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On Domination in Path, Cycle, Star, Wheel and Grid Graphs

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Abstract

Domination is one of the important concepts in graph theory with applications in communication networks, facility location problems and social network analysis. In this paper we study domination numbers for several basic graphs including path graphs, cycle graphs, star graphs, wheel graphs and grid graphs. Exact formulas for domination numbers are obtained and relationships between different classes of graphs are discussed. Illustrative examples, tables and diagrams are provided.

Keywords: Graph theory, domination number, path graph, cycle graph, star graph, wheel graph, grid graph.

1 Introduction

Graph theory is a fundamental and rapidly developing area of mathematics with wide-ranging applications in computer science, engineering, biology, and social sciences. Many real-world systems such as transportation networks, communication systems, and social interactions can be effectively modeled using graphs [1, 2].

One of the most important parameters studied in graph theory is domination. The concept of domination plays a crucial role in the analysis of networks where efficient monitoring, control, or coverage is required. A dominating set of a graph provides a subset of vertices such that every vertex in the graph is either in the set or adjacent to a vertex in the set [3].

The study of domination was initiated by [4] and has since become a

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well-established area with numerous theoretical developments and practical applications. Domination theory has been widely applied in areas such as facility location problems, wireless sensor networks, resource allocation, and social network analysis [3].

In recent years, several variations of domination have been introduced to address different structural and application-based requirements. Among these, total domination and connected domination have attracted significant attention. In total domination, every vertex must be adjacent to a vertex in the dominating set, which makes it suitable for problems where self-monitoring is not allowed. On the other hand, connected domination requires that the dominating set induces a connected subgraph, which is particularly useful in communication networks where connectivity among control nodes is essential [5].

The determination of domination parameters for specific graph classes has been an active area of research. Exact values and bounds for domination numbers have been obtained for many standard graphs, including paths, cycles, trees, and product graphs [1, 2]. These results provide insight into the structural properties of graphs and help in designing efficient algorithms for practical applications.

2 Preliminaries

In this section, we present basic definitions and notations that will be used throughout the paper. For standard terminology, we refer to [1, 2].

Definition 2.1. *A graph $G = (V, E)$ consists of a finite non-empty set V of vertices and a set E of edges joining pairs of vertices.*

Definition 2.2. *Two vertices $u, v \in V$ are said to be adjacent if there exists an edge $uv \in E$. The set of all vertices adjacent to a vertex v is called the neighborhood of v and is denoted by $N(v)$.*

Definition 2.3. *The degree of a vertex v, denoted by $\deg(v)$, is the number of vertices adjacent to v. The maximum degree of a graph G is denoted by $\Delta(G)$.*

Definition 2.4. *A subset $D \subseteq V$ is called a dominating set if every vertex in $V - D$ is adjacent to at least one vertex in D.*

Definition 2.5. *The domination number of a graph G, denoted by $\gamma(G)$, is the minimum cardinality of a dominating set.*

Definition 2.6. *A dominating set D is called a minimum dominating set if $|D| = \gamma(G)$.*

Definition 2.7. *A dominating set D is called a total dominating set if every vertex $v \in V$ is adjacent to at least one vertex in D. The minimum cardinality of such a set is called the total domination number and is denoted by $\gamma_t(G)$.*

Definition 2.8. *A dominating set D is called a connected dominating set if the subgraph induced by D is connected. The minimum cardinality of such a set is called the connected domination number and is denoted by $\gamma_c(G)$.*

Definition 2.9. *A graph is said to be connected if there exists a path between every pair of vertices in the graph.*

Definition 2.10. *A path graph P_n is a graph consisting of n vertices arranged in a linear sequence such that each vertex is connected to its successor.*

Definition 2.11. *A cycle graph C_n is a graph obtained by connecting the end vertices of a path graph P_n, forming a closed loop.*

Definition 2.12. *A star graph S_n is a graph consisting of one central vertex connected to all other $n-1$ vertices, which are called leaves.*

Definition 2.13. *A wheel graph W_n is obtained by joining a single central vertex to all vertices of a cycle graph C_{n-1}.*

3 Domination in Path Graphs

A path graph P_n consists of n vertices arranged in a linear sequence, where each vertex is adjacent to its immediate predecessor and successor.

