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Improved stability analysis for Markovian Jump Static Neural Networks with Mode-Dependent Time-Varying Delays

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Abstract

This paper investigates the problem of delay-dependent stability analysis for Markovian jump static neural networks (MJSNNs) with mode-dependent time-varying delays. The fundamental objective of this paper is to create novel stability criterion for the considered MJSNNs with less conservatism. A suitable Lyapunov-Krasovskii functional (LKF) is constructed with more system information. By employing integral inequality, a novel delay-dependent sufficient condition is obtained to ensure the asymptotically stability of the equilibrium point. The obtained stability condition is derived and entrenched in terms of linear matrix inequality (LMI) which can be clearly checked by MATLAB LMI control toolbox. At long last, two benchmark illustrative case are given to show the effectiveness of the theoretical result.

Key words: MJSNNs; LKF; Mode-dependent time-varying delays; Integral inequality.

1. Introduction

Over the past few decades, MJNNs have been broadly investigated. The reason is mostly that MJNNs is a suitable scientific model to represent a class of dynamic systems subject to arbitrary unexpected changes in their structures, and discovered their applications in numerous areas, such as target tracking problems, fault-tolerant systems, manufactory processes and so on [1], [2]. In the view of mathematical point, MJNNs can be regarded as a special class of stochastic neural networks with system matrices changed randomly at discrete time instances governed by a Markov process. On the other hand, the phenomena of time delay has frequently encountered in dynamic systems, and it is often a major cause of instability, oscillation and poor performance [3], [4], [5]. Thus, the investigation of the stability properties with time delay is always important issue. Consequently, much attention has been dedicated

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to the examination of the stability of neural networks (NNs). Recently, numerous outcomes in regards to NNs with Markovian jump parameters have been explored in the works [1], [2], [3], [4], [5], [6], [7].

Nowadays, the Lyapunov method is powerful tool to study the stability of dynamic systems. To find out the less conservative results of such dynamic systems is one of the primary difficulties in the issue of stability of NNs. Generally speaking that the less conservative stability conditions can be derived from two aspects. That is, choosing augmented LKF involving more system information and using tighter bound integral inequalities to estimate the derivatives of LKF which are actually well supported to reduce the conservatism. Thus, much effort has been invested in reducing the conservatism of delay-dependent stability criteria. For example, in [7], [19], [20], [21], different sorts of LKF has been well considered to reduce the conservatism of developed stability conditions. Moreover, to develop a novel integral inequality for quadratic functions plays a significant role in the issue of stability of NNs. Recently, various effective integral inequalities have been introduced to pick up an improved stability conditions, for example [22], [23], [24], [25], [26], and reference therein.

On the other hand, there has been rapidly growing interest have been paid in the issue of MJNNs [1]- [7]. For example, in [3], delay-dependent stability analysis for MJSNNs including the relative information on the interval time-varying delay is considered into the framework and derived delay-dependent sufficient conditions are expressed in the form of LMIs. Equally, some various stability results have been examined in the recent works [4], [6]. However, to the best of our knowledge, there are few results on the stability for MJSNNs with time-varying delays. In this manner, it is noteworthy to study the stability analysis for MJSNNs with mode-dependent time-varying delays.

Motivated by the above discussions, in this paper, our research efforts are mainly focused on developing a new approach to analyze the stability MJSNNs with mode-dependent time-varying delays. This paper begins with by building up a new single integral inequality, then to show the effectiveness of proposed integral inequality, a suitable LKF including some delay-dependent terms is introduced. By employing developed integral inequality, novel delay-dependent sufficient condition is obtained to ensure the asymptotically stability. The obtained less conservative stability condition is derived and entrenched in terms of LMI technique, subsequent to utilizing MATLAB LMI control toolbox, a feasible solution can be easily obtained. Finally, numerical examples are given to show the effectiveness of the proposed result.

Notations: Throughout this paper, the following notions are used. Let the

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symbols $\mathbb{R}^n$ and $\mathbb{R}^{m\times n}$ are the n-dimensional Euclidean space with vector norm $\|\cdot\|$ and the set of $m \times n$ real matrices over the real field $\mathbb{R}$, respectively. The superscripts X^T and X^{-1} represents the transpose and inverse of the matrix X, respectively. $(\Omega, \mathscr{F}, \mathscr{P})$ is a probability space, where Ω is the sample space, $\mathscr{F}$ is the σ-algebra of subsets of the sample space and $\mathscr{P}$ is the probability measure on $\mathscr{F}$. $\mathcal{Q} > 0$ means that $\mathcal{Q}$ is the symmetric positive definite matrix. ✠ indicates the elements below the main diagonal of a symmetric matrix. I_n stands for the identity matrix with appropriate dimensions. The notation $\langle x^T \odot \mathcal{R}\rangle$ is defined as $\langle x^T \odot \mathcal{R}\rangle = x^T\mathcal{R}x$.

