



A Short Note on Pairwise Fuzzy Irresolvable Spaces

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Abstract

In this paper, the concept of irresolvability of fuzzy bitopological spaces have been studied in detail. Some of their characteristics together with the other fuzzy topological spaces have been established.

Key words: Pairwise fuzzy open set, pairwise fuzzy dense set, pairwise fuzzy first category set, pairwise fuzzy irresolvable space, pairwise fuzzy Baire space, pairwise fuzzy D-Baire space, pairwise fuzzy hyper connected space and pairwise fuzzy sub maximal space.

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1. Introduction

Till 1965, Mathematicians were concerned only about “well-defined” things and smartly avoided any other possibilities which are more realistic in nature. In 1965, L.A.Zadeh [15] introduced the concept of fuzzy set to accommodate real life situations by giving partial membership to each element of a situation under consideration. Fuzzy sets allow everyone us to represent vague concepts expressed in natural language. C.L.Chang [2] introduced the concept of fuzzy topological spaces in 1968 as a generalization of topological spaces. Since then many topologists have contributed to the theory of fuzzy topological spaces. In 1989, A.Kandil [5] introduced the concept of fuzzy bitopological spaces.

The systematic study of resolvability in classical topology began with the works of E.Hewitt [4] and M.Katetov [6]. The concepts of resolvability and irresolvability in topological spaces were introduced and studied by E.Hewitt [4] in 1943. Since then

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several mathematicians found interest in the study of resolvable and irresolvable spaces. In 1993, C.Chattopadhyay and C.Bandyopadhyay [3] extended the study of resolvable and irresolvable spaces to the bitopological spaces. The concepts of resolvability and irresolvability of fuzzy bitopological spaces were introduced by G.Thangaraj [7]. In this paper, the concept of irresolvability of fuzzy bitopological spaces have been studied in detail. Some of their characteristics together with the other fuzzy topological spaces have been established.

2. Preliminaries

Now we give some basic notions and results used in the sequel. In this work by (X, T) or simply by X , we will denote a fuzzy topological space due to Chang (1968). By a fuzzy bitopological space (Kandil, 1989) we mean an ordered triple (X, T_1, T_2) , where T_1 and T_2 are two fuzzy topologies on a non-empty set X . Throughout this paper, the indices i and j take values in $\{1, 2\}$ and $i \neq j$.

Definition 2.1 [2] Let (X, T) be any fuzzy topological space and λ be any fuzzy set in (X, T) . The closure and interior of a fuzzy set λ in a fuzzy topological space (X, T) are respectively denoted as $cl(\lambda)$ and $int(\lambda)$ are defined as

- (1) $cl(\lambda) = \wedge\{\mu \mid \lambda \leq \mu, 1 - \mu \in T\}$ and
- (2) $int(\lambda) = \vee\{\mu \mid \mu \leq \lambda, \mu \in T\}$.

Lemma 2.2 [1] For a fuzzy set λ of a fuzzy space X ,

- (a) $1 - cl(\lambda) = int(1 - \lambda)$ and
- (b) $1 - int(\lambda) = cl(1 - \lambda)$.

Lemma 2.3 [1] For a family $\mathcal{A} = \{\lambda_\alpha\}$ of fuzzy sets of a fuzzy space X . Then, $\vee cl(\lambda_\alpha) \leq cl(\vee \lambda_\alpha)$. In case \mathcal{A} is a finite set, $\vee cl(\lambda_\alpha) = cl(\vee \lambda_\alpha)$. Also $\vee int(\lambda_\alpha) \leq int(\vee \lambda_\alpha)$.

Theorem 2.4 [14] Let X be a fuzzy topological space and λ, μ be fuzzy sets in X . Then we have

- (1). λ is fuzzy closed (resp., fuzzy open) $\Leftrightarrow cl(\lambda) = \lambda$ (resp., $int(\lambda) = \lambda$);
- (2). $\lambda \leq \mu \Rightarrow cl(\lambda) \leq cl(\mu)$ ($int(\lambda) \leq int(\mu)$);
- (3). $cl cl(\lambda) = cl(\lambda)$ ($int int(\lambda) = int(\lambda)$);
- (4). $cl(\lambda) \vee cl(\mu) = cl(\lambda \vee \mu)$;

- (5). $cl(\lambda) \wedge cl(\mu) \geq cl(\lambda \wedge \mu)$;
- (6). $int(\lambda) \vee int(\mu) \leq int(\lambda \vee \mu)$;
- (7). $int(\lambda) \wedge int(\mu) = int(\lambda \wedge \mu)$;

Definition 2.5 [15] A fuzzy set λ in a set X is a function from X to $[0, 1]$, that is., $\lambda : X \rightarrow [0, 1]$.