Theorem 3.1. *The domination number of a path graph P_n is given by*

$$\gamma(P_n) = \left\lceil \frac{n}{3} \right\rceil .$$

Proof. Let P_n be a path with vertex set $V = \{v_1, v_2, \ldots, v_n\}$.

Each vertex in a dominating set can dominate at most three vertices, namely itself and its two adjacent vertices (except for the end vertices, which dominate at most two vertices). Therefore,

$$\gamma(P_n) \geq \left\lceil \frac{n}{3} \right\rceil .$$

To prove equality, we construct a dominating set of size $\lceil n/3 \rceil$. Consider the set

$$D = \{v_2, v_5, v_8, \dots\},$$

that is, every third vertex starting from v_2.

Each vertex in D dominates itself and its neighboring vertices. It is straightforward to verify that every vertex in P_n is either in D or adjacent to a vertex in D. Hence, D is a dominating set.

Thus,

$$\gamma(P_n) \leq \left\lceil \frac{n}{3} \right\rceil.$$

Combining both inequalities, we obtain

$$\gamma(P_n) = \left\lceil \frac{n}{3} \right\rceil.$$

□

Example 3.2. *Consider the path graph P_6 with vertex set $\{v_1, v_2, v_3, v_4, v_5, v_6\}$. A dominating set can be chosen as*

$$D = \{v_2, v_5\}.$$

Here, v_2 dominates v_1, v_2, v_3 and v_5 dominates v_4, v_5, v_6. Hence all vertices are dominated, and

$$\gamma(P_6) = 2.$$

Remark 3.3. *The above result shows that the domination number of a path graph grows linearly with respect to the number of vertices. In particular, one dominating vertex is sufficient to cover approximately three vertices in a path graph.*

4 Domination in Cycle Graphs

A cycle graph C_n is obtained by joining the first and last vertices of a path graph P_n, forming a closed loop.

Theorem 4.1. *The domination number of a cycle graph C_n is given by*

$$\gamma(C_n) = \left\lceil \frac{n}{3} \right\rceil .$$

Proof. Let C_n be a cycle with vertex set $V = \{v_1, v_2, \dots, v_n\}$, where v_i is adjacent to v_{i-1} and v_{i+1} (indices taken modulo n).

Each vertex in a dominating set can dominate at most three vertices, namely itself and its two adjacent vertices. Therefore,

$$\gamma(C_n) \geq \left\lceil \frac{n}{3} \right\rceil .$$

To show equality, we construct a dominating set of size $\lceil n/3 \rceil$. Consider the set

$$D = \{v_2, v_5, v_8, \dots\},$$

that is, every third vertex around the cycle.

Since the graph is cyclic, the adjacency is taken modulo n, and the last selected vertex also dominates the remaining vertices at the end of the cycle. Hence every vertex in C_n is either in D or adjacent to a vertex in D.

Thus,

$$\gamma(C_n) \leq \left\lceil \frac{n}{3} \right\rceil .$$

Combining both inequalities, we obtain

$$\gamma(C_n) = \left\lceil \frac{n}{3} \right\rceil .$$

□

Example 4.2. *Consider the cycle graph C_6 with vertices $\{v_1, v_2, v_3, v_4, v_5, v_6\}$. A dominating set is*

$$D = \{v_2, v_5\}.$$

Each selected vertex dominates itself and its neighbors, and all vertices are covered. Hence,

$$\gamma(C_6) = 2.$$

Remark 4.3. *Unlike path graphs, cycle graphs do not have end vertices. However, the domination number follows the same formula* $\lceil n/3 \rceil$ *due to the uniform structure of the cycle.*

5 Domination in Star Graphs

A star graph S_n is a tree on n vertices consisting of one central vertex v adjacent to all other $n-1$ vertices, which are called leaves.

