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On Direct Sum of Four Intuitionistic Fuzzy Graphs

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Abstract

In this paper, the direct sum $G_A \oplus G_B \oplus G_C \oplus G_D$ of four intuitionistic fuzzy graphs (IFGs) $G_A, G_B, G_C\ and\ G_D$ is defined. The Direct sum of regular,connected and effective IFG are also investigated. The degree of the vertices in $G_A \oplus G_B \oplus G_C \oplus G_D$ of the IFGs $G_A, G_B, G_C and G_D$ is calculated.

Key Words: Fuzzy graph, direct sum, IFG, degree of vertices in IFG, regular IFG, connected IFG and effective IFG.

AMS Classification: 05C72

1 Introduction

Azriel Rosenfeld introduced the concept of fussy graph theory in 1975. Dr.K. Radha and Mr.S.Arumugam were defined the direct sum of two fuzzy graphs. K.T.Atanassor introduced the concept of intuitionistic fuzzy graphs. R.Parvathi and M.G.Karunambigai gave the definition of intuitionistic fuzzy graph(IFG). A.Nagoorgani and S.Shajitha Begum defined the various types of degree of vertices in intuitionistic fuzzy graph.Dr.S.Karthikeyan and Mrs.K.Lakshmi defined the direct sum of two IFG. KR.Balasubramanian, K.Sakthivel and N.Arul Pandiyan defined the direct sum of three IFG.

Definition 1.1 An Intuitionistic Fuzzy Graph is of the form G=(V,E), where

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(i)$V = v_1, v_2, \ldots, v_n$ such that $\mu_1: V \to [0,1]$ and $\gamma_1: V \to [0,1]$ denote the degree of membership and non-membership of element $v_i \in V$ respectively and $0 \le \mu_1(v_i) + \gamma_1(v_i) \le 1$, for every $v_i \in V (i=1,2,3,\ldots,n)$.

(ii)$E \subset V \times V$ where $\mu_2: V \times V \to [0,1]$ and $\gamma_2: V \times V \to [0,1]$ are such that $\mu_2(v_i,v_j) \le \min[\mu_1(v_i),\mu_1(v_j)]$, $\gamma_2(v_i,v_j) \le \max[\gamma_1(v_i),\gamma_1(v_j)]$ and $0 \le \mu_2(v_i,v_j) + \gamma_2(v_i,v_j) \le 1$ for every $(v_i,v_j) \in V$.

Here the triple $(v_i, \mu_{1i}, \gamma_{1i})$ denotes the degree of membership and degree of non-membership of the vertex v_i. The triple $(e_{ij}, \mu_{2ij}, \gamma_{2ij})$ denotes the degree of membership and degree of non-membership of the edge $e_{ij} = (v_i, v_j)$ on V.

2 Direct sum $G_A \oplus G_B \oplus G_C \oplus G_D$

Definition 2.1 Let $G_A: [(v_i, \mu_{1iA}, \gamma_{1iA}), (e_{ij}, \mu_{2ijA}, \gamma_{2ijA})]$, $G_B: [(v_i, \mu_{1iB}, \gamma_{1iB}), (e_{ij}, \mu_{2ijB}, \gamma_{2ijB})]$, $G_C: [(v_i, \mu_{1iC}, \gamma_{1iC}), (e_{ij}, \mu_{2ijC}, \gamma_{2ijC})]$ and $G_D: [(v_i, \mu_{1iD}, \gamma_{1iD}), (e_{ij}, \mu_{2ijD}, \gamma_{2ijD})]$ denote four IFGs with underlying crisp graphs $G_A^*: (V_1, E_1)$, $G_B^*: (V_2, E_2)$, $G_C^*: (V_3, E_3)$ and $G_D^*: (V_4, E_4)$ respectively. Let $v \in V_1 \cup V_2 \cup V_3 \cup V_4$ and let $E = \{uv/u, v \in V, uv \in E_1$ or $uv \in E_2$ or $uv \in E_3$ or $uv \in E_4 \}$

