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Application of Dynamic Programming Technique to Reliability Model in Space Mission

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Abstract

The research demonstrates how dynamic programming can be used to improve the reliability of space mission systems. Spacecraft systems require traditional reliability models to operate because their multiple interdependent subsystems face three strict boundaries of cost, weight and power. This method helps the system in modelling as a multistage decision process, applying optimal redundancy at each stage and maximizing the mission success probability. In this paper the recursive relation method is applied to determine the most efficient subsystem configuration based on available resources. A numerical example demonstrates how effective the technique functions. The study concludes that dynamic programming provides a systematic and efficient framework for enhancing reliability in complex, high-risk space missions.

Key Words: Reliability of series configuration, Reliability of parallel configuration,Dynamic programming problem.

1 Introduction

Space mission works with complex operating system in extreme unpredictable environment. The success of the mission depends on the reliability of subsystems, namely propulsion, power production, communication, guidance and control, thermal regulation, and payload instrumentation.

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Reliability of the subsystem refers the probability that each subsystem works without failure for a specified period. Subsystems of space mission are interconnected. A failure of one system may lead to partial or complete mission failure. Thus, engineers focus on reliability modelling and redundancy techniques to improve the performance of the subsystem of space mission. Subsystems are connected by multiple configurations such as series connected system, parallel connected system and series-parallel connected system.

Reliability modelling provides a mathematical framework to analyse and predict the performance of complex space systems. Traditional reliability models often represent systems as series, parallel, or series-parallel configurations and compute overall system reliability based on component reliabilities. Space missions require more advanced approaches because space missions consist of three critical subsystems namely propulsion, navigation and life support. To complete a successful mission these subsystems should perform efficiently. Including the redundancy will increase the system weight cost and power consumption. The proposed model maximizes the probability of mission success by satisfying the constraints on total cost using the Dynamic Programming Technique.

2 SERIES CONFIGURATION

The simplest combination of units that form a system is a series combination. This is also one of the most commonly used structures. In this case, the system consists of n units which are connected in series. Let the successful operation of these individual units be represented by $X_1, X_2, X_3, ...X_n$ and their respective probabilities by $P(X_1), P(X_2), P(X_3), ..., P(X_n)$. For the successful operation of the system, it is necessary that all units function satisfactorily. Hence, the probability of the simultaneous successful operation of all the units is $P(X_1 \textit{ and } X_2 \textit{ and } X_3 ... \textit{and } X_n)$.

3 PARALLEL CONFIGURATION

Several systems exist in which successful operation depends on the satisfactory functioning of any one of their n sub-systems or elements. These are said to be connected in parallel. We can also have a system in which several signal paths perform the same operation, and the satisfactory performance of any one of these paths is sufficient to ensure the successful operation of the system. The elements for such a system are also said to be connected in parallel. Let $X_1, X_2, \cdots, X_n$ represent the successful operation of units $1, 2, \cdots, n,$ respectively. Similarly, let $\overline{X_1}, \overline{X_2}, \cdots, \overline{X_n}$, respectively, represent their unsuccessful operation.

4 GENERAL SERIES-PARALLEL CONFIGURATION

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The system consists of stage $1,2,3,\ldots,k$ connected in series. Each stage contains a number of redundant elements, stage i consisting of n_i redundant elements connected in parallel. The reliability of the system is the product of the reliabilities of each stage. Stage i with n_i elements will have the reliability,

$$R_i = 1-[1-P(X_{i1})][1-P(X_{i2})]\cdots[1-P(X_{in_i})]$$

$$=1-\prod_{j=1}^{n_i}\left[1-P\left(X\_\{ij\}\right)\right]$$

$$R(S)=R_1R_2\ldots R_k=\prod_{i=1}^{k}\left\{\prod_{j=1}^{n_i}\left[1-P\left(X\_\{ij\}\right)\right]\right\}$$

5 REDUNDANCY

By using the technique of establishing redundancies, we may increase system reliability. This entails the system's new parallel routes being purposefully created. The probability $P\left(a\ or\ b\right)=P\left(a\right)+P\left(b\right)-P\left(a\ and\ b\right)$ equals $P(a\,or\,b)=P(a)+P(b)-P(a)\times P(b)$, if two elements with probabilities of success P(a) and P(b) are connected in parallel. Since $P\left(a\right)$ and $P\left(b\right)$ are both less than one on their own, their product is always smaller than $P\left(a\right)\ or\ P\left(b\right)$, assuming the elements are independent. As a result, $P\left(a\ or\ b\right)$ is always higher than $P\left(a\right)\ or\ P\left(b\right)$. We purposefully use both components to raise the probability of success, which makes the system redundant, even though either one is adequate for the system to function successfully.