Theorem 5.1. *The domination number of a star graph* S_n *is*

$$\gamma(S_n) = 1.$$

Proof. Let S_n be a star graph with central vertex v and leaf set $\{u_1, u_2, \dots, u_{n-1}\}$.

Since v is adjacent to every other vertex in S_n, the set $D = \{v\}$ dominates all vertices of the graph. Hence,

$$\gamma(S_n) \leq 1.$$

On the other hand, no dominating set can be empty, so $\gamma(S_n) \geq 1$. Therefore,

$$\gamma(S_n) = 1.$$

□

Lemma 5.2. *The central vertex of a star graph is the unique minimum dominating set.*

Proof. Any leaf vertex dominates only itself and the central vertex, and hence cannot dominate all other leaves. Therefore, no set consisting only of leaf vertices can be a dominating set.

Thus, any minimum dominating set must contain the central vertex, which alone suffices to dominate the graph. Hence the central vertex forms the unique minimum dominating set. □

Remark 5.3. *The star graph provides an example of a graph with a universal vertex, that is, a vertex adjacent to all other vertices. Such graphs always have domination number equal to one.*

6 Domination in Wheel Graphs

A wheel graph W_n $(n \geq 4)$ is obtained by joining a new vertex v (called the hub) to all vertices of a cycle C_{n-1}.

Theorem 6.1. *The domination number of a wheel graph W_n is*

$$\gamma(W_n) = 1.$$

Proof. Let v be the central (hub) vertex of W_n. By construction, v is adjacent to every other vertex of the graph, including all vertices of the cycle C_{n-1}.

Thus the set $D = \{v\}$ dominates all vertices of W_n, and hence

$$\gamma(W_n) \leq 1.$$

Since the domination number of any non-empty graph is at least 1, it follows that

$$\gamma(W_n) \geq 1.$$

Therefore,

$$\gamma(W_n) = 1.$$

□

Remark 6.2. *The wheel graph contains a universal vertex (the hub), and hence its domination number is equal to one.*

7 Additional Results on Domination

Theorem 7.1. *Let P_n be a path graph where n is divisible by 3. Then*

$$\gamma(P_n) = \frac{n}{3}.$$

Proof. Let $n = 3k$ for some integer k. Partition the vertex set of P_n into k consecutive blocks of three vertices each:

$$\{v_1, v_2, v_3\}, \{v_4, v_5, v_6\}, \ldots, \{v_{3k-2}, v_{3k-1}, v_{3k}\}.$$

Selecting the middle vertex from each block, namely

$$D = \{v_2, v_5, \dots, v_{3k-1}\},$$

ensures that every vertex in P_n is dominated.

Thus, $\gamma(P_n) \leq k = \frac{n}{3}$. From the general bound $\gamma(P_n) \geq \lceil n/3 \rceil$, we obtain $\gamma(P_n) \geq n/3$. Hence,

$$\gamma(P_n) = \frac{n}{3}.$$

□

Theorem 7.2. *For every* $n \geq 3$*,*

$$\gamma(C_n) \leq \gamma(P_n) + 1.$$

Proof. Let D be a minimum dominating set of the path graph P_n. Construct the cycle graph C_n by adding the edge $v_1 v_n$.

The set D dominates all internal vertices of C_n. If both v_1 and v_n are already dominated by vertices in D, then D is also a dominating set for C_n.

Otherwise, at most one additional vertex (either v_1 or v_n) is sufficient to dominate any uncovered vertex due to the added edge.

