



Dissipativity and Passivity analysis of neural networks with mixed-time-varying delays

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Abstract

This paper focuses on the problem of Dissipativity and Passivity analysis of NNs with mixed-time-varying delays. By employing Lyapunov functional approach, some sufficient conditions are derived to guarantee that the considered NNs are strictly $(\mathcal{Q}, \mathcal{S}, \mathcal{R})-\gamma$ -Dissipative and Passivity. Based on Lyapunov stability theory, proper Lyapunov-Krasovskii functional (LKF) with some new terms is constructed, and estimating their derivative by using newly developed single integral inequality that includes Jensens inequality which can be easily checked by applying MATLAB LMT toolkit. Three Numerical example is finally provided to demonstrate the effectiveness and advantages of the proposed method.

Key words: Dissipativity Analysis, Passivity analysis, Neural networks, Lyapunov-Krasovskii functional, Mixed time delays.

1. Introduction

During the past decades, NNs have been attracted many researchers attention for their extensive successful applications in several areas, such as associative memory, static image processing, combinatorial optimization, signal processing and pattern recognition [1]-[3]. Moreover, all these applications deeply depend on characteristics behavior of the dynamical system. It is well-known concept, the stability analysis is a fundamental property of dynamical system [4], because the unstable system there no practical application sense. In dynamical systems, the existing time delay may cause unstable, poor performance, and oscillation of the behaviors of the system [12]. Therefore, most researchers has mainly focused on to find the maximum delay upper bounds for the problem of NNs with time delay. Hence, the study of NNs with time delay gained considerable attention in the last few decades [1]-[15].

On the other hand, NNs character can be identify based on system performance,

such as H_∞ performance, passivity performance, dissipativity analysis, and $L_2 - L_\infty$ performance. In modern physical engineering, the dissipativity analysis present an important role, because it provides more flexible results on robust a control system, electrotechnical system, and circuit theory. For this purpose, recently, various results about dissipativity criteria been reported on NNs with time-varying delay components in the literature [6]-[8]. Motivated by the above discussion, in this article, we develop a new delay-dependent dissipativity criterion for NNs with mixed time -varying delays. The Lyapunov stability theory, inequality technique and LMI method are utilized to obtain the required main results. However, this proposed result is different from some existing previous literature on NNs with time delays [1]-[15].

On the other hand, dissipativity analysis performing one of the hot topic on the study of NNs, it is an essential property of physical systems, which is closely related with the intuitive phenomenon of loss or dissipation of energy. J. C. Willems initially introduced the dissipativity concept on the dynamical systems. After that, the concept generalized to nonlinear systems in the work [12]. The dissipative theory which offers a fundamental idea to energy-related input-output description model for the design and analysis of control systems [12]. It gives more flexible results on the modern control applications like robotics, active vibration damping, electromechanical systems, combustion engines, circuit theory. In addition, the theory of dissipative analysis acting a general case of some few known results, which contained the passivity theorem, bounded real lemma and some other results. Recently, the $(\mathcal{Q}, \mathcal{S}, \mathcal{R}) - \gamma$ - dissipativity, where γ is dissipativity performance level has been investigated in [13]-[19], [21]. By adjusting the weighting matrices in $(\mathcal{Q}, \mathcal{S}, \mathcal{R}) - \gamma$ - dissipativity, the concept play an unified model for passivity and H performance [15]. As a consequence, the dissipativity analysis numerous takes advantage on the issues of NNs with time delays [12]-[21]

As we know as, in recent years, significant efforts have been paid to the issues of passivity analysis, which is well established based on the circuit theory model. The passivity system is acting as one of the most efficient tools for studying the stability analysis of NNs, nonlinear control model, especially for the higher-order systems. The concept of passivity as a part of the general theory of dissipative systems, it has been found in many applications in the different areas such as stability, complexity, chaos control, synchronization and so on. Thus, the concept becomes one of the most critical areas of research and receives a great deal of devotion on the researcher society [22]-[30]. For example, in work [24], delay-dependent passivity criterion was achieved by applying integral inequality methods for uncertain continuous-time delayed NNs. The delay-independent passivity of NNs was established in the literature [30]

The structure of this paper as follows. In Section 2, some necessary assumptions, definitions, and lemmas are given. The main results of this article are presented in Section 3. In Section 4, three numerical cases with simulation are verified. Finally, the general conclusions are reported in Section 5.

