A Few Separation Axioms on Na*AS - closed Sets in Nano Topological Space

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Abstract

This work goal is to gain the relation between a number of the existing sets using inducing and investigating the capabilities of Na*AS -T0 space, Na*AS-T1 space, Na*AS -T2 space.

Key Words: Nano-T0 space, Na*AS -T0 space, nano-T1 space, Na*AS -T1 space, nano-T2 space, Na*AS -T2 space.

AMS Classification: 54A05

1 Introduction

Lellis Thivagar and Richard introduced the idea of nano topology initially. In nano topological spaces, Nasef et al. have found multiple really open sets. Anbarasi Rodrigo and Sahaya Dani developed the concept of Na*AS- closed sets. Sathishmohan et al. introduce the idea of nano neighborhoods in nano topological spaces. This motivates the author to present and examine the properties of Na*AS -T0 space, Na*AS -T1 space, and Na*AS -T2 space in Nano topological spaces.

2 Groundworks

Exegesis 2.1: For c, d ∈ U and c ≠ d, ∃ a nano-open set G such that c ∈ G and d ∈ G, U is referred to as nano-T0 (or N-T0) space.

Exegesis 2.2: For c, d ∈ U and c ≠ d, ∃ a nano semi-open [resp. nano pre-open] set G such that c ∈ G and d ∈ G, a space U is referred to as a nano semi-T0 (or NS-T0) [resp. nano pre-T0 (or NP-T0)] space.

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3 Na*_{AS} - T_0 space

Exegesis 3.1: A space U is referred to as Na*_{AS} -T_0 space for c, d ∈ U and c ≠ d, ∃ a Na*_{AS} - closed set G such that c ∈ G and d ∉ G.

Illustration 3.2: Let X = {1, 2, 3} with τ = {X, ∅, {1}, {2}, {3}, {1,2}, {1,3}, {2,3}}, let σ = {X, ∅, {1}, {2}, {3}}. Then Na*_{AS} - closed units are = {X, ∅, {1}, {2}, {1,2}, {1,3}, {2,3}}. permit c = {2} and d = {3} wherein c, d ∈ X and c ≠ d. Let G = {1, 2}, as a consequence c ∈ G however d ∉ G.

Theorem 3.3: Permits (U, τ_R(X)) to be a nano topological space, then every N-T_0 space is Na*_{AS} -T_0 space but now not conversely.

Illustration 3.4: In illustration 3.2 nano closed sets are = {X, ∅, {1}, {2}, {3}, {1,2}, {1,3}, {2,3}}. let c = {2} and d = {3} then it's miles Na*_{AS} - T_0 space but no longer conversely.

Theorem 3.5: Each NS-T_0 (resp. NP-T_0, Nα-T_0) space is Na*_{AS} -T_0 space but no longer conversely.

Example 3.6: From illustration 3.2, Let c = {2} and d = {3} then it's miles Na*_{AS} -T_0 space but no longer NS-T_0 space.

Illustration 3.7: From illustration 3.2, permit c = {2} and d = {3} then it's miles Na*_{AS} -T_0 space but no longer NP-T_0 space.

Illustration 3.8: From illustration 3.2, Let c = {2} and d = {3} then it's far Na*_{AS} - T_0 space however not Nα-T_0 space.

Theorem 3.9: A nano topological space U is Ncl{c} iff Na*_{AS} cl{c} ≠ Na*_{AS} cl{d} for c ≠ d in U.

proof: Let c, d ∈ U and c ≠ d with U as Na*_{AS} -T_0 space. We should show that Na*_{AS} cl{c} ≠ Na*_{AS} cl{d}. don't forget the set A = U -{c}, it's far clear that Ncl(A) is both A or U. If Ncl(A) = A then A is nano-closed and consequently Na*_{AS} -closed. consequently U - A = {c} is a Na*_{AS} -open set which includes c but no longer d. So, c ∉ Na*_{AS} cl{d}; however, c ∈ Na*_{AS} cl{c} and for this reason Na*_{AS} cl{c} ≠ Na*_{AS} cl{d}. If Ncl(A) = U, then A is Na*_{AS} -open and so U - A = {c} is Na*_{AS} -closed. therefore Na*_{AS} cl{c} = {c}. considering that d ∉ {c} and d ∈ Na*_{AS} cl{d}, it follows that Na*_{AS} cl{c} ≠ Na*_{AS} cl{d}.