Define $G = G_A \oplus G_B \oplus G_C \oplus G_D$ by

$$(\mu_1,\gamma_1)(\mathrm{u}) = \begin{cases} (\mu_{1A},\gamma_{1A}) & if \quad u \in V_1 \\ (\mu_{1B},\gamma_{1B}) & if \quad u \in V_2 \\ (\mu_{1C},\gamma_{1C}) & if \quad u \in V_3 \\ (\mu_{1D},\gamma_{1D}) & if \quad u \in V_4 \\ ((\mu_{1A} \vee \mu_{1B} \vee \mu_{1C} \vee \mu_{1D}), (\gamma_{1A} \wedge \gamma_{1B} \wedge \gamma_{1C} \wedge \gamma_{1D})) & if \quad u \in V_1 \cap V_2 \cap V_3 \cap V_4 \end{cases}$$

and

$$(\mu_2,\gamma_2)(uv) \le \begin{cases} (\mu_{1A}(u) \wedge \mu_{1A}(v), \gamma_{1A}(u) \vee \gamma_{1A}(v)) & if \quad uv \in E_1 \\ (\mu_{1B}(u) \wedge \mu_{1B}(v), \gamma_{1B}(u) \vee \gamma_{1B}(v)) & if \quad uv \in E_2 \\ (\mu_{1C}(u) \wedge \mu_{1C}(v), \gamma_{1C}(u) \vee \gamma_{1C}(v)) & if \quad uv \in E_3 \\ (\mu_{1D}(u) \wedge \mu_{1D}(v), \gamma_{1D}(u) \vee \gamma_{1D}(v)) & if \quad uv \in E_4 \end{cases}$$

Therefore G is called the direct sum of four IFGs G_A, G_B, G_C and G_D.

Example 2.2 The following figure shows the direct sum $G_A \oplus G_B \oplus G_C \oplus G_D$ of three intuitionistic fuzzy graphs G_A, G_B, G_C and G_D which have distinct edge set.

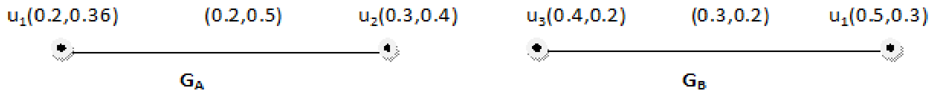

Figure 1:

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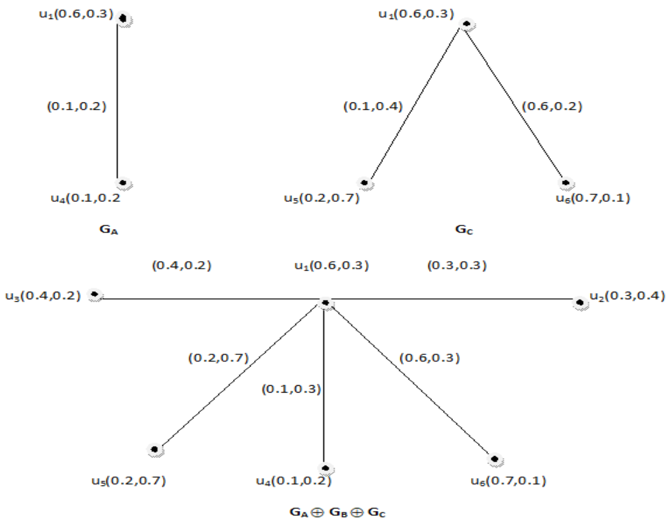

Figure 2

Example 2.3 The following figure shows the direct sum$G_A \oplus G_B \oplus G_C \oplus G_D$ of four intuitionistic fuzzy graphs G_A, G_B, G_C and G_D which edge set are not disjoint.