Notations: Throughout this paper, the superscripts \mathcal{D}^{-1} and \mathcal{D}^T stand for the inverse and transpose of matrix \mathcal{D} , respectively. \mathbb{R}^n denotes the n dimensional Euclidean space, $\mathbb{R}^{n \times m}$ is the set of all $n \times m$ real matrices. A real symmetric matrix $\mathcal{P}_1 > 0$, ($\mathcal{P}_1 \geq 0$) ($\mathcal{P}_1 < 0$) denotes P being a positive definite (positive semi-definite) and (negative definite) matrix, respectively. The symmetric terms in a symmetric matrix are denoted by $*$. I is an appropriately dimensioned identity matrix.

2. Problem formulation and preliminaries

In this paper, we consider the following NNs with mixed time-varying delays

$$\begin{cases} \dot{z}(t) = -\mathcal{D}z(t) + \mathcal{A}g(z(t)) + \mathcal{B}g(z(t-d(t))) + \mathcal{C} \int_{t-\tau(t)}^t g(z(s))ds + \omega(t), \\ y(t) = g(z(t)), \\ z(t) = \varphi(t), t \in [-\delta, 0] \end{cases} \quad (1)$$

where $z(t) = [z_1(t), \dots, z_n(t)]^T \in \mathbb{R}^n$ is the neuron state vector, $g(z(t)) = [g(z_1(t)), \dots, g(z_n(t))]^T \in \mathbb{R}^n$ denotes the neuron activation function and $\omega(t)$ is the noise input vector, which belonging to $\mathcal{L}_2[0, \infty)$. $\mathcal{D} = \text{diag}\{d_1, \dots, d_n\}$ is a positive diagonal matrix with $d_i > 0, i = 1, 2, \dots, n$ and $\mathcal{A} \in \mathbb{R}^n, \mathcal{B} \in \mathbb{R}^n$ and $\mathcal{C} \in \mathbb{R}^n$ are connection weight matrix, discrete delayed connection weight matrix and distributed delayed connection weight matrix, respectively. $\varphi(t)$ is assumed to be continuously differentiable on $[-\delta, 0]$, where $\delta = \max\{d, \tau\}$.

Assumption 1: The delays $d(t)$ and $\tau(t)$ are continuous time-varying delay components its satisfies

$$0 \leq d(t) \leq d, \dot{d}(t) \leq \mu, \quad 0 \leq \tau(t) \leq \tau, \quad (2)$$

where d, τ, μ are being real constants.

Assumption 2: Each $g_i(\cdot)$ is continuous and bounded, and there exist constants l_i^-

and l_i^+ , such that

$$l_i^- \leq \frac{g_i(a_1) - g_i(a_2)}{(a_1 - a_2)} \leq l_i^+, \quad (3)$$

where $a_1, a_2 \in \mathbb{R}, a_2 \neq a_1$

Definition 2.1 [12] The system (1) is said to be strictly $(\mathcal{Q}, \mathcal{S}, \mathcal{R}) - \gamma$ -dissipative if, for $\gamma > 0$, the following inequality

$$\mathcal{G}(\omega, y, t^*) \geq \gamma \langle \omega, \omega \rangle_{t^*}, \forall t^* \geq 0 \quad (4)$$

holds under zero initial condition.

Remark 2.2 The property of dissipativity, let us define an energy supply function as follow:

$$\mathcal{G}(\omega, y, t^*) = \langle y, \mathcal{Q}y \rangle_{t^*} + 2\langle y, \mathcal{S}\omega \rangle_{t^*} + \langle \omega, \mathcal{R}\omega \rangle_{t^*}, \forall t^* \geq 0, \quad (5)$$

where \mathcal{Q}, \mathcal{S} and \mathcal{R} are real matrices with \mathcal{Q}, \mathcal{R} symmetric, $\langle a, b \rangle_{t^*} = \int_0^{t^*} a^T b dt$. It is assumed that $\mathcal{Q} \leq 0$ and denoted that $-\mathcal{Q} = \mathcal{Q}_-^T \mathcal{Q}_-$ for some \mathcal{Q}_- .

Definition 2.3 [12] The system (1) is called passive if there exists a scalar $\gamma \geq 0$ such that for all $t_p \geq 0$

$$2 \int_0^{t_p} y^T(s)\omega(s)ds \geq -\gamma \int_0^{t_p} \omega^T(s)\omega(s)ds. \quad (6)$$

under the zero initial condition.