Definition 2.6 [8] A fuzzy set λ in a fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy open set if $\lambda \in T_i$, ($i = 1, 2$).

Definition 2.7 [8] A fuzzy set λ in a fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy closed set if $1 - \lambda \in T_i$, ($i = 1, 2$).

Definition 2.8 [7] A fuzzy set λ in a fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy dense set if $cl_{T_1}cl_{T_2}(\lambda) = 1$ and $cl_{T_2}cl_{T_1}(\lambda) = 1$.

Definition 2.9 [10] A fuzzy set λ in a fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy nowhere dense set if $int_{T_1}cl_{T_2}(\lambda) = 0 = int_{T_2}cl_{T_1}(\lambda)$.

Definition 2.10 [10] A fuzzy set λ in a fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy first category set if $\lambda = \bigvee_{k=1}^{\infty}(\lambda_k)$, where (λ_k) 's are pairwise fuzzy nowhere dense sets in (X, T_1, T_2) . Any other fuzzy set in (X, T_1, T_2) is said to be a pairwise fuzzy second category set.

Definition 2.11 [10] A fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy Baire space if $int_{T_i}(\bigvee_{k=1}^{\infty}(\lambda_k)) = 0$, ($i = 1, 2$), where (λ_k) 's are pairwise fuzzy nowhere dense sets in (X, T_1, T_2) .

Definition 2.12 [12] A fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy D-Baire space if every pairwise fuzzy first category set in (X, T_1, T_2) is a pairwise fuzzy nowhere dense set in (X, T_1, T_2) . That is, (X, T_1, T_2) is a pairwise fuzzy D-Baire space if $int_{T_1}cl_{T_2}(\lambda) = 0$ and $int_{T_2}cl_{T_1}(\lambda) = 0$, for each pairwise fuzzy first category set λ in (X, T_1, T_2) .

Definition 2.13 [11] A fuzzy bitopological space (X, T_1, T_2) is said to be a pairwise fuzzy submaximal space if for each fuzzy set λ in (X, T_1, T_2) such that $cl_{T_i}(\lambda) = 1$, ($i = 1, 2$), λ is a pairwise fuzzy open set in (X, T_1, T_2) .

Definition 2.14 [9] A fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy hyperconnected space if λ is a pairwise fuzzy open set in (X, T_1, T_2) , then $cl_{T_i}(\lambda) = 1$, $(i = 1, 2)$.

Theorem 2.15 [13] If λ is a pairwise fuzzy nowhere dense set in a pairwise fuzzy irresolvable space (X, T_1, T_2) , then λ is not a pairwise fuzzy dense set in (X, T_1, T_2) .

Theorem 2.16 [10] Let (X, T_1, T_2) be a fuzzy bitopological space. Then the following are equivalent:

- (1). (X, T_1, T_2) is a pairwise fuzzy Baire space.
- (2). $int_{T_i}(\lambda) = 0$, $(i = 1, 2)$, for every pairwise fuzzy first category set λ in (X, T_1, T_2) .
- (3). $cl_{T_i}(\mu) = 1$, $(i = 1, 2)$, for every pairwise fuzzy residual set μ in (X, T_1, T_2) .

3. Pairwise fuzzy irresolvable spaces and other fuzzy bitopological spaces

The following proposition gives a condition for a pairwise fuzzy irresolvable space not to be a pairwise fuzzy Baire space.

Proposition 3.1 If each pairwise fuzzy first category set λ is a pairwise fuzzy dense set in a pairwise fuzzy irresolvable space (X, T_1, T_2) , then (X, T_1, T_2) is not a pairwise fuzzy Baire space.

