for those graphs. One of the captivating subfields of graph theory is graph labeling, which has numerous applications in areas like X-ray crystallography, communication networks, and coding theory.

2. PRELIMINARIES

2.1 A Graph G is composed of a finite, non-empty set V and a specified set E containing unordered pairs of unique V members. Each component of V is known as a vertex or point of

¹ Department of Mathematics, Research Scholar, Jamal Mohamed College (Autonomous), Bharathidasan University Tamil Nadu, South India. Email: senthamizh16@gmail.com.
² Department of Mathematics, Associate Professor, Jamal Mohamed College (Autonomous), Bharathidasan University Tamil Nadu, South India. Email: hssn_jhr@yahoo.com
G. Any pair (u, v) of vertices in E is a G edge or line. The edge e = (u, v) is said to connect the vertices u and v. If the vertex pairs u and v are close together and e = (u, v) is a G edge.

2.2 The graph called $G^+ = G \hat{\otimes} K_1$ is created by linking every vertex of a graph G with exactly one pendant edge.

2.3 The sun graph $S_n$ is a series of cycles on n vertices having an edge attached to each cycle vertex that terminates at a vertex of degree 1.

2.4 If A and B are the greatest and smallest observations respectively in a distribution, then its co-efficient of range is given by

$$\text{co-efficient of range (or)C. R} = \frac{x_{\max} - x_{\min}}{x_{\max} + x_{\min}} = \frac{A-B}{A+B}$$

2.5 If $G(V, E)$ be a finite graph. If a function $\alpha : V(G) \rightarrow \{1, 2, 3, \ldots, n\}$ is called a co-efficient of range labeling if the edge labels are used

$\beta : E(G) \rightarrow \{1, 2, 3, \ldots, (n-1)\}$ is defined by

$$\beta[E(G)] = \text{Decrease Numerator of} \left[ \frac{\text{Maximum value}[\alpha(u), \alpha(v)] - \text{minimum value}[\alpha(u), \alpha(v)]}{\text{Maximum value}[\alpha(u), \alpha(v)] + \text{minimum value}[\alpha(u), \alpha(v)]} \right] + 3 \text{minimum value } [\alpha(u), \alpha(v)]$$

by $\text{Maximum value}[\alpha(u), \alpha(v)] - \text{minimum value}[\alpha(u), \alpha(v)]$

(or)

$$\beta[E(G)] = \text{Decrease Numerator of} \left[ \frac{\text{MAX} V[\alpha(u), \alpha(v)] - \text{MIN} V[\alpha(u), \alpha(v)]}{\text{MAX} V[\alpha(u), \alpha(v)] + \text{MIN} V[\alpha(u), \alpha(v)]} \right] + 3 \text{MIN} V[\alpha(u), \alpha(v)]$$

by $\text{MAX} V[\alpha(u), \alpha(v)] - \text{MIN} V[\alpha(u), \alpha(v)]$

If a graph G admits co-efficient of Range labeling, we say G is co-efficient of Range graph.

2.6 Let $G = (V, E)$ be a co-efficient of Range graph, then the co-efficient of Range value is

$$\text{Co-efficient of Range value of } G = \frac{\text{Maximum edge value of } G - \text{Minimum edge value of } G}{\text{Maximum edge value of } G + \text{Minimum edge value of } G}$$

(or)
COERV(G) = \frac{MAXEV(G) - MINEV(G)}{MAXEV(G) + MINEV(G)}

3. MAIN RESULTS

**Theorem 3.1** The Path graph $P_n$ admits co-efficient of Range labeling for every value $n$.

**Proof:** Let $G$ be a path graph.

Where $V(G) = \{v_i, 1 \leq i \leq n\}$ and $E(G) = \{v_i v_{i+1}, 1 \leq i \leq n-1\}$

Let $\alpha : V(G) \rightarrow \{1, 2, 3, ..., n\}$

Such that $\alpha(v_i) = i$ for $1 \leq i \leq n$

We have $\beta(e_i)$

\[
\beta(e_i) = \text{Decrease Numerator of } \left[ \frac{\text{Maximum Value } \alpha(v_i), \alpha(v_{i+1}) - \text{Minimum Value } \alpha(v_i), \alpha(v_{i+1})}{\text{Maximum Value } \alpha(v_i), \alpha(v_{i+1}) + \text{Minimum Value } \alpha(v_i), \alpha(v_{i+1})} \right]
+ 3 \text{ Minimum Value } \alpha(v_i), \alpha(v_{i+1})
\]

by $\text{Maximum Value } \alpha(v_i), \alpha(v_{i+1}) - \text{Minimum Value } \alpha(v_i), \alpha(v_{i+1})$