Lemma 2.4 [15] For any constant matrix $\mathcal{U} \in \mathbb{C}^{n \times n}$ and $\mathcal{U} > 0$, scalars $d_M > d_m > 0$, such that the following integration is well defined, then

$$-(d_M - d_m) \int_{t-d_M}^{t-d_m} z^T(s)\mathcal{U}z(s)ds \leq -\left(\int_{t-d_M}^{t-d_m} z(s)ds \right)^T \mathcal{U} \left(\int_{t-d_M}^{t-d_m} z(s)ds \right).$$

Lemma 2.5 [15] For any constant matrices $\mathcal{X} \in \mathbb{R}^{n \times n}$ and positive matrix $\mathcal{R} \in \mathbb{R}^{n \times n}$,

$$\begin{bmatrix} \mathcal{R} & \mathcal{X} \\ \mathcal{X}^T & \mathcal{R} \end{bmatrix}, \text{ scalars } 0 \leq d_m \leq d(t) \leq d_M, \text{ and vector function } \dot{z} : [-d_M, -d_m] \rightarrow \mathbb{R}^n,$$

such that the following integrations are well defined, then

$$-(d_M - d_m) \int_{t-d_M}^{t-d_m} \dot{z}^T(s) \mathcal{R} \dot{z}(s) ds = - \begin{bmatrix} z(t-d_M) - z(t-d(t)) \\ z(t-d(t)) - z(t-d_m) \end{bmatrix}^T \begin{bmatrix} \mathcal{R} & \mathcal{X} \\ \mathcal{X}^T & \mathcal{R} \end{bmatrix} \begin{bmatrix} z(t-d_M) - z(t-d(t)) \\ z(t-d(t)) - z(t-d_m) \end{bmatrix}.$$

3. Main Results

In this section, we first present delay-dependent global asymptotic stability criteria for NNs with mixed delays. For simplicity, we denote the matrix and vector representation, $e_i \in \mathbb{R}^{8n \times n}$ ($i = 1, 2, \dots, 8$) are defined as block entry matrices (for example $e_4 = [0_n, 0_n, 0_n, I_n, 0_n, 0_n, 0_n, 0_n]^T$).

Theorem 3.1 For given scalars d, τ and μ system (1) is globally asymptotic stable with $\omega(t) = 0$, if there exist symmetric positive definite matrices $\mathcal{P}_1, \mathcal{P}_2, \mathcal{P}_3, \mathcal{P}_4, \mathcal{P}_5$, positive diagonal matrix $\Lambda_1, \Lambda_2, \mathcal{N}_1, \mathcal{N}_2$, and any matrix $\mathcal{H}, \mathcal{G}_1, \mathcal{G}_2$ such that the following LMI holds:

$$\Omega = \Omega_1 + \Omega_2 < 0, \tag{7}$$

where

$$\begin{aligned} \Omega_1 = & 2e_1 \mathcal{P}_1 e_7^T + 2[e_4 - \mathcal{L}_1 e_1]^T \Lambda_1 e_7 + 2[\mathcal{L}_2 e_1 - e_4]^T \Lambda_2 e_7 + e_1 \mathcal{P}_2 e_1^T - e_3 \mathcal{P}_2 e_3 + e_1 \mathcal{P}_3 e_1^T \\ & - (1 - \mu) e_2 \mathcal{P}_3 e_2^T + d^2 e_7 \mathcal{P}_4 e_7^T - \begin{bmatrix} (e_1 - e_2) \\ (e_2 - e_3) \end{bmatrix} \begin{bmatrix} \mathcal{P}_4 & \mathcal{H} \\ \mathcal{H}^T & \mathcal{P}_4 \end{bmatrix} \begin{bmatrix} (e_1 - e_2) \\ (e_2 - e_3) \end{bmatrix}^T + \tau^2 e_4 \mathcal{P}_5 e_4^T \\ & - e_6 \mathcal{P}_5 e_6^T + 2[e_1 \mathcal{G}_1 + e_7 \mathcal{G}_2] [-e_7 - \mathcal{D} e_1 + \mathcal{A} e_4 + \mathcal{B} e_5 + \mathcal{C} e_6]^T, \\ \Omega_2 = & -2e_4 \mathcal{N}_1 e_4^T + 2e_1 (\mathcal{L}_1 + \mathcal{L}_2) \mathcal{N}_1 e_4^T - 2e_1 \mathcal{L}_1 \mathcal{N}_1 \mathcal{L}_2 e_1^T - 2e_5 \mathcal{N}_2 e_5^T + 2e_2 (\mathcal{L}_1 + \mathcal{L}_2) \mathcal{N}_2 e_5^T \\ & - 2e_2 \mathcal{L}_1 \mathcal{N}_2 \mathcal{L}_2 e_2^T. \end{aligned}$$