Conversely: For c, d ∈ U and c ≠ d. permit Na*_{AS} cl{c} ≠ Na*_{AS} cl{d}. consequently ∃ a factor z in U such that z ∈ Na*_{AS} cl{c} however z ∉ Na*_{AS} cl{d}. If we suppose that c ∈ Na*_{AS} cl{d} then Na*_{AS} cl{c} ⊂ Na*_{AS} cl{d} and this means z ∈ Na*_{AS} cl{d} that is a contradiction. consequently, our supposition is incorrect, i.e., c ∈ Na*_{AS} cl{d} implies c ∈ U - Na*_{AS} cl{d} and Na*_{AS} cl{d} is a Na*_{AS} - closed set containing c but no longer d. this means U is Na*_{AS} -T_0 space.

4 Na*_{AS} - T1 space

Exegesis 4.1: A space U is called nano α-T1 (or Naα-T1) space for c, d ∈ U and c ≠ d, ∃ a Na-open sets G and H such that c ∈ G, d ∉ G and d ∈ H, c ∉ H.
**Exegesis 4.2:** A space $U$ is referred to as $\mathcal{N}_{\alpha}*A_{S}$-$T_{1}$ space for $c, d \in U$ and $c \neq d$, $\exists$ a $\mathcal{N}_{\alpha}*A_{S}$ -closed units $G$ and $H$ such that $c \in G$, $d \notin G$ and $d \in H$, $c \notin H$.

**Theorem 4.3:** Each nano-$T_{1}$ (resp. $\mathcal{N}_{S}-T_{1}$, $\mathcal{N}_{P}-T_{1}$, $\mathcal{N}_{\alpha}-T_{1}$) space is $\mathcal{N}_{\alpha}*A_{S}$-$T_{1}$ space however not conversely.

**Illustration 4.4:** Let $X = \{1, 2, 3, 4\}$, $\tau_{R}(X) = \{U, \varnothing, \{1\}, \{1, 2, 4\}, \{2, 4\}\}$ be a nano topology on $X$, we've got Let $c = \{2\}$ and $d = \{3\}$ then it's far $\mathcal{N}_{\alpha}*A_{S}$-$T_{1}$ space but now not $\mathcal{N}_{S}$-$T_{1}$ space.

**Illustration 4.5:** From illustration 4.4, let $c = \{1\}$ and $d = \{3\}$ then it is $\mathcal{N}_{\alpha}*A_{S}$-$T_{1}$ space however no longer $\mathcal{N}_{S}$-$T_{1}$ space.

**Lemma 4.7:** Let $C$ and $D$ be the subsets of $U$ such that $C \subset D$ and $D$ is $\mathcal{N}_{\alpha}*A_{S}$ -closed, then $C$ is $\mathcal{N}_{\alpha}*A_{S}$ -closed subset of $D$ if $C$ is $\mathcal{N}_{\alpha}*A_{S}$ -closed subset of $U$.

**Lemma 4.8:** For any subset $A$ of $U$

(1) $\mathcal{N}_{\alpha}*A_{S}$ int $(\mathcal{N}_{\alpha}*A_{S}$ cl $(A)) = \mathcal{N}_{\alpha}*A_{S}$ cl $(\mathcal{N}_{\alpha}*A_{S}$ int $(A))$.