If $\alpha(v_i)$ is a maximum value and $\alpha(v_{i+1})$ is a minimum value

\[
\beta(e_i) = \text{Decrease Numerator of } \left[ \frac{\alpha(v_i) - \alpha(v_{i+1})}{\alpha(v_i) + \alpha(v_{i+1}) + 3\alpha(v_{i+1})} \right]
\]

by $[\alpha(v_i) - \alpha(v_{i+1})]$

\[
= \frac{3 \alpha(v_{i+1})(\alpha(v_i) + \alpha(v_{i+1}))}{\alpha(v_i) + \alpha(v_{i+1})} = 3 \alpha(v_{i+1})
\]

Suppose $\alpha(v_{i+1})$ is a maximum value and $\alpha(v_i)$ is a minimum value

\[
\beta(e_i) = \text{Decrease Numerator of } \left[ \frac{\alpha(v_{i+1}) - \alpha(v_i)}{\alpha(v_i) + \alpha(v_{i+1})} + 3\alpha(v_i) \right]
\]

by $[\alpha(v_{i+1}) - \alpha(v_i)]$

\[
= \frac{3 \alpha(v_i)[\alpha(v_{i+1}) + \alpha(v_i)]}{\alpha(v_{i+1}) + \alpha(v_i)} = 3 \alpha(v_i)
\]

Hence, every path graph $P_n$ admits co-efficient of Range labeling.
Therefore, any path graph is a co-efficient of range graph.

**EXAMPLE 3.2**

![Diagram of a path graph](image)

Figure 3.1 co-efficient of range labeling $P_7$

\[
\alpha(e_1) = \text{Decrease Numerator of } \frac{2 - 1}{2 + 1} + 3(1) \text{ by } (2 - 1) \\
= \text{Decrease Numerator of } \frac{1}{3} + 3 \text{ by } 1 \\
= \text{Decrease Numerator of } \frac{1 + 3(3)}{3} \text{ by } 1 \\
= \frac{3(3)}{3} \\
\alpha(e_1) = 3
\]

\[
\alpha(e_6) = \text{Decrease Numerator of } \frac{7 - 6}{7 + 6} + 3(6) \text{ by } (7 - 6) \\
= \text{Decrease Numerator of } \frac{1}{13} + 3(6) \text{ by } 1 \\
= \text{Decrease Numerator of } \frac{1 + 3(6)(13)}{13} \text{ by } 1 \\
= \frac{3(6)(13)}{13} = 18
\]

**EXAMPLE 3.3**
REMARK 3.4

If $P_7$ and $P_{10}$ is a Co-efficient of Range graph then we can also find Co-efficient of Range value $P_7$ and $P_{10}$

Co-efficient of Range value $P_7$ and $P_{10}$

$$Co_{-Efficient\;Range\;Value\;of\;(P_7)} = \frac{Maximum\;Edge\;Value\;of\;(P_7) - Minimum\;Edge\;Value\;of\;(P_7)}{Maximum\;Edge\;Value\;of\;(P_7) + Minimum\;Edge\;Value\;of\;(P_7)}$$

$$= \frac{18-3}{18+3} = \frac{15}{21}$$

$CoERV(P_7) = 0.7$

$$Co_{-Efficient\;Range\;Value\;of\;(P_{10})} = \frac{Maximum\;Edge\;Value\;of\;(P_{10}) - Minimum\;Edge\;Value\;of\;(P_{10})}{Maximum\;Edge\;Value\;of\;(P_{10}) + Minimum\;Edge\;Value\;of\;(P_{10})}$$

$$= \frac{27-3}{27+3} = \frac{24}{30}$$

THEOREM 3.5

The cycle graph $C_n$ admits co-efficient of Range labeling for every value $n$.