Proof: We choose the following LKF for NNs (1)

$$\mathbb{V}(t) = \sum_{i=1}^6 \mathbb{V}_i(t)$$

where

$$\begin{aligned} \mathbb{V}_1(t) &= z^T(t)\mathcal{P}_1z(t) \\ \mathbb{V}_2(t) &= 2\sum_{i=1}^n\lambda_{1i}\int_0^{z_i(t)}(g_i(s)-l_i^-(s))ds+2\sum_{i=1}^n\lambda_{2i}\int_0^{z_i(t)}(l_i^+(s)-g_i(s))ds \\ \mathbb{V}_3(t) &= \int_{t-d}^tz^T(s)\mathcal{P}_2z(s)ds \\ \mathbb{V}_4(t) &= \int_{t-d(t)}^tz^T(s)\mathcal{P}_3z(s)ds \\ \mathbb{V}_5(t) &= d\int_{-d}^0\int_{t+\alpha}^t\dot{z}^T(s)\mathcal{P}_4\dot{z}(s)dsd\alpha \\ \mathbb{V}_6(t) &= \tau\int_{-t}^0\int_{t+\alpha}^tg^T(z(s))\mathcal{P}_5g(z(s))dsd\alpha. \end{aligned}$$

Calculating the derivative of $\mathbb{V}(t)$ we get

$$\dot{\mathbb{V}}_1(t) = 2z^T(t)\mathcal{P}_1\dot{z}(t) \tag{8}$$

$$\dot{\mathbb{V}}_2(t) = 2[g(z(t)) - \mathcal{L}_1z(t)]^T\Lambda_1\dot{z}(t) + 2[\mathcal{L}_2z(t) - g(z(t))]^T\Lambda_2\dot{z}(t) \tag{9}$$

$$\dot{\mathbb{V}}_3(t) = z^T(t)\mathcal{P}_2z(t) - z^T(t-d)\mathcal{P}_2z(t-d) \tag{10}$$

$$\dot{\mathbb{V}}_4(t) = z^T(t)\mathcal{P}_3z(t) - (1-\mu)z^T(t-d(t))\mathcal{P}_3z(t-d(t)) \tag{11}$$

$$\dot{\mathbb{V}}_5(t) = d^2\dot{z}^T(t)\mathcal{P}_4\dot{z}(t) - d\int_{t-d}^t\dot{z}^T(s)\mathcal{P}_4\dot{z}(s)ds \tag{12}$$

$$\dot{\mathbb{V}}_6(t) = \tau^2g^T(z(t))\mathcal{P}_5g(z(t)) - \tau\int_{t-\tau}^tg^T(z(s))ds\mathcal{P}_5g(z(s))ds. \tag{13}$$

Applying Lemma (2.4) and (2.5), we get

$$-d \int_{t-d}^t \dot{z}^T(s) \mathcal{P}_4 \dot{z}(s) ds \leq - \begin{bmatrix} z(t) - z(t-d(t)) \\ z(t-d(t)) - z(t-d) \end{bmatrix}^T \begin{bmatrix} \mathcal{P}_4 & \mathcal{H} \\ \mathcal{H}^T & \mathcal{P}_4 \end{bmatrix} \begin{bmatrix} z(t) - z(t-d(t)) \\ z(t-d(t)) - z(t-d) \end{bmatrix}, \quad (14)$$

$$-\tau \int_{t-\tau(t)}^t g^T(z(s)) ds \mathcal{P}_5 g(z(s)) ds \leq - \left(\int_{t-\tau(t)}^t z(s) ds \right)^T \mathcal{P}_5 \left(\int_{t-\tau(t)}^t z(s) ds \right). \quad (15)$$

On the other hand, for any matrices \mathcal{G}_1 and \mathcal{G}_2 with appropriate dimensions, it is true that,

$$0 = 2[z^T(t) \mathcal{G}_1 + \dot{z}^T(t) \mathcal{G}_2] [-\dot{z}(t) - \mathcal{D}z(t) + \mathcal{A}g(z(t)) + \mathcal{B}g(z(t-d(t)))] + \mathcal{C} \int_{t-\tau(t)}^t g(z(s)) ds. \quad (16)$$