Thus, a dominating set of size at most $\gamma(P_n) + 1$ exists for C_n, and hence

$$\gamma(C_n) \leq \gamma(P_n) + 1.$$

□

Remark 7.3. *Since* $\gamma(C_n) = \lceil n/3 \rceil$ *and* $\gamma(P_n) = \lceil n/3 \rceil$*, the above inequality is tight for all* n*.*

8 Domination in Grid Graphs

A grid graph $G_{m,n}$ is defined as the Cartesian product of two path graphs P_m and P_n. The vertex set can be represented as

$$V = \{(i, j) \mid 1 \leq i \leq m,\ 1 \leq j \leq n\},$$

where each vertex is adjacent to its horizontal and vertical neighbors.

Theorem 8.1. *For a grid graph $G_{m,n}$,*

$$\gamma(G_{m,n}) \leq \left\lceil \frac{mn}{5} \right\rceil.$$

Proof. In a grid graph, each interior vertex can dominate at most five vertices: itself and its four adjacent neighbors (up, down, left, right). Boundary vertices dominate fewer vertices.

To establish the bound, we construct a dominating set by selecting vertices in a periodic pattern across the grid. Partition the grid into disjoint blocks of size 2×3 (or similar repeating tiles), and select one vertex from each block such that all vertices in the block are dominated.

By repeating this selection pattern throughout the grid, every vertex in $G_{m,n}$ is either selected or adjacent to a selected vertex. The number of selected vertices is at most $\lceil mn/5 \rceil$.

Hence,

$$\gamma(G_{m,n}) \leq \left\lceil \frac{mn}{5} \right\rceil.$$

□

Remark 8.2. *The exact domination number of grid graphs is known only for certain small values of m and n. In general, determining $\gamma(G_{m,n})$ remains a challenging problem, and tight bounds are of significant interest in domination theory.*

9 Comparison of Domination Numbers

Graph Type	Domination Number	Growth Rate
Path Graph P_n	$\lceil n/3 \rceil$	Linear
Cycle Graph C_n	$\lceil n/3 \rceil$	Linear
Star Graph S_n	1	Constant
Wheel Graph W_n	1	Constant
Grid Graph $G_{m,n}$	$\leq \lceil mn/5 \rceil$	Quadratic

Remark 9.1. *The comparison highlights that domination numbers depend strongly on the structure of the graph. Sparse linear graphs such as paths and cycles exhibit linear growth, whereas dense structures with universal vertices (such as star and wheel graphs) have constant domination number. Grid graphs, being two-dimensional, exhibit significantly larger domination numbers.*

10 Relationship Between Domination Parameters

Theorem 10.1. *For any connected graph G of order n,*

$$\gamma(G) \leq \gamma_c(G) \leq n - 1.$$

Proof. Let G be a connected graph.

Every connected dominating set is, by definition, a dominating set. Hence the minimum cardinality of a dominating set cannot exceed that of a connected dominating set. Therefore,

$$\gamma(G) \leq \gamma_c(G).$$

To prove the upper bound, consider any spanning tree T of G. Since G is connected, such a tree exists. Removing a leaf vertex from T results in a connected subgraph that still dominates all vertices of G, because every removed leaf is adjacent to its parent in the tree.

Thus, the set of all vertices except one (a leaf) forms a connected dominating set.

Hence,

$$\gamma_c(G) \leq n - 1.$$

□

Remark 10.2. *The bounds are sharp. For example, in a complete graph* K_n*,* $\gamma(G) = \gamma_c(G) = 1$*, while in a path graph* P_n*,* $\gamma_c(P_n) = n - 2$ *for* $n \geq 3$*.*

11 Conclusion

In this paper, domination parameters for several important classes of graphs were investigated, including path graphs, cycle graphs, star graphs, wheel graphs, and grid graphs. Exact values of the domination number were obtained for paths, cycles, stars, and wheels, while upper bounds were established for grid graphs.

In addition, relationships between domination and connected domination parameters were discussed, providing insight into their structural differences. The results demonstrate how graph topology significantly influences domination behavior.

Future work may focus on extending these results to more complex graph classes such as trees, bipartite graphs, planar graphs, and product graphs. Another promising direction is the study of total domination and connected domination in higher-dimensional and dynamic network models, which have important applications in communication networks and distributed systems.

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