Proof: Let λ be a pairwise fuzzy first category set in a fuzzy bitopological space (X, T_1, T_2) such that $cl_{T_1}cl_{T_2}(\lambda) = 1 = cl_{T_2}cl_{T_1}(\lambda)$. Since (X, T_1, T_2) is a pairwise fuzzy irresolvable space, $cl_{T_1}cl_{T_2}(1 - \lambda) \neq 1$ and $cl_{T_2}cl_{T_1}(1 - \lambda) \neq 1$, for the pairwise fuzzy dense λ in (X, T_1, T_2) and hence $1 - int_{T_1}int_{T_2}(\lambda) \neq 1$ and $1 - int_{T_2}int_{T_1}(\lambda) \neq 1$, in (X, T_1, T_2) . This implies that $int_{T_1}int_{T_2}(\lambda) \neq 0$ and $int_{T_2}int_{T_1}(\lambda) \neq 0$, in (X, T_1, T_2) . Then, $int_{T_1}(\lambda) \neq 0$ and $int_{T_2}(\lambda) \neq 0$, in (X, T_1, T_2) . [For, if $int_{T_1}(\lambda) = 0$ and $int_{T_2}(\lambda) = 0$ in (X, T_1, T_2) , then $int_{T_1}int_{T_2}(\lambda) = int_{T_1}(0) = 0$ and $int_{T_2}int_{T_1}(\lambda) = int_{T_2}(0) = 0$ in (X, T_1, T_2) , a contradiction]. Hence, for the pairwise fuzzy first category set λ , $int_{T_1}(\lambda) \neq 0$ and $int_{T_2}(\lambda) \neq 0$, in (X, T_1, T_2) . Then, by theorem 2.16, that (X, T_1, T_2) is not a pairwise fuzzy Baire space.

Proposition 3.2 If a fuzzy bitopological space (X, T_1, T_2) is a pairwise fuzzy Baire and pairwise fuzzy irresolvable space and λ is a pairwise fuzzy first category set in (X, T_1, T_2) , then λ is not a pairwise fuzzy dense set in (X, T_1, T_2) .

Proof: Let (X, T_1, T_2) be a pairwise fuzzy Baire and pairwise fuzzy irresolvable space and λ be a pairwise fuzzy first category set in (X, T_1, T_2) . Since (X, T_1, T_2) is a pairwise fuzzy Baire space, by theorem 2.16, $int_{T_i}(\lambda) = 0$ ($i = 1, 2$), in (X, T_1, T_2) . Then, $int_{T_j}int_{T_i}(\lambda) = int_{T_j}(0) = 0$ ($i \neq j$ and $i, j = 1, 2$), in (X, T_1, T_2) . That is, $int_{T_j}int_{T_i}(\lambda) = 0$, in (X, T_1, T_2) . Then, $1 - int_{T_j}int_{T_i}(\lambda) = 1 - 0 = 1$, in (X, T_1, T_2) and hence $cl_{T_j}cl_{T_i}(1 - \lambda) = 1$, in (X, T_1, T_2) . Then $1 - \lambda$ is a pairwise fuzzy dense set in (X, T_1, T_2) . Since (X, T_1, T_2) is a pairwise fuzzy irresolvable space, $cl_{T_j}cl_{T_i}(1 - (1 - \lambda)) \neq 1$, in (X, T_1, T_2) , for the pairwise fuzzy dense set $1 - \lambda$ in (X, T_1, T_2) . Then, $cl_{T_j}cl_{T_i}(\lambda) \neq 1$, in (X, T_1, T_2) . Hence λ is not a pairwise fuzzy dense set in (X, T_1, T_2) .

Proposition 3.3 If λ is a pairwise fuzzy first category set in a pairwise fuzzy D-Baire and pairwise fuzzy irresolvable space (X, T_1, T_2) , then λ is not a pairwise fuzzy dense set in (X, T_1, T_2) .

Proof: Let λ be a pairwise fuzzy first category set in (X, T_1, T_2) . Since (X, T_1, T_2) is a pairwise fuzzy D-Baire space, by definition 2.12, the pairwise fuzzy first category set λ is a pairwise fuzzy nowhere dense set in (X, T_1, T_2) . Since (X, T_1, T_2) is a pairwise fuzzy irresolvable space, by theorem 2.15, λ is not a pairwise fuzzy dense set in (X, T_1, T_2) . Hence, the pairwise fuzzy first category set in a pairwise fuzzy D-Baire and pairwise fuzzy irresolvable space is not a pairwise fuzzy dense set in (X, T_1, T_2) .