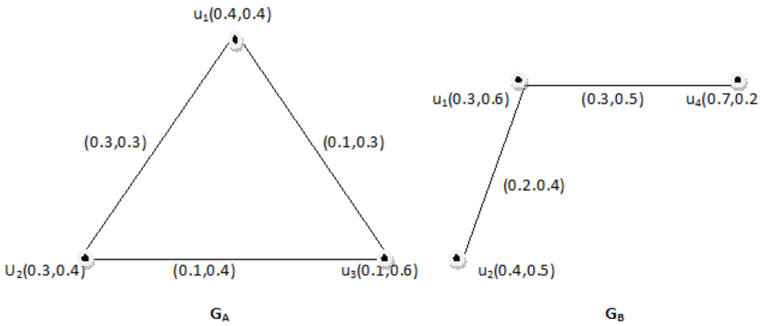

Figure 3

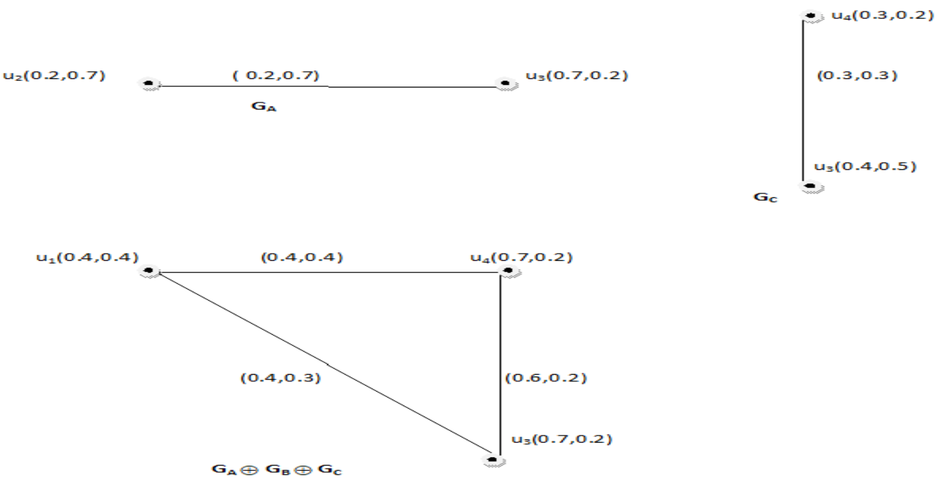

Figure 4

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3 Direct sum of four regular IFGs

Definition 3.1 If the four graphs $G_A: ((v_i, \mu_{1iA}, \gamma_{1iA}), (e_{ij}, \mu_{2ijA}, \gamma_{2ijA}))$, $G_B: ((v_i, \mu_{1iB}, \gamma_{1iB}), (e_{ij}, \mu_{2ijB}, \gamma_{2ijB}))$, $G_C: ((v_i, \mu_{1iC}, \gamma_{1iC}), (e_{ij}, \mu_{2ijC}, \gamma_{2ijC}))$ and $G_D: [(v_i, \mu_{1iD}, \gamma_{1iD}), (e_{ij}, \mu_{2ijD}, \gamma_{2ijD})]$ are four regular intuitionistic fuzzy graphs then their direct sum $G_A \oplus G_B \oplus G_C \oplus G_D$ need not be a regular intuitionistic fuzzy graph.

Example 3.2 Following figure is the direct sum of four regular IFGs.