Furthermore, from (3), for the diagonal matrix, $\mathcal{N}_1, \mathcal{N}_2$, we can achieve the following inequalities

$$-2g^T(z(t)) \mathcal{N}_1 g(z(t)) + 2z^T(t) (\mathcal{L}_1 + \mathcal{L}_2) \mathcal{N}_1 g(z(t)) - 2z^T(t) \mathcal{L}_1 \mathcal{N}_1 \mathcal{L}_2 z(t) \geq 0, \quad (17)$$

$$\begin{aligned} & -2g^T(z(t-d(t))) \mathcal{N}_2 g(z(t-d(t))) + 2z^T(t-d(t)) (\mathcal{L}_1 + \mathcal{L}_2) \mathcal{N}_2 g(z(t-d(t))) \\ & - 2z^T(t-d(t)) \mathcal{L}_1 \mathcal{N}_2 \mathcal{L}_2 z(t-d(t)) \geq 0. \end{aligned} \quad (18)$$

From (9)-(18), we can get

$$\dot{\mathbb{V}}(t) \leq \xi^T(t) (\Omega_1 + \Omega_2) \xi(t), \quad (19)$$

If $\Omega < 0$, then $\dot{\mathbb{V}}(t) < 0$. This means that the system (1) is globally asymptotically stable. The proof is completed ■

The other notations are defined as

$$\begin{aligned} \mathcal{L}_1 &= \text{diag}\{l_1^-, l_2^-, \dots, l_n^-\}, \quad \mathcal{L}_2 = \text{diag}\{l_1^+, l_2^+, \dots, l_n^+\}, \\ \xi(t) &= \begin{bmatrix} z^T(t) & z^T(t-d(t)) & z^T(t-d) & g^T(z(t)) & g^T(z(t-d(t))) & \int_{t-\tau(t)}^t g^T(z(s)) ds & \dot{z}^T(t) \end{bmatrix}^T \end{aligned}$$

Theorem 3.2 For given scalars d, τ and μ system (1) with $\omega(t) \neq 0$ is strictly $(\mathcal{Q}, \mathcal{S}, \mathcal{R}) - \gamma$ -dissipativity, if there exist symmetric positive definite matrices $\mathcal{P}_1, \mathcal{P}_2, \mathcal{P}_3, \mathcal{P}_4, \mathcal{P}_5$, positive diagonal matrix matrix $\Lambda_1, \Lambda_2, \mathcal{N}_1, \mathcal{N}_2$, and any matrix $\mathcal{H}, \mathcal{G}_1, \mathcal{G}_2$ and positive scalar γ such that the following LMI holds:

$$\Xi = \begin{bmatrix} \Xi_{11} & \Xi_{12} \\ \Xi_{12}^T & \Xi_{22} \end{bmatrix} < 0, \quad (20)$$

where $\Xi_{11} = \Omega - e_4 \mathcal{Q} e_4^T$, $\Xi_{12} = e_1 \mathcal{G}_1 e_8^T + e_7 \mathcal{G}_2 e_8^T - e_4 \mathcal{S} e_8^T$, $\Xi_{22} = e_8 (-\mathcal{R} + \gamma I) e_8^T$, and Ω is defined the same as in Theorem (3.1).

Proof: To show the dissipativity, we choose the same LKF and define the following performance index for NNs (1)

$$\mathcal{J}_{\gamma, t^*} = \int_0^{t^*} (y(t)^T \mathcal{Q} y(t) + 2y(t)^T \mathcal{S} \omega(t) + \omega(t)^T (\mathcal{R} - \gamma I) \omega(t)) dt. \quad (21)$$

Following the proof of Theorem (3.1), we get

$$\int_0^{t^*} \dot{V}(t) dt - \mathcal{J}_{\gamma, t^*} \leq \int_0^{t^*} \eta(t)^T \Xi \eta(t) dt, \quad (22)$$

where $\eta(t) = [\xi^T(t) \ \omega^T(t)]^T$. It can be deduced from (4) that

$$\int_0^{t^*} \dot{V}(t) dt \leq \mathcal{J}_{\gamma, t^*}. \quad (23)$$

Under the zero initial condition, we can conclude that (4) holds, which means NNs (1) is strictly $(\mathcal{Q}, \mathcal{S}, \mathcal{R}) - \gamma$ -dissipative. This complete the proof. ■