(2) $\mathcal{N}$ int $(\mathcal{N}_{\alpha}*A_{S}$ cl $(A)) = \mathcal{N}$ cl $(\mathcal{N}_{\alpha}*A_{S}$ int $(A))$.

(3) $\mathcal{N}$ cl $(\mathcal{N}_{\alpha}*A_{S}$ int $(A)) = \mathcal{N}$ int $(\mathcal{N}_{\alpha}*A_{S}$ cl $(A))$.

**Lemma 4.9:** If $f : (U, \tau_{R}(X)) \rightarrow (V, \tau_{R}(Y))$ is nano-open and nano-non-stop then for any subset $A$ of $U$ then

(1) $f(\mathcal{N}$ int $(A)) \subset \mathcal{N}$ int $(f(A))$.

(2) $f(\mathcal{N}$ cl $(A)) \subset \mathcal{N}$ cl $(f(A))$.

**Theorem 4.10:** Let $(U, \tau_{R}(X))$ be a nano topological space, then for each $\mathcal{N}_{\alpha}*A_{S}$-$T_{1}$ (resp. $\mathcal{N}_{S}$-$T_{1}$, $\mathcal{N}_{P}$-$T_{1}$) space is $\mathcal{N}_{\alpha}*A_{S}$-$T_{0}$ space.

**Illustration 4.11:** Let $U = \{1, 2, 3, 4\}$, and $\tau_{R}(X) = \{U, \varnothing, \{1\}, \{1, 2, 4\}, \{2, 4\}\}$ be a nano topology on $U$, we've got let $G = \{1, 4\}$ and $H = \{2, 3\}$. Let $c = \{1\}$ and $d = \{3\}$, $c, d \in U$ and $c \neq d$, then it's miles clean that $c \in G$, $d \notin G$ and $d \in H$ and $c H$. Then we can say that it's miles $\mathcal{N}_{\alpha}*A_{S}$-$T_{0}$ space.

**Theorem 4.12:** For a topological space $U$, each of the subsequent statements is equal

(a) $U$ is $\mathcal{N}_{\alpha}*A_{S}$-$T_{1}$ space.

(b) Every one-factor set is $\mathcal{N}_{\alpha}*A_{S}$ -closed in $U$.

(c) Every subset of $U$ is the intersection of all $\mathcal{N}_{\alpha}*A_{S}$ -open sets containing it.

(d) The intersection of all $\mathcal{N}_{\alpha}*A_{S}$ -open sets containing the point $\{c\}$ in $U$ is $\{c\}$.

**Proof:** (a) $\Rightarrow$ (b): Let $c \in U$, for this reason for and $d \in V$, $d \neq c \exists$ a $\mathcal{N}_{\alpha}*A_{S}$ -open set $G$ containing $d$ but not $c$. for this reason $d \in G \subseteq \{c\}$. Truly $\{c\} = S \{G: d \in \{c\} \} c$ so $\{c\}$ being a union of $\mathcal{N}_{\alpha}*A_{S}$ -open set is $\mathcal{N}_{\alpha}*A_{S}$-open $\Rightarrow \{c\}$ is $\mathcal{N}_{\alpha}*A_{S}$ -closed.

(b) $\Rightarrow$ (c): Suppose each one-factor set is $\mathcal{N}_{\alpha}*A_{S}$ -closed. let $A \subset U$, then for each $d \in A \exists$ a subset $\{d\}$ such that $a \in \{d\}$ and each of those units $\{d\}$ is $\mathcal{N}_{\alpha}*A_{S}$ -open. as a result, $A = \{\{d\}: d \in A\}$ in order that the intersection of all $\mathcal{N}_{\alpha}*A_{S}$ -open sets containing $A$ is the set $A$ itself.
Lemma 4.13: Let f: (U, τ(X)) → (V, τ(Y)) be nano-open and nano-continuous then for each Na*AS-open set A of U, f(A) is Na*AS-open subset of D.