Proposition 3.4 If λ is a pairwise fuzzy open set in a pairwise fuzzy hyperconnected and pairwise fuzzy irresolvable space (X, T_1, T_2) , then $cl_{T_1}cl_{T_2}(\lambda) = 1 = cl_{T_2}cl_{T_1}(\lambda)$ and $cl_{T_1}cl_{T_2}(1 - \lambda) \neq 1$ and $cl_{T_2}cl_{T_1}(1 - \lambda) \neq 1$ in (X, T_1, T_2) .

Proof: Let λ be a pairwise fuzzy open set in (X, T_1, T_2) . Since (X, T_1, T_2) is a pairwise fuzzy hyperconnected space, λ is a pairwise fuzzy dense set in (X, T_1, T_2) . Since (X, T_1, T_2) is a pairwise fuzzy irresolvable space, for the pairwise fuzzy dense set λ in (X, T_1, T_2) , $cl_{T_1}cl_{T_2}(1 - \lambda) \neq 1$ and $cl_{T_2}cl_{T_1}(1 - \lambda) \neq 1$ in (X, T_1, T_2) . Hence, if λ is a pairwise fuzzy open set in (X, T_1, T_2) , then $cl_{T_1}cl_{T_2}(\lambda) = 1 = cl_{T_2}cl_{T_1}(\lambda)$ and $cl_{T_1}cl_{T_2}(1 - \lambda) \neq 1$ and $cl_{T_2}cl_{T_1}(1 - \lambda) \neq 1$ in (X, T_1, T_2) .

Proposition 3.5 If μ is a pairwise fuzzy closed set in a pairwise fuzzy hyperconnected and pairwise fuzzy irresolvable space (X, T_1, T_2) , then μ is not a pairwise fuzzy dense set in (X, T_1, T_2) .

Proof: Let μ be a pairwise fuzzy closed set in (X, T_1, T_2) . Then $1 - \mu$ is a pairwise fuzzy open set in (X, T_1, T_2) . Since (X, T_1, T_2) is a pairwise fuzzy hyperconnected and pairwise fuzzy irresolvable space, by proposition 3.4, $cl_{T_1}cl_{T_2}(1 - \mu) = 1 = cl_{T_2}cl_{T_1}(1 - \mu)$ and $cl_{T_1}cl_{T_2}(1 - (1 - \mu)) \neq 1$ and $cl_{T_2}cl_{T_1}(1 - (1 - \mu)) \neq 1$ in (X, T_1, T_2) . That is, $cl_{T_1}cl_{T_2}(\mu) \neq 1$ and $cl_{T_2}cl_{T_1}(\mu) \neq 1$ in (X, T_1, T_2) . Thus μ is not a pairwise fuzzy dense set in (X, T_1, T_2) .

The following proposition shows that the pairwise fuzzy submaximality implies the pairwise fuzzy irresolvability of fuzzy bitopological spaces.

Proposition 3.6 If a fuzzy bitopological space (X, T_1, T_2) is a pairwise fuzzy submaximal space, then (X, T_1, T_2) is a pairwise fuzzy irresolvable space.

Proof: Let (X, T_1, T_2) be a pairwise fuzzy submaximal space and λ be a pairwise fuzzy dense set in (X, T_1, T_2) . Since (X, T_1, T_2) is a pairwise fuzzy submaximal space, the pairwise fuzzy dense set λ in (X, T_1, T_2) is a pairwise fuzzy open set in (X, T_1, T_2) . Then $int_{T_1}(\lambda) = \lambda$ and $int_{T_2}(\lambda) = \lambda$ in (X, T_1, T_2) . Thus, $cl_{T_1}cl_{T_2}(1 - \lambda) = 1 - int_{T_1}int_{T_2}(\lambda) = 1 - int_{T_1}(\lambda) = 1 - \lambda \neq 1$ and $cl_{T_2}cl_{T_1}(1 - \lambda) = 1 - int_{T_2}int_{T_1}(\lambda) = 1 - int_{T_2}(\lambda) = 1 - \lambda \neq 1$, in (X, T_1, T_2) . Hence, for the pairwise fuzzy dense set λ in (X, T_1, T_2) , $cl_{T_1}cl_{T_2}(1 - \lambda) \neq 1$ and $cl_{T_2}cl_{T_1}(1 - \lambda) \neq 1$, in (X, T_1, T_2) . Therefore, (X, T_1, T_2) is a pairwise fuzzy irresolvable space.