Theorem 3.3 For given scalars d, τ and μ system (1) passive, if there exist symmetric positive definite matrices $\mathcal{P}_1, \mathcal{P}_2, \mathcal{P}_3, \mathcal{P}_4, \mathcal{P}_5$, positive diagonal matrix matrix $\Lambda_1, \Lambda_2, \mathcal{N}_1, \mathcal{N}_2$, and any matrix $\mathcal{H}, \mathcal{G}_1, \mathcal{G}_2$ and positive scalar γ such that the following LMI holds:

$$\Omega = \Omega_1 + \Omega_2 < 0, \quad (24)$$

where

$$\begin{aligned} \Omega_1 = & 2e_1\mathcal{P}_1e_7^T + 2[e_4 - \mathcal{L}_1e_1]^T\Lambda_1e_7 + 2[\mathcal{L}_2e_1 - e_4]^T\Lambda_2e_7 + e_1\mathcal{P}_2e_1^T - e_3\mathcal{P}_2e_3 + e_1\mathcal{P}_3e_1^T \\ & - (1 - \mu)e_2\mathcal{P}_3e_2^T + d^2e_7\mathcal{P}_4e_7^T - \begin{bmatrix} (e_1 - e_2) \\ (e_2 - e_3) \end{bmatrix} \begin{bmatrix} \mathcal{P}_4 & \mathcal{H} \\ \mathcal{H}^T & \mathcal{P}_4 \end{bmatrix} \begin{bmatrix} (e_1 - e_2) \\ (e_2 - e_3) \end{bmatrix}^T + \tau^2e_4\mathcal{P}_5e_4^T \\ & - e_6\mathcal{P}_5e_6^T + 2[e_1\mathcal{G}_1 + e_7\mathcal{G}_2][-e_7 - \mathcal{D}e_1 + \mathcal{A}e_4 + \mathcal{B}e_5 + \mathcal{C}e_6]^T, \\ \Omega_2 = & -2e_4\mathcal{N}_1e_4^T + 2e_1(\mathcal{L}_1 + \mathcal{L}_2)\mathcal{N}_1e_4^T - 2e_1\mathcal{L}_1\mathcal{N}_1\mathcal{L}_2e_1^T - 2e_5\mathcal{N}_2e_5^T + 2e_2(\mathcal{L}_1 + \mathcal{L}_2)\mathcal{N}_2e_5^T \\ & - 2e_2\mathcal{L}_1\mathcal{N}_2\mathcal{L}_2e_2^T. \end{aligned}$$

Proof: To show the passivity, we choose the same LKF and define the following performance index for NNs (1)

From (8)-(19), we can get

$$\dot{\mathbb{V}}(t) - 2y^T(t)\omega(t) - \gamma\omega^T(t)\omega(t) \leq 0, \quad (25)$$

By integrating (25) with respect to t over the time period from 0 to t_p , we know that under zero initial conditions

$$\begin{aligned} 2 \int_0^{t_p} y^T(s)\omega(s)ds & \geq \mathbb{V}(x(t_p)) - \mathbb{V}(x(0)) - \gamma \int_0^{t_p} \omega^T(s)\omega(s)ds \\ & \geq -\gamma \int_0^{t_p} \omega^T(s)\omega(s)ds. \end{aligned} \quad (26)$$

If $\Omega < 0$, then $\dot{\mathbb{V}}(t) < 0$. This means that the system (1) with discrete time-varying delay is passive in the sense of Definition 2.3. This completes the proof. ■

4. Numerical example

In this section, we give an illustrative example to demonstrate the less conservatism of our result and the effectiveness of the proposed method.