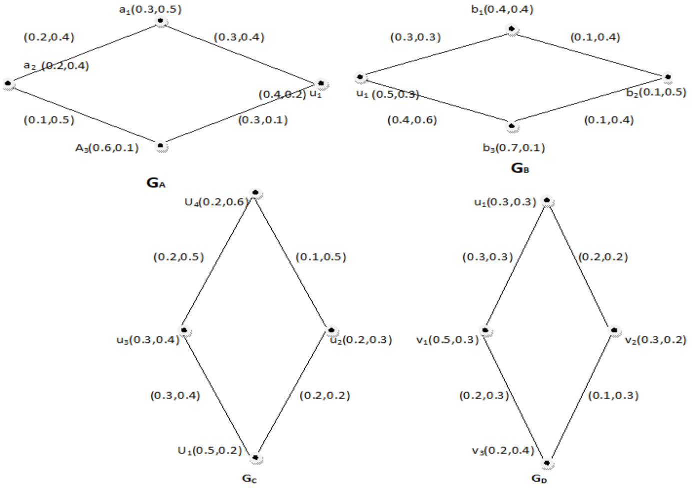

Figure 5

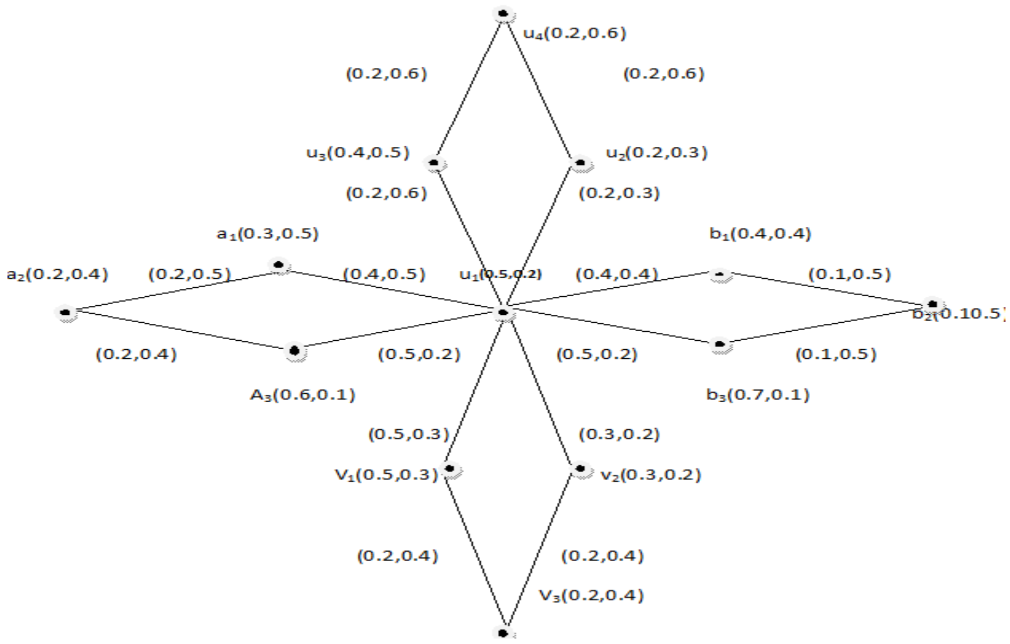

Figure 6

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4 Direct sum of four connected IFGs

Definition 4.1 If the four graphs $G_A: ((v_i, \mu_{1iA}, \gamma_{1iA}), (e_{ij}, \mu_{2ijA}, \gamma_{2ijA}))$, $G_B: ((v_i, \mu_{1iB}, \gamma_{1iB}), (e_{ij}, \mu_{2ijB}, \gamma_{2ijB}))$, $G_C: ((v_i, \mu_{1iC}, \gamma_{1iC}), (e_{ij}, \mu_{2ijC}, \gamma_{2ijC}))$ and $G_D: [(v_i, \mu_{1iD}, \gamma_{1iD}), (e_{ij}, \mu_{2ijD}, \gamma_{2ijD})]$ are four disconnected intuitionistic fuzzy graphs then their direct sum $G_A \oplus G_B \oplus G_C \oplus G_D$ can be a connected intuitionistic fuzzy graph.

Example 4.2 The following figure shows the direct sum of four disconnected IFG.