2. Problem formulation and preliminaries

Given a probability space $(\Omega, \mathscr{F}, \mathscr{P})$, we consider the following Markovian jump static neural networks with mode-dependent time-varying delays

$$\begin{cases} \dot{x}(t) = & -\mathcal{A}(r_t)x(t) + f(\mathcal{M}(r_t)x(t-d_{r_t}(t))) + \mathcal{J} \\ x(t) = & \phi(t),\ t \in [-d, 0] \end{cases} \tag{1}$$

where $x(t) = [x_1(t), x_2(t), ...x_n(t)]^T \in \mathbb{R}^n$ is the neuron state vector at time $t \geq 0$, n corresponds to the number of neurons, $f(x(t)) = [f_1(x(t)), f_2(x(t)), ...f_n(x(t))]^T \in \mathbb{R}^n$ denotes the continuous activation function, $\mathcal{J} = [\mathcal{J}_1, \mathcal{J}_2, ..., \mathcal{J}_n]^T$ is a constant input vector. Also, $\{r_t\}$ is assumed to be a continuous-time Markov process that takes values in the finite discrete set $\mathcal{S} = \{1, 2, ..., N\}$ and has the transition probabilities

$$\Pr\{r_{t+\triangle} = j | r_t = i\} = \begin{cases} \gamma_{ij}\triangle + o(\triangle), & \text{if } i \neq j \\ 1 + \gamma_{ii}\triangle + o(\triangle), & \text{if } i = j \end{cases} \tag{2}$$

where $\triangle > 0$ and $\lim\limits_{\triangle\to 0^+} \frac{o(\triangle)}{\triangle} = 0$, $\gamma_{ij} \geq 0$, $\forall i \neq j$ and $\gamma_{ii} = -\sum\limits_{i\neq j}\gamma_{ij}$. The transition probability matrix is denoted by $\Gamma = [\gamma_{ij}]_{N\times N}$. $\phi(t)$ is a vector-valued initial continuous function defined on the interval $[-d, 0], d = max\{d_{r_t}, r_t \in \mathcal{S}\}$. $\mathcal{A}(r_t) = diag\{a_1, a_2, ...a_n\} > 0$ with $a_i > 0$, $i = 1, 2, ..., n$ is a diagonal matrix representing self-feedback term. $\mathcal{M}(r_t) = [\mathcal{M}_{ij}]$ is the delayed connection weight matrix. The delay, $d_{r_t}(t)$ is a time-varying continuous function satisfying

$$0 \leq d_{r_t}(t) \leq d_{r_t},\ \dot{d_{r_t}} \leq \mu < \infty. \tag{3}$$

where, d_{r_t} and μ are known real constants.

Assumption 1: Each neuron activation function, $f_i(\cdot), i = 1, 2, ..., n$ in (1) is

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continuous, bounded and satisfies,

$$\mathcal{K}_i^- \leq \frac{f_i(\alpha_1) - f_i(\alpha_2)}{(\alpha_1 - \alpha_2)} \leq \mathcal{K}_i^+, \tag{4}$$

$f_i(0) = 0, \alpha_1, \alpha_2 \in \mathbb{R}, \alpha_1 \neq \alpha_2$ and $\mathcal{K}_i^-$ and $\mathcal{K}_i^+$ are known real constants matrices. Under Assumption 1, there exists an equilibrium x^* of (1). By defining $y^* = x(\cdot) - x^*$, then the system (1) can be transformed into

$$\begin{cases} \dot{y}(t) = & -\mathcal{A}(r_t)y(t) + g(\mathcal{M}(r_t)y(t - d_{r_t}(t))) \\ y(t) = & \psi(t),\ t \in [-d, 0] \end{cases} \tag{5}$$