Example 4.1 Consider NNs (1) with the following parameters:

$$\mathcal{D} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}, \quad \mathcal{A} = \begin{bmatrix} -1 & 0.5 \\ 0.5 & -1.5 \end{bmatrix}, \quad \mathcal{B} = \begin{bmatrix} -2 & 0.5 \\ 0.5 & -2 \end{bmatrix}, \quad \mathcal{C} = \begin{bmatrix} 0.25 & 0 \\ 0 & 0.25 \end{bmatrix}.$$

The activation function satisfies

$$\mathcal{L}_1 = \begin{bmatrix} 0.3680 & 0 \\ 0 & 0.1795 \end{bmatrix}, \mathcal{L}_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

By using the Matlab LMI toolbox, we solve the LMI for $d = 0.5, \tau = 0.2$ and $\mu = 0.1$ the feasible solutions are

$$\begin{aligned} \mathcal{P}_1 &= \begin{bmatrix} 484.4975 & 0.0664 \\ 0.0664 & -1.3836 \end{bmatrix}, \mathcal{P}_2 = \begin{bmatrix} 0.7589 & 0.0170 \\ 0.0170 & 0.7564 \end{bmatrix}, \mathcal{P}_3 = \begin{bmatrix} -1.4003 & 0.0241 \\ 0.0241 & -1.4263 \end{bmatrix}, \\ \mathcal{P}_4 &= \begin{bmatrix} -0.0531 & 0.0002 \\ 0.0002 & -0.0428 \end{bmatrix}, \mathcal{P}_5 = \begin{bmatrix} 1.0226 & 0.0000 \\ 0.0000 & 1.0226 \end{bmatrix}, \mathcal{N}_1 = \begin{bmatrix} 0.4297 & -0.0031 \\ -0.0031 & 0.4989 \end{bmatrix}, \\ \mathcal{N}_2 &= \begin{bmatrix} 0.4023 & 0.0000 \\ 0.0000 & 0.4804 \end{bmatrix}, \mathcal{G}_1 = \begin{bmatrix} 0.0559 & 0.0144 \\ 0.0144 & 0.0430 \end{bmatrix}, \mathcal{G}_2 = \begin{bmatrix} 0.0639 & 0.0260 \\ 0.0260 & 0.0627 \end{bmatrix}, \\ \Lambda_1 &= \begin{bmatrix} 1.3161 & 0.0000 \\ 0.0000 & -0.0086 \end{bmatrix}, \Lambda_2 = \begin{bmatrix} 1.3160 & -0.0000 \\ -0.0000 & -0.0087 \end{bmatrix}, \mathcal{H} = \begin{bmatrix} 0.0018 & -0.0015 \\ -0.0015 & 0.0064 \end{bmatrix}. \end{aligned}$$

Therefore, the concerned neural networks with time-varying delays is globally asymptotic stable.

Example 4.2 Consider NNs (1) with the following parameters:

$$\mathcal{D} = \begin{bmatrix} 2 & 0 \\ 0 & 1.5 \end{bmatrix}, \mathcal{A} = \begin{bmatrix} -1 & 1 \\ 0.5 & -1 \end{bmatrix}, \mathcal{B} = \begin{bmatrix} -0.5 & 0.6 \\ 0.7 & 0.8 \end{bmatrix}, \mathcal{C} = \begin{bmatrix} 1 & -0.6 \\ 0 & 1 \end{bmatrix}.$$

$$l_1^+ = l_2^+ = 0.9, l_1^- = l_2^- = -0.1$$

For choose the concept of dissipativity

$$\mathcal{Q} = \begin{bmatrix} -0.9 & 0 \\ 0 & -0.9 \end{bmatrix}, \mathcal{S} = \begin{bmatrix} 0.5 & 0 \\ 0.3 & 1 \end{bmatrix}, \mathcal{R} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}.$$

For given scalar τ , the MAUBs d is attained for Theorem (3.2) under various μ , which is shown in Table 1. By using the Matlab LMI control Toolbox to solve the

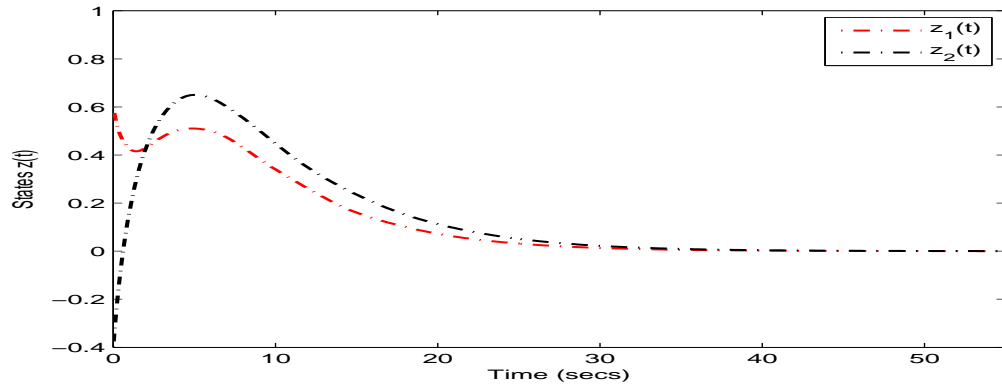


Figure 1: State response of the NNs (1)

LMIs (25) in Theorem (3.2) is feasible with given above parameters is asymptotically stable. Under the initial conditions $\psi(t) = [0.6, -0.4]^T$, the numerical illustration of state trajectories $z_1(t), z_2(t)$ of the system (1) is depicted in Fig 1.