PROOF

Let $G$ be a cycle graph. Where, $V(G) = \{v_i, 1 \leq i \leq n\}$ and $E(G) = \{v_i v_{i+1}, 1 \leq i \leq n - 1\} \cup \{v_n v_1\}$

Let $\alpha : V(G) \to \{1, 2, 3, ..., n\}$

Such that $\beta(v_i) = i$ for $1 \leq i \leq n$

We have $\beta(e_i)$
If \( \alpha(v_i) \) is a maximum value and \( \alpha(v_{i+1}) \) is a minimum value

\[
\beta(e_i) = \text{Decrease Numerator of } \left[ \frac{\alpha(v_i) - \alpha(v_{i+1})}{\alpha(v_i) + \alpha(v_{i+1})} + 3\alpha(v_{i+1}) \right]
\]

\[
\text{by } [\alpha(v_i) - \alpha(v_{i+1})]
\]

\[
= \frac{3\alpha(v_{i+1})(\alpha(v_i) + \alpha(v_{i+1}))}{\alpha(v_i) + \alpha(v_{i+1})} = 3\alpha(v_{i+1})
\]

Suppose \( \alpha(v_{i+1}) \) is a maximum value and \( \alpha(v_i) \) is a minimum value

\[
\beta(e_i) = \text{Decrease Numerator of } \left[ \frac{\alpha(v_{i+1}) - \alpha(v_i)}{\alpha(v_i) + \alpha(v_{i+1})} + 3\alpha(v_i) \right]
\]

\[
\text{by } [\alpha(v_{i+1}) - \alpha(v_i)]
\]

\[
= \frac{3\alpha(v_i)[\alpha(v_{i+1}) + \alpha(v_i)]}{\alpha(v_{i+1}) + \alpha(v_i)} = 3\alpha(v_i)
\]

\[\beta(e_i)\]

\[
= \text{Decrease Numerator of } \left[ \frac{\text{Maximum Value } [\alpha(v_n), \alpha(v_1)] - \text{Minimum Value } [\alpha(v_n), \alpha(v_1)]}{\text{Maximum Value } [\alpha(v_n), \alpha(v_1)] + \text{Minimum Value } [\alpha(v_n), \alpha(v_1)]} \right]
\]

\[
+ 3 \text{ Minimum Value } [\alpha(v_n), \alpha(v_1)]
\]

\[
\text{by Maximum Value } [\alpha(v_n), \alpha(v_1)] - \text{Minimum Value } [\alpha(v_n), \alpha(v_1)]
\]

If \( \alpha(v_n) \) is a maximum value and \( \alpha(v_1) \) is a minimum value

\[
\beta(e_i) = \text{Decrease Numerator of } \left[ \frac{\alpha(v_n) - \alpha(v_1)}{\alpha(v_n) + \alpha(v_1)} + 3\alpha(v_1) \right]
\]

\[
[\alpha(v_n) - \alpha(v_1)] = \frac{3\alpha(v_1)(\alpha(v_n) + \alpha(v_1))}{\alpha(v_n) + \alpha(v_1)} = 3\alpha(v_1)
\]

Suppose \( \alpha(v_1) \) is a maximum value and \( \alpha(v_n) \) is a minimum value
Hence, every cycle graph \( C_n \) admits co-efficient of range labeling.

Therefore, any cycle graph is a co-efficient of Range graph.

**EXAMPLE 3.6**

![Figure 3.3 co-efficient of range labeling \( C_6 \)](image)

**REMARK 3.7**

If \( C_6 \) is a co-efficient of range graph then we can also find co-efficient of range value,

\[
\text{Co-efficient of Range Value (} C_6 \text{)} = \frac{\text{Maximum Edge Value of } (C_6) - \text{Minimum Edge Value of } (C_6)}{\text{Maximum Edge Value of } (C_6) + \text{Minimum Edge Value of } (C_6)}
\]

\[
= \frac{15 - 3}{15 + 3} = \frac{12}{18} = 0.7
\]

**THEOREM 3.8**

The sun graph \( S_n \) admits co-efficient of range labeling for every value \( n \).