Remark 3.7 The converse of the above proposition need not be true. That is, a pairwise fuzzy irresolvable space need not be a pairwise fuzzy submaximal space.

Example 3.8 Let $X = \{a, b, c\}$. The fuzzy sets λ, μ, δ are defined on X as follows:

$$\lambda : X \rightarrow [0, 1] \text{ is defined as } \lambda(a) = 0.25; \quad \lambda(b) = 0; \quad \lambda(c) = 0$$

$$\mu : X \rightarrow [0, 1] \text{ is defined as } \mu(a) = 0.75; \quad \mu(b) = 0.5; \quad \mu(c) = 0$$

$$\delta : X \rightarrow [0, 1] \text{ is defined as } \delta(a) = 0.8; \quad \delta(b) = 0; \quad \delta(c) = 1.$$

Clearly, $T_1 = \{0, \lambda, \mu, 1\}$ and $T_2 = \{0, \lambda, \delta, 1\}$ are fuzzy topologies on X . On computation, one can see that $cl_{T_1}(\delta) = 1$ and $cl_{T_2}(\delta) = 1$ and hence $cl_{T_1}cl_{T_2}(\delta) = cl_{T_1}(1) = 1$ and $cl_{T_2}cl_{T_1}(\delta) = cl_{T_2}(1) = 1$. Thus, δ is a pairwise fuzzy dense set in (X, T_1, T_2) and $cl_{T_1}cl_{T_2}(1 - \delta) = 1 - \lambda \neq 1$ and $cl_{T_2}cl_{T_1}(1 - \delta) = 1 - \lambda \neq 1$ and hence (X, T_1, T_2) is a pairwise fuzzy irresolvable space. Now $int_{T_1}(\delta) = \lambda$ and $int_{T_2}(\delta) = \delta$ and hence $int_{T_1}(\delta) \neq int_{T_2}(\delta)$, implies that δ is not a pairwise fuzzy open set in (X, T_1, T_2) . Therefore, (X, T_1, T_2) is not a pairwise fuzzy submaximal space.

Definition 3.9 A fuzzy bitopological space (X, T_1, T_2) is said to be a pairwise fuzzy door space if every fuzzy subset of (X, T_1, T_2) is either pairwise fuzzy open or pairwise fuzzy closed.

The following proposition shows that a pairwise fuzzy door space is a pairwise fuzzy submaximal space.

Proposition 3.10 If a fuzzy bitopological space (X, T_1, T_2) is a pairwise fuzzy door space, then (X, T_1, T_2) is a pairwise fuzzy submaximal space.

Proof: Let $\lambda (\neq 1)$ be a pairwise fuzzy dense set in (X, T_1, T_2) . Then $cl_{T_1}cl_{T_2}(\lambda) = 1 = cl_{T_2}cl_{T_1}(\lambda)$, in (X, T_1, T_2) . It has to be proved that λ is a pairwise fuzzy open set in (X, T_1, T_2) . Assume the contrary. Suppose that λ is not a pairwise fuzzy open set in (X, T_1, T_2) . Since (X, T_1, T_2) is a pairwise fuzzy door space, λ is a pairwise fuzzy closed set in (X, T_1, T_2) . (since λ is not a pairwise fuzzy open set in (X, T_1, T_2)). Then $cl_{T_i}(\lambda) = \lambda$, for $i = 1, 2$, in (X, T_1, T_2) . Thus, $cl_{T_1}cl_{T_2}(\lambda) = cl_{T_1}(\lambda) = \lambda \neq 1$ and $cl_{T_2}cl_{T_1}(\lambda) = cl_{T_2}(\lambda) = \lambda \neq 1$, a contradiction to λ being a pairwise fuzzy dense set in (X, T_1, T_2) . Hence, the assumption that λ is not a pairwise fuzzy open set in (X, T_1, T_2) , does not hold. Therefore, each pairwise fuzzy dense set λ is a pairwise fuzzy open set in a pairwise fuzzy door space (X, T_1, T_2) and hence (X, T_1, T_2) is a pairwise fuzzy submaximal space.