6 DYNAMIC PROGRAMMING FORMULATION TO A RELIABILITY MODEL

Reliability is a most important requirement for many systems, such as those designed for multistage operation systems. Such systems can be considered as a series of "black boxes" or subsystems. If by chance a subsystem fails, that is to say, increase the reliability of the whole system, redundancy in some or all of the systems is incorporated. This means that instead of using just one subsystems in stage j, two or more are connected in parallel along with switching circuits in such a way that if one operating subsystem fails, one of the redundant subsystem is automatically switched on.

Notations:

- $c_j -$ Cost of one subsystem in stage j.
- M- Total Money available for the entire system.
- $1+x_j -$ The number of subsystems in stage j.
- $p_j(x_j) -$ The probability that the j^{th} stage operates successfully.

- $x_j -$ is the redundancy in the stage

where $x_j \geq 0, x_j \leq m_j$ some given number.

Assuming that $p_j(x_j)$ is independent of what is done for other stages, the reliability of the whole system is $p_1(x_1)\times p_2(x_2)\times\cdots\times p_n(x_n)$. The total cost involved is $\sum_{j=1}^{n} c_j(1+x_j)$.

7 APPLICATION TO A RELIABILITY MODEL

As a numerical example, we apply this of Dynamic Programming Technique to calculate the Reliability of in Space Mission.

Space Missions: Subsystem reliability

A spacecraft has three critical subsystems (eg., life support, navigation, population). Redundant components can be added, each with weight and cost implications.

Objective: Maximize mission success probability while respecting total weight/cost limits.

Objective: Maximize $Z = p_1(x_1)\times p_2(x_2)\times p_3(x_3)$

Constraints: $c_1x_1 + c_2x_2 + c_3x_3 \leq 1{,}30{,}000$ where $x_j \in \{0,1,2\}$

x_j	$p_1(x_1)$	$p_2(x_2)$	$p_3(x_3)$
0	0.6	0.8	0.7
1	0.7	0.8	0.8
2	0.99	0.9	0.9

Solution:

The system reliability is

$$R(S) = \prod_{i=1}^{k}\left[1-\prod_{j=1}^{n_i}(1-P(x_{ij}))\right] \quad (1)$$

WKT, for Maximum Reliability,

$$Z = p_1(x_1)\times p_2(x_2)\times p_3(x_3)$$

$$\sum_{j=1}^{n} c_j x_j \leq M - \sum_{j=1}^{n} c_j = b. \quad (2)$$

$$0 \leq x_j \leq m_j$$

The optimal values in the 'n' stages are given by the recursive relations

$$\xi_{k-1} = \xi_k - c_k x_k, (k = n, n-1, \cdots, 2) \text{ with } \xi_n = b \quad (3)$$

Here, $M = 13,\ c_1 = 2,\ c_2 = 1,\ c_3 = 3$

Sub. $n = 3$ in (2),

$$\Rightarrow M - \sum_{j=1}^{3} c_j = b$$

$$13-(c_1+c_2+c_3)=b$$

$$b=7$$

$$\xi_3 = 7 \text{ by (3)}$$

Now substitute k=3 in (3),

$$\xi_2 = \xi_3 - c_3 x_3$$

$$\xi_2 = 7-3x_3 \qquad (4)$$

k=2 in (3)

$$\xi_1 = \xi_2 - c_2 x_2$$

$$= (7-3x_3)-x_2$$

$$\xi_1 = \xi_2 - x_2 \qquad (5)$$

The admissible values of ξ_2 and ξ_1 are

x_3	0	1	2
ξ_2	7	4	1

And

x_2	0	0	0	1	1	1	2
x_3	0	1	2	0	1	2	0
ξ_1	7	4	1	6	3	0	5

WKT,

$$F_k(\xi_k) = \max_{0 \le x_k \le min\{m_k, [\xi_k / c_k]\}} \{ f_k(x_k) \times F_{k-1}(\xi_k - c_k x_k) \}$$

For $k = n, n-1, \cdots, 2$

At first stage,

$$F_1(\xi_1) = \max_{0 \le x_1 \le min\{2, [\xi_1/2]\}} \{ f_1(x_1) \}$$

Where $\xi_1 = 0,1,3,4,5,6,7$

$\xi_1 = 0, x_1 = 0,1,2$

$$F_1(0) = \max_{0 \le x_1 \le min\{2,0\}} \{ f_1(0) \} = 0.6$$

$\xi_1 = 1$

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$$F_1(1) = \max_{0\le x_1 \le min\{2,[1/2]\}} \{ f_1(0), f_1(1) \} = 0.7$$

$$= max\{0.6, 0.7\}$$

$\xi_1 = 3$

$$F_1(3) = \max_{0\le x_1 \le min\{2,[3/2]\}} \{ f_1(0), f_1(1) \} = 0.7$$

$$= max\{0.6, 0.7\}$$

		$f_1(x_1)$		
ξ_1 x_1	0	1	2	$F_1(\xi_1)$
0	0.6			0.6
1	0.6	0.7		0.7
≥ 3	0.6	0.7		0.7