**PROOF**

Let \( G = (V,E) \) be a graph. Let \( v_1, v_2, ..., v_n \) is a vertex of cycle \( S_n \) and let \( u_1, u_2, ..., u_n \) is a end vertices of every fixed to \( v_1, v_2, ..., v_n \)

Let \( \alpha : V(G) \to \{1,2,3,4, ..., n\} \)

\( \alpha_1 : U(G) \to \{n+1, n+2, ..., 2n\} \)
Such that $\alpha(v_i) = i$ for $1 \leq i \leq n$

$\alpha_1(u_i) = i$ for $n + 1 \leq i \leq 2n$

we have $\beta(e_i)$

$$= \text{Decrease Numerator of } \left[ \frac{\text{Maximum Value } [\alpha(v_i), \alpha(v_{i+1})] - \text{Minimum Value } [\alpha(v_i), \alpha(v_{i+1})]}{\text{Maximum Value } [\alpha(v_i), \alpha(v_{i+1})] + \text{Minimum Value } [\alpha(v_i), \alpha(v_{i+1})]} \right]$$

$$+ 3 \text{ Minimum Value } [\alpha(v_i), \alpha(v_{i+1})]$$

by $\text{Maximum Value } [\alpha(v_i), \alpha(v_{i+1})] - \text{Minimum Value } [\alpha(v_i), \alpha(v_{i+1})]$

If $\alpha(v_i)$ is a maximum value and $\alpha(v_{i+1})$ is a minimum value

$$\beta(e_i) = \text{Decrease Numerator of } \left[ \frac{\alpha(v_i) - \alpha(v_{i+1})}{\alpha(v_i) + \alpha(v_{i+1})} + 3\alpha(v_{i+1}) \right]$$

by $[\alpha(v_i) - \alpha(v_{i+1})]$

$$= \frac{3}{\alpha(v_{i+1}) + \alpha(v_i)} = 3\alpha(v_{i+1})$$

Suppose $\alpha(v_{i+1})$ is a maximum value and $\alpha(v_i)$ is a minimum value

$$\beta(e_i) = \text{Decrease Numerator of } \left[ \frac{\alpha(v_{i+1}) - \alpha(v_i)}{\alpha(v_{i+1}) + \alpha(v_i)} + 3\alpha(v_i) \right]$$

by $[\alpha(v_{i+1}) - \alpha(v_i)]$

$$= \frac{3}{\alpha(v_{i+1}) + \alpha(v_i)} = 3\alpha(v_i)$$

$\beta(e_{i1})$

$$= \text{Decrease Numerator of } \left[ \frac{\text{Maximum value } [\alpha(v_i), \alpha_1(u_i)] - \text{Minimum value } [\alpha(v_i), \alpha_1(u_i)]}{\text{Maximum value } [\alpha(v_i), \alpha_1(u_i)] + \text{Minimum value } [\alpha(v_i), \alpha_1(u_i)]} \right]$$

$$+ 3 \text{ Minimum value } [\alpha(v_i), \alpha_1(u_i)]$$

By $\text{Maximum value } [\alpha(v_i), \alpha_1(u_i)] - \text{Minimum value } [\alpha(v_i), \alpha_1(u_i)]$

If $\alpha(v_i)$ is a maximum value and $\alpha_1(u_i)$ is a minimum value

$$\beta(e_{i1}) = \text{Decrease Numerator of } \left[ \frac{\alpha(v_i) - \alpha_1(u_i)}{\alpha(v_i) + \alpha_1(u_i)} + 3\alpha_1(u_i) \right]$$

If $\alpha(v_i)$ is a maximum value and $\alpha(v_{i+1})$ is a minimum value
\[
\beta(e_i) = \text{Decrease Numerator of } \left[ \frac{\alpha(v_i) - \alpha(v_{i+1})}{\alpha(v_i) + \alpha(v_{i+1}) + 3\alpha(v_{i+1})} \right] \\
\text{by } [\alpha(v_i) - \alpha(v_{i+1})] \\
= \frac{3\alpha(v_{i+1})(\alpha(v_i) + \alpha(v_{i+1}))}{(\alpha(v_i) + \alpha(v_{i+1}))} = 3\alpha(v_{i+1})
\]

Suppose \(\alpha(v_{i+1})\) is a maximum value and \(\alpha(v_i)\) is a minimum value

\[
\beta(e_i) = \text{Decrease Numerator of } \left[ \frac{\alpha(v_{i+1}) - \alpha(v_i)}{\alpha(v_i) + \alpha(v_{i+1}) + 3\alpha(v_i)} \right] \\
\text{by } [\alpha(v_{i+1}) - \alpha(v_i)] \\
= \frac{3\alpha(v_i)[\alpha(u_i) + \alpha(v_i)]}{[\alpha(u_i) + \alpha(v_i)]} = 3\alpha(v_i)
\]

Hence, every sun graph admits co-efficient of range labeling.