Theorem 4.14: The belongings of being Na*AS-T1 space is a nano-topological asset.

Theorem 4.15: Each open subspace of a Na*AS-T1 space is Na*AS-T1 space.

Theorem 4.16: Let U be Na-T1 space and f: (U, τR(X)) → (V, τR(Y)) be Na*-closed surjection then D is Na*AS-T1 space.

5 Na*AS-T2 space

Exegesis 5.1: A space U is known as nano α-T2 (or Na-T2) space for c, d ∈ U and c ≠ d, ∃ disjoint Na-open units G and H such that c ∈ G and d ∈ H.

Exegesis 5.2: A space U is known as Na*AS-T2 space for c, d ∈ U and c ≠ d, ∃ disjoint Na*AS-open units G and H such that c ∈ G and d ∈ H.

Lemma 5.3: If A is nano open in U and V is Na*AS-open in U then A ∩ V is Na*AS-open in U.

Lemma 5.4: If f: (U, τR(C)) → (V, τR(D)) is Na-open and Na*AS-continuous, then inverse image of Na*AS-open set is Na*AS-open.

Theorem 5.5: Every nano-T2 space is Na*AS-T2 space however, not conversely.

Illustration 5.6: From the illustration 4.4, permit c = {2} and d = {3} then it is Na*AS-T2 space however now not Na-T2 space.

Theorem 5.7: Every NS-T2 space is Na*AS-T2 space however no longer conversely.

Proof: same as Theorem 5.5

Illustration 5.8: From illustration 4.4, permit c = {1} and d = {3} then it's far Na*AS-T2 space but no longer NS-T2 space.

Theorem 5.9: Every Na-T2 space is Na*AS-T2 (resp. NS-T2, NP-T2) space however no longer conversely.

Proof: identical to Theorem 5.5

Illustration 5.10: From illustration 4.4, Let c = {b} and d = {c} then it's miles Na*AS-T2 space however no longer Na-T2 space.

Theorem 5.11: For the nano topological space U the subsequent are equal (a) U is Na*AS-T2 space.
(b) If c ∈ U, then for every d ≠ c ∃ a Na*AS-neighbourhood G of c such that d ∉ Na*AS cl (G). (c) For each c ∈ U, {Na*AS cl(G): G is a Na*AS-neighbourhood of c} = {c}.

Theorem 5.12: The assets of being Na*AS-T2 space is nano topological a belonging.

Proof: Identical to Theorem 4.14
Theorem 5.13: Every nano-open subspace of $\mathcal{N}_{\alpha}^{* AS}$-T$_2$ space is $\mathcal{N}_{\alpha}^{* AS}$-T$_2$ space.

Theorem 5.14: If $f: (U, \tau_{\mathcal{R}}(C)) \rightarrow (V, \tau_{\mathcal{R}}(D))$ is injective, $\mathcal{N}_{\alpha}$-open, $\mathcal{N}_{\alpha}^{* AS}$-non-stop and $V$ is $\mathcal{N}_{\alpha}^{* AS}$-T$_2$ space then $U$ is $\mathcal{N}_{\alpha}^{* AS}$-T$_2$ space.

Theorem 5.15: If $f: (U, \tau_{\mathcal{R}}(C)) \rightarrow (V, \tau_{\mathcal{R}}(D))$ is injective, $\mathcal{N}_{\alpha}^{* AS}$-continuous and $V$ is nano-T$_2$ space then $U$ is $\mathcal{N}_{\alpha}^{* AS}$-T$_2$ space.

6 Conclusion

Using a few deductions to prove, we have explored and analyzed the resources of the aforementioned arguments on $\mathcal{N}_{\alpha}^{* AS}$-T$_0$, $\mathcal{N}_{\alpha}^{* AS}$-T$_1$, and $\mathcal{N}_{\alpha}^{* AS}$-T$_2$ spaces with some of the prevailing sets.

References