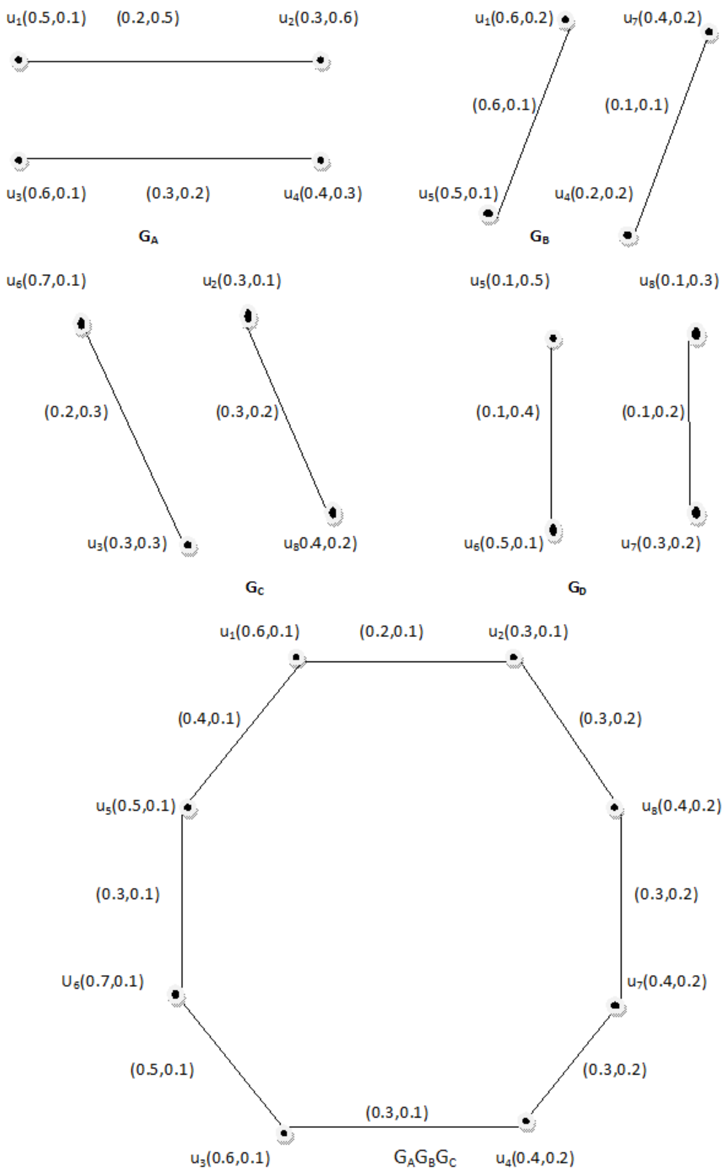

Figure 7

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5 Direct Sum Of Four Effective Intuitionistic Fuzzy Graphs

Definition 5.1 An intuitionistic fuzzy graph G is an effective intuitionistic fuzzy graph if

$$\mu_2(uv) = \mu_1(u) \wedge \mu_1(v) \text{and} \gamma_2(uv) = \gamma_1(u) \wedge \gamma_1(v)$$

for all $u, v \in E$.

Example 5.2

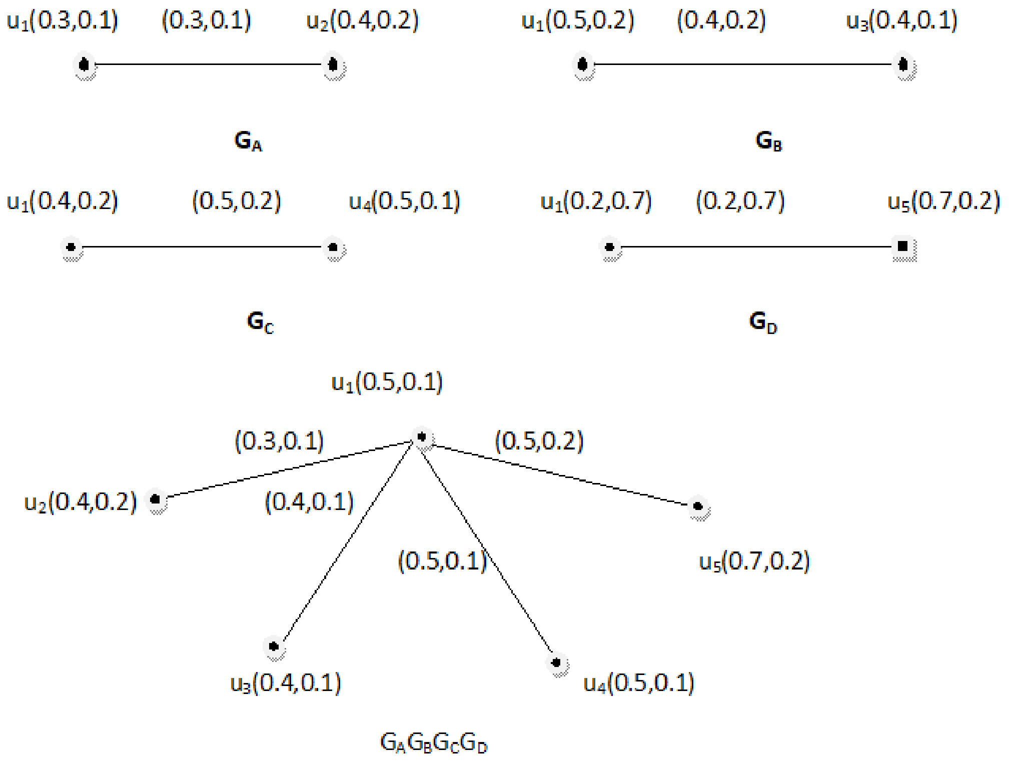

Figure 8

6 Degree of vertex in $G_A \oplus G_B \oplus G_C \oplus G_D$

In this section, we find the degree of vertices in direct sum $G_A \oplus G_B \oplus G_C \oplus G_D$ of four IFGs G_A, G_B, G_C and G_D in terms of degrees of the vertices in the intuitionistic fuzzy graphs G_A, G_B, G_C and G_D .