Therefore, Any Sun graph is a co-efficient of range labeling.

**EXAMPLE 3.9**

![Co-efficient of range labeling of S_6](image)

**Figure 3.4 Co-efficient of range labeling of S_6**

**REMARK 3.10**

If \(S_6\) is co-efficient of range graph then we can also find co-efficient of range value.
Co - Efficient Range Value \((S_6)\)
\[
\text{Co - Efficient Range Value (}\ S_6\ )
= \frac{\text{Maximum edge value of (}S_6\text{) - Minimum edge value of (}S_6\text{)}}{\text{Maximum edge value of (}S_6\text{) + Minimum edge value of (}S_6\text{)}}
= \frac{18 - 3}{10 + 3} = \frac{15}{21} = 0.7
\]

**THEOREM 3.11**

The double star graph \(S_{m,n}\) admits co-efficient of range labeling for every value \(n\).

**PROOF**

Let \(G\) be a double star graph.

It is fixed by \(S_{m,n}\) and \(v_1, v_2\) these two vertices in \(S_{m,n}\) Type equation here which are not pendant. Consider \(u_i\)’s are \(m\) pendant vertices to \(v_1\) and \(w_j\)’s are \(n\) pendant vertices to \(v_2\).

Let \(\alpha : V(G) \rightarrow \{1,2,3,\ldots,n\}\)

Such that \(\alpha(v_1) = 1\)
\[
\alpha(v_2) = 2
\]
\[
\alpha(u_i) = i \text{ if } 3 \leq i \leq m
\]
\[
\alpha(w_j) = i \text{ if } m + 1 \leq i \leq n
\]

We have
\[
\beta(e_i)
= \text{Decrease Numerator of } \left[\frac{\text{Maximum value}[\alpha(v_1), \alpha(v_2)] - \text{Minimum value}[\alpha(v_1), \alpha(v_2)]}{\text{Maximum value}[\alpha(v_1), \alpha(v_2)] + \text{Minimum value}[\alpha(v_1), \alpha(v_2)]}\right]
+ 3\text{Minimum value}[\alpha(v_1), \alpha(v_2)]
\]

by \(\text{Maximum value}[\alpha(v_1), \alpha(v_2)] - \text{Minimum value}[\alpha(v_1), \alpha(v_2)]\)

If \(g(v_1)\) is a maximum value and \(\alpha(v_2)\) is a minimum value
\[
\beta(e_i) = \text{Decrease Numerator of } \left[\frac{\alpha(v_1) - \alpha(v_2)}{\alpha(v_1) + \alpha(v_2)} + 3\alpha(v_2)\right]
\]

by \(\alpha(v_1) - \alpha(v_2)\)
\[
= \frac{3\alpha(v_2)[\alpha(v_1) + \alpha(v_2)]}{\alpha(v_1) + \alpha(v_2)} = 3\alpha(v_2)
\]

Suppose \(\alpha(v_2)\) is a maximum value and \(\alpha(v_1)\) is a minimum value
\[ \beta(e_i) = \text{Decrease Numerator of } \left[ \frac{\alpha(v_2) - \alpha(v_1)}{\alpha(v_2) + \alpha(v_1)} + 3\alpha(v_1) \right] \]

by \( [\alpha(v_2) - \alpha(v_1)] \)

\[ = \frac{3\alpha(v_1)[(\alpha(v_1)+\alpha(v_2)]}{\alpha(v_1)+\alpha(v_2)} = 3\alpha(v_1) \]

\[ \beta(e_{1i}) = \text{Decrease Numerator of } \left[ \text{Maximum value}[\alpha(v_1),\alpha(u_i)] - \text{Minimum value}[\alpha(v_1),\alpha(u_i)] \right] \]

\[ + 3\text{Minimum value}[\alpha(v_1),\alpha(u_i)] \]

By \( \text{Maximum value}[\alpha(v_1),\alpha(u_i)] - \text{Minimum value}[\alpha(v_1),\alpha(u_i)] \)