At second stage,

$$F_2(\xi_2) = \max_{0\le x_2 \le min\{2,[\xi_2/1]\}} \{ f_2(x_2) \times F_1(\xi_1) \}$$

Here, $\xi_2 = 7, 4, 1$

$$F_2(\xi_2) = \max_{0\le x_2 \le min\{2,[\xi_2/1]\}} \{ f_2(x_2) \times F_1(\xi_2 - x_2) \}$$

At $\xi_2 = 7$

$$F_2(7) = \max_{0\le x_2 \le min\{2,[7/1]\}} \{ f_2(x_2) \times F_1(\xi_2 - x_2) \}$$

$$= max\{0.56, 0.56, 0.63\}$$

$$= 0.63$$

At $\xi_2 = 4$

$$F_2(4) = \max_{0\le x_2 \le min\{2,[4/1]\}} \{ f_2(x_2) \times F_1(\xi_2 - x_2) \}$$

$$= max\{ f_2(0) \times F_1(4), f_2(1) \times F_1(3), f_2(2) \times F_1(2)\}$$

$$= max\{0.56, 0.56, 0\} = 0.56$$

$\xi_2 = 1$

$$F_2(4) = \max_{0\le x_2 \le min\{2,[4/1]\}} \{ f_2(x_2) \times F_1(\xi_2 - x_2) \}$$

$$= max\{ f_2(0) \times F_1(4), f_2(1) \times F_1(3), f_2(2) \times F_1(2)\}$$

$$= max\{0.56, 0.56, 0\} = 0.56$$

$At\, \xi_2 = 1$

$$F_2(1) = \max_{0 \le x_2 \le \min\{2,[1/1]\}} \{ f_2(x_2) \times F_1(\xi_2 - x_2) \}$$

$$= \max\{ f_2(0) \times F_1(1),\ f_2(1) \times F_1(0) \}$$

$$= \max\{0.56, 0.48\} = 0.56$$

		$f_2(x_2) \times F_1(\xi_2 - x_2)$		
$\xi_2\ x_2$	0	1	2	$F_2(\xi_2)$
1	0.8×0.7	0.8×0.6		0.56
4	0.8×0.7	0.8×0.7		0.56
7	0.8×0.7	0.8×0.7	0.9×0.7	0.63

At final stage,

$$F_3(\xi_3) = \max_{0 \le x_2 \le \min\{2,[\xi_3/3]\}} \{ f_3(x_3) \times F_2(\xi_2) \}$$

$$F_3(7) = \max_{0 \le x_2 \le \min\{2,[7/3]\}} \{ f_3(x_3) \times F_2(7 - 3x_3) \}$$

At $\xi_3 = 7$

$$F_3(7) = \max\{ f_3(0) \times F_2(7),\ f_3(1) \times F_2(4),\ f_3(2) \times F_2(1) \}$$

$$= \max\{0.7 \times 0.63, 0.8 \times 0.56, 0.9 \times 0.56\} = 0.504$$

		$f_3 \times F_2(7 - 3x_3)$		
$\xi_3\ x_3$	0	1	2	$F_3(\xi_3)$
7	0.7×0.63	0.8×0.56	0.9×0.56	0.504

From the table, we get the maximum value for $x_3 = 2, x_2 = 2, x_1 = 1$

Maximum reliability is 0.504 at a cost of

$$\sum_{j=1}^{3} c_j(1 + x_j) = c_1(1 + x_1) + c_2(1 + x_2) + c_3(1 + x_3) = 16$$

8 Conclusion

This research has successfully examined and demonstrated the use of the dynamic programming technique for dependability modelling in space missions. When decision-making is sequential, mission stages are interdependent, and system configurations vary over time, traditional reliability analysis techniques become insufficient. By breaking down the mission into phases and optimizing reliability choices at each level, dynamic programming successfully tackles these issues.

The dynamic programming-based reliability model offers a practical approach for improving mission success in space applications. Its ability to handle sequential decisions, and complex system interactions makes it a valuable methodology for spacecraft design, mission planning, and autonomous fault management.

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