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Example 6.1 The following illustrate the degree of vertices in $G_A \oplus G_B \oplus G_C \oplus G_D$.

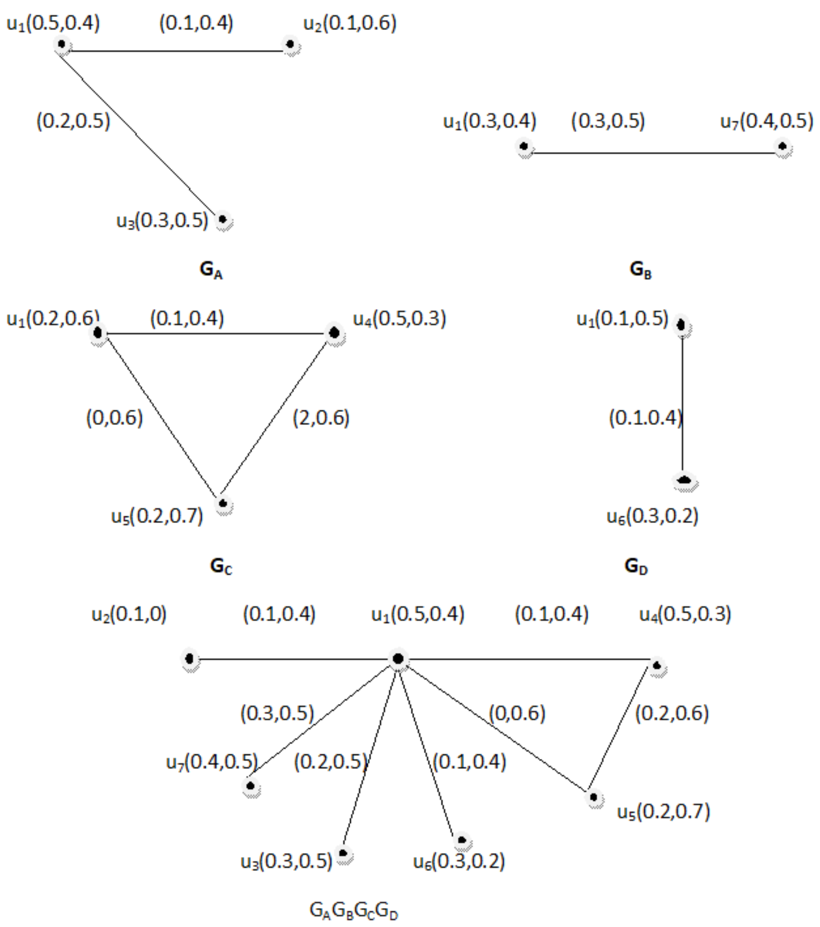

Figure 9

$$d_{G_A} \oplus d_{G_B} \oplus d_{G_C} \oplus d_{G_D}(u) = d_{G_A}(u) + d_{G_B}(u) + d_{G_C}(u) + d_{G_D}(u)$$

For u_1;

(0.1,0.4)+(0.2,0.5)+(0.1,0.4)+(0,0.6)+(0.1,0.4)+(0.3,0.5)

=(0.3,0.5)+(0.1,0.4)+(0,0.6)+(0.2,0.5)+(0.1,0.4)+(0.1,0.4) (0.8,2.8)=(0.8,2.8)

Similarly for all vertices.

7 Conclusion

The direct sum of 4 IFGs were illustrated.The special cases regular, connected and effective IFGs are discussed with examples . Degree of vertices in direct sum of four IFGs are calculated with example.

Therefore, this work will be further developed to various special intutionistic fuzzy graphs based on the researchers interest and on the researchers field.

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