If \( \alpha(v_1) \) is a maximum value and \( \alpha(u_i) \) is a minimum value

\[ \beta(e_{1i}) = \text{Decrease Numerator of } \left[ \frac{\alpha(v_1) - \alpha(u_i)}{\alpha(v_1) + \alpha(u_i)} + 3\alpha(u_i) \right] \]

By \( [\alpha(v_1) - \alpha(u_i)] \)

\[ = \frac{3\alpha(u_i)[\alpha(v_1)+\alpha(u_i)]}{\alpha(v_1)+\alpha(u_i)} = 3\alpha(u_i) \]

Suppose \( \alpha(u_i) \) is a maximum value and \( \alpha(v_1) \) is a minimum value.

\[ \beta(e_{1i}) = \text{Decrease Numerator of } \left[ \frac{\alpha(u_i) - \alpha(v_1)}{\alpha(u_i) + \alpha(v_1)} + 3\alpha(v_1) \right] \]

By \( [\alpha(u_i) - \alpha(v_1)] \)

\[ = \frac{3\alpha(v_1)[\alpha(v_1)+\alpha(u_i)]}{\alpha(v_1)+\alpha(u_i)} = 3\alpha(v_1) \]

\[ \beta(e_{2i}) = \text{Decrease Numerator of } \left[ \frac{\alpha(v_2) - \alpha(v_1)}{\alpha(v_2) + \alpha(v_1)} + 3\alpha(v_1) \right] \]

by \( [\alpha(v_2) - \alpha(v_1)] \)

\[ = \text{Decrease Numerator of } \left[ \text{Maximum value}[\alpha(v_2),\alpha_1(w_j)] - \text{Minimum value}[\alpha(v_2),\alpha_1(w_j)] \right] \]

\[ + 3\text{Minimum value}[\alpha(v_2),\alpha_1(w_j)] \]

by \( \text{Maximum value}[\alpha(v_2),\alpha_1(w_j)] - \text{Minimum value}[\alpha(v_2),\alpha_1(w_j)] \)
If $\alpha(v_2)$ is a maximum value and $\alpha(w_j)$ is a minimum value

$$
\beta(e_{2i}) = \text{Decrease Numerator of } \left[ \frac{\alpha(v_2) - \alpha(w_j)}{\alpha(v_2) + \alpha(w_j)} + 3\alpha(w_j) \right]
$$

by $[\alpha(v_2) - \alpha(w_j)]$

$$
= \frac{3\alpha(w_j)[\alpha(v_2)+\alpha(w_j)]}{\alpha(v_2)+\alpha(w_j)} = 3\alpha(w_j)
$$

Suppose $\alpha_1(w_j)$ is a maximum value and $\alpha(v_2)$ is a minimum value,

$$
\beta(e_{2i}) = \text{Decrease Numerator of } \left[ \frac{\alpha_1(w_j) - \alpha(v_2)}{\alpha_1(w_j) + \alpha(v_2)} + 3\alpha(v_2) \right]
$$

by $[\alpha_1(w_j) - \alpha(v_2)]$

$$
= \frac{3\alpha(v_2)[\alpha_1(w_j)+\alpha(v_2)]}{\alpha_1(w_j)-\alpha(v_2)} = 3\alpha(v_2)
$$

Hence, Every Double star graph admits co-efficient of range labeling.

Therefore, Any Double star graph is a co-efficient of range graph.

**EXAMPLE 3.12**

![Figure 3.5 Co-efficient of range labeling of $S_{3,3}$](image.png)

**REMARK 3.13**

If $S_{3,3}$ is a co-efficient of range graph, then we can also find co-efficient of range value.

$$
\text{Co - Efficient of Range Value } (S_{3,3})
= \frac{\text{Maximum edge value of } (S_{3,3}) - \text{Minimum edge value of } (S_{3,3})}{\text{Maximum edge value of } (S_{3,3}) + \text{Minimum edge value of } (S_{3,3})}
= \frac{6-3}{6+3} = \frac{3}{9} = 0.3
$$
4 Conclusion

This research has significantly advanced coefficient of range labeling, a novel form of labeling. In this work, the ideas behind coefficient of range labeling of graphs have been covered. We concentrated on demonstrating the coefficient of range graph status of the path graph, cycle graph, sun graph, double star graphs. moreover, we may discover the coefficient of range value of those graphs. There are still a few additional families that may be investigated further.

References