Table-1: Maximum allowable upper bounds of d for different μ in Example 1.

| τ | $\frac{\mu \rightarrow}{Methods \downarrow}$ | 0.2 | 0.4 | 0.6 | 0.8 |
|--------|--|--------|--------|--------|--------|
| 0.3 | Theorem (3.1) | 2.2507 | 1.8472 | 1.6271 | 1.0836 |

Example 4.3 Consider NNs (1) with the following parameters:

$$\mathcal{D} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \mathcal{A} = \begin{bmatrix} 0.3 & 0.1 \\ 0.1 & 0.3 \end{bmatrix}, \quad \mathcal{B} = \begin{bmatrix} 0.1 & 0.16 \\ 0.05 & 0.1 \end{bmatrix}, \quad \mathcal{C} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.3 \end{bmatrix}.$$

The activation function satisfies

$$\mathcal{L}_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \mathcal{L}_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

By using the Matlab LMI toolbox, we solve the LMI for $d = 0.5, \tau = 0.2, \gamma = 0.5$ and

$\mu = 0.1$ the feasible solutions are

$$\begin{aligned} \mathcal{P}_1 &= \begin{bmatrix} -0.0691 & 0.0033 \\ 0.0033 & -0.0561 \end{bmatrix}, \mathcal{P}_2 = \begin{bmatrix} 0.7845 & -0.0651 \\ -0.0651 & 0.7823 \end{bmatrix}, \mathcal{P}_3 = \begin{bmatrix} -2.1859 & -0.5441 \\ -0.5441 & -2.1977 \end{bmatrix}, \\ \mathcal{P}_4 &= \begin{bmatrix} -0.2717 & -0.0626 \\ -0.0626 & -0.1303 \end{bmatrix}, \mathcal{P}_5 = \begin{bmatrix} 0.9756 & 0.0107 \\ 0.0107 & 1.0009 \end{bmatrix}, \mathcal{N}_1 = \begin{bmatrix} 0.0814 & 0.0730 \\ 0.0730 & 0.2643 \end{bmatrix}, \\ \mathcal{N}_2 &= \begin{bmatrix} 0.0498 & 0.0649 \\ 0.0649 & 0.2912 \end{bmatrix}, \mathcal{G}_1 = \begin{bmatrix} 0.2725 & -0.3204 \\ -0.3204 & -0.0351 \end{bmatrix}, \mathcal{G}_2 = \begin{bmatrix} 0.3480 & -0.0694 \\ -0.0694 & 0.3010 \end{bmatrix}, \\ \Lambda_1 &= \begin{bmatrix} -0.0004 & 0.0002 \\ 0.0002 & -4.8579 \end{bmatrix}, \Lambda_2 = \begin{bmatrix} -0.0004 & 0.0002 \\ 0.0002 & -4.8578 \end{bmatrix}, \mathcal{H} = \begin{bmatrix} -0.1046 & -0.0069 \\ -0.0069 & -0.0228 \end{bmatrix}, \\ \mathcal{I} &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}. \end{aligned}$$

Therefore, the concerned neural networks with time-varying delays is passivity.

5. Conclusion

This paper focuses on the problem of Dissipativity and Passivity analysis of NNs with mixed-time-varying delays. By employing Lyapunov functional approach, some sufficient conditions are derived to guarantee that the considered NNs are strictly $(\mathcal{Q}, \mathcal{S}, \mathcal{R}) - \gamma$ -Dissipative and Passivity. Based on Lyapunov stability theory, proper Lyapunov-Krasovskii functional (LKF) with some new terms is constructed, and estimating their derivative by using newly developed single integral inequality that includes Jensens inequality which can be easily checked by applying MATLAB LMT toolkit. Three Numerical example is finally provided to demonstrate the effectiveness and advantages of the proposed method.

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