where $y(t) = [y_1(t), y_2(t), ...y_n(t)]^T \in \mathbb{R}^n$ is the state vector; $\psi(t) = \phi(t) - x^*$ is the initial condition. The transformed activation functions $g(\mathcal{M}(r_t)y(\cdot)) = f(\mathcal{M}(r_t)x(\cdot) + \mathcal{M}(r_t)x^* + \mathcal{J}) - f(\mathcal{M}(r_t)x^* + \mathcal{J})$ satisfying the following assumption.
Assumption 2: The transformed activation neuron activation function, $g_i(\cdot), i = 1, 2, ..., n$ in (5) is continuous, bounded and satisfies,

$$\mathcal{K}_i^- \leq \frac{g_i(\alpha_1) - g_i(\alpha_2)}{(\alpha_1 - \alpha_2)} \leq \mathcal{K}_i^+, \tag{6}$$

$g_i(0) = 0, \alpha_1, \alpha_2 \in \mathbb{R}, \alpha_1 \neq \alpha_2$ and $\mathcal{K}_i^-$ and $\mathcal{K}_i^+$ are known real constants matrices.
For each $r_t = i, i \in \mathcal{S}$, for example, the matrix $\mathcal{A}(r_t)$ will be denoted by $\mathcal{A}_i$ and so on.
Now, the system in (5) can be converted to

$$\begin{cases} \dot{y}(t) = & -\mathcal{A}_i y(t) + g(\mathcal{M}_i y(t - d_i(t))) \\ y(t) = & \psi(t),\ t \in [-d, 0],\ d = max\{d_i, i \in \mathcal{S}\} \end{cases} \tag{7}$$

The following lemma which will be used in the proof of our main result.

Lemma 2.1 For a given matrix $\mathcal{Z}$, $\theta_1 \leq \theta(t) \leq \theta_2$, and any appropriate dimension matrix $\mathcal{X}$, which satisfies $\begin{bmatrix} \hat{\mathcal{Z}} & \mathcal{X} \\ \mathcal{X}^T & \hat{\mathcal{Z}} \end{bmatrix} \geq 0$. Then, the following inequality holds for all

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continuously differentiable function $x(t)$:

$$-(\theta_2-\theta_1)\int_{t-\theta_2}^{t-\theta_1}\langle\dot{x}^T(s)\odot\mathcal{Z}\rangle ds\leq-\left\langle\mathscr{C}^T(t)\odot\begin{bmatrix}\hat{\mathcal{Z}} & \mathcal{X}\\ \mathcal{X}^T & \hat{\mathcal{Z}}\end{bmatrix}\right\rangle \quad (8)$$

where $\mathscr{C}(t)=col[\mathscr{C}_1(t)\ \ \mathscr{C}_2(t)\ \ \mathscr{C}_3(t)\ \ \mathscr{C}_4(t)\ \ \mathscr{C}_5(t)\ \ \mathscr{C}_6(t)]$ $\mathscr{C}_1(t)=[x(t-\theta_1)-x(t-\theta(t))]$,
$\mathscr{C}_2(t)=[x(t-\theta_1)+x(t-\theta(t))-\frac{2}{(\theta(t)-\theta_1)}\int_{t-\theta(t)}^{t-\theta_1}x(s)ds]$,
$\mathscr{C}_3(t)=[x(t-\theta_1)-x(t-\theta(t))+\frac{6}{(\theta(t)-\theta_1)}\int_{t-\theta(t)}^{t-\theta_1}x(s)ds-\frac{12}{(\theta(t)-\theta_1)^2}\int_{t-\theta(t)}^{t-\theta_1}\int_{\alpha}^{t}x(s)dsd\alpha]$,
$\mathscr{C}_4(t)=[x(t-\theta(t))-x(t-\theta_2)]$, $\mathscr{C}_5(t)=[x(t-\theta(t))+x(t-\theta_2)-\frac{2}{(\theta_2-\theta(t))}\int_{t-\theta_2}^{t-\theta(t)}x(s)ds]$,
$\mathscr{C}_6(t)=[x(t-\theta(t))-x(t-\theta_2)+\frac{6}{(\theta_2-\theta(t))}\int_{t-\theta_2}^{t-\theta(t)}x(s)ds-\frac{12}{(\theta_2-\theta(t))^2}\int_{t-\theta_2}^{t-\theta(t)}\int_{\alpha}^{t}x(s)dsd\alpha]$,

$$\hat{\mathcal{Z}}=\begin{bmatrix}\mathcal{Z} & 0 & 0\\ 0 & 3\mathcal{Z} & 0\\ 0 & 0 & 5\mathcal{Z}\end{bmatrix},\ \mathcal{X}=\begin{bmatrix}\mathcal{X}_1 & \mathcal{X}_2 & \mathcal{X}_3\\ \mathcal{X}_4 & \mathcal{X}_5 & \mathcal{X}_6\\ \mathcal{X}_7 & \mathcal{X}_8 & \mathcal{X}_9\end{bmatrix}.$$