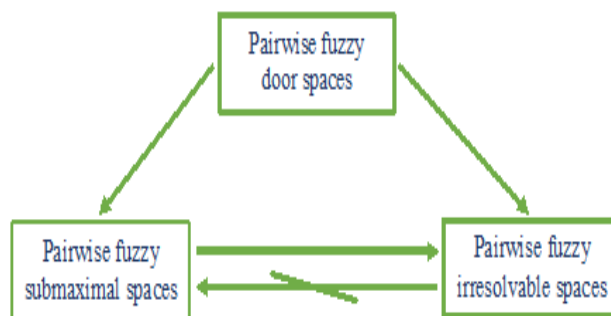
Proposition 3.11 If a fuzzy bitopological space (X, T_1, T_2) is a pairwise fuzzy door space, then (X, T_1, T_2) is a pairwise fuzzy irresolvable space.

Proof: Let (X, T_1, T_2) be a pairwise fuzzy door space. Then, by proposition 3.10, (X, T_1, T_2) is a pairwise fuzzy submaximal space. By proposition 3.6, (X, T_1, T_2) is a pairwise fuzzy irresolvable space.

Remark 3.12 Inter-relations between pairwise fuzzy door spaces, pairwise fuzzy submaximal spaces, pairwise fuzzy resolvable spaces and pairwise fuzzy irresolvable spaces can be summarized as follows:

4. Conclusion

The concept of irresolvability of fuzzy bitopological spaces have studied in detail and also some of their characteristics together with the other fuzzy topological spaces have established in this paper.



References

- [1] Azad KK, On fuzzy semicontinuity, fuzzy almost continuity and fuzzy weakly continuity, J. Math. Anal. Appl., 82, 1981, 14-32.
- [2] Chang CL, Fuzzy topological spaces, J.Math. Anal. Appl., 24, 1968, 182-190.
- [3] Chattopadhyay C, Bandyopadhyay C, Resolvability and irresolvability in bitopological spaces, Soochow. J.Math., 19(4), 1993, 435-442.
- [4] Hewitt E, A problem in set theoretic topology, Duke Math. J., 10, 1943, 309-333.
- [5] Kandil A, Biproximities and fuzzy bitopological spaces, Simen Stevin, 63, 1989, 45-66.
- [6] Katetov M, On topological spaces containing no disjoint dense subsets, Math. Sbornik. N.S., 21(63), 1947, 03-12.
- [7] Thangaraj G, On pairwise fuzzy resolvable and fuzzy irresolvable spaces, Bull. Cal. Math. Soc., 102(1), 2010, 59-68.
- [8] Thangaraj G, Chandiran V, On pairwise fuzzy Volterra spaces, Ann. Fuzzy Math. Inform., 6(7), 2014, 1005-1012.
- [9] Thangaraj G, Chandiran V, A note on pairwise fuzzy Volterra spaces, Ann. Fuzzy Math. Inform., 9(3), 2015, 365-372.
- [10] Thangaraj G, Sethuraman S, On pairwise fuzzy Baire spaces, Gen. Math. Notes, 20(2), 2014, 12-21.
- [11] Thangaraj G, Sethuraman S, A note on pairwise fuzzy Baire spaces, Ann. Fuzzy Math. Inform., 8(5), 2014, 729-737.
- [12] Thangaraj G, Sethuraman S, On pairwise fuzzy D-Baire spaces, (Communicated to Scientia Magna).

- [13] Thangaraj G, Vivakanandan P, Chandiran V, A short note on pairwise fuzzy irresolvable spaces and pairwise fuzzy open hereditarily irresolvable spaces, (Communicated to Journal of Computational Mathematica).
- [14] Warren RH, Neighborhoods bases and continuity in fuzzy topological spaces, Rocky Mountain J. Math., 8(3), 1978, 459-470.
- [15] Zadeh LA, Fuzzy sets, Information and Control, 8, 1965, 338-353.