Proof: When $\theta_1\leq\theta(t)\leq\theta_2$, together with Auxiliary function-based integral inequality proposed in [24], we can obtain as follows:

$$\begin{aligned}
-(\theta_2-\theta_1)\int_{t-\theta_2}^{t-\theta_1}\langle\dot{x}^T(s)\odot\mathcal{Z}\rangle ds &= -(\theta_2-\theta_1)\int_{t-\theta(t)}^{t-\theta_1}\langle\dot{x}^T(s)\odot\mathcal{Z}\rangle ds\\
&\quad -(\theta_2-\theta_1)\int_{t-\theta_2}^{t-\theta(t)}\langle\dot{x}^T(s)\odot\mathcal{Z}\rangle ds\\
&\leq -\tfrac{\theta_2-\theta_1}{\theta(t)-\theta_1}\{\langle\mathscr{C}_1^T(t)\odot\mathcal{Z}\rangle+\langle\mathscr{C}_2^T(t)\odot 3\mathcal{Z}\rangle+\langle\mathscr{C}_3^T(t)\odot 5\mathcal{Z}\rangle\}\\
&\quad -\tfrac{\theta_2-\theta_1}{\theta_2-\theta(t)}\{\langle\mathscr{C}_4^T(t)\odot\mathcal{Z}\rangle+\langle\mathscr{C}_5^T(t)\odot 3\mathcal{Z}\rangle+\langle\mathscr{C}_6^T(t)\odot 5\mathcal{Z}\rangle\}\\
&= -\tfrac{\theta_2-\theta_1}{\theta(t)-\theta_1}\left\langle\begin{bmatrix}\mathscr{C}_1(t)\\ \mathscr{C}_2(t)\\ \mathscr{C}_3(t)\end{bmatrix}^T\odot\hat{\mathcal{Z}}\right\rangle-\tfrac{\theta_2-\theta_1}{\theta_2-\theta(t)}\left\langle\begin{bmatrix}\mathscr{C}_4(t)\\ \mathscr{C}_5(t)\\ \mathscr{C}_6(t)\end{bmatrix}^T\odot\hat{\mathcal{Z}}\right\rangle\\
&= -\left\langle\begin{bmatrix}\mathscr{C}_1(t)\\ \mathscr{C}_2(t)\\ \mathscr{C}_3(t)\end{bmatrix}^T\odot\hat{\mathcal{Z}}\right\rangle-\tfrac{\theta_2-\theta(t)}{\theta(t)-\theta_1}\left\langle\begin{bmatrix}\mathscr{C}_1(t)\\ \mathscr{C}_2(t)\\ \mathscr{C}_3(t)\end{bmatrix}^T\odot\hat{\mathcal{Z}}\right\rangle\\
&\quad -\left\langle\begin{bmatrix}\mathscr{C}_4(t)\\ \mathscr{C}_5(t)\\ \mathscr{C}_6(t)\end{bmatrix}^T\odot\hat{\mathcal{Z}}\right\rangle-\tfrac{\theta(t)-\theta_1}{\theta_2-\theta(t)}\left\langle\begin{bmatrix}\mathscr{C}_4(t)\\ \mathscr{C}_5(t)\\ \mathscr{C}_6(t)\end{bmatrix}^T\odot\hat{\mathcal{Z}}\right\rangle
\end{aligned}$$

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if $\begin{bmatrix} \hat{\mathcal{Z}} & \mathcal{X} \\ \mathcal{X}^T & \hat{\mathcal{Z}} \end{bmatrix} > 0$ the following inequality is satisfied by [26].

$$\left\langle \begin{bmatrix} \sqrt{\frac{\theta_2-\theta(t)}{\theta(t)-\theta_1}} \begin{bmatrix} \mathscr{C}_1(t) \\ \mathscr{C}_2(t) \\ \mathscr{C}_3(t) \end{bmatrix} \\ \sqrt{\frac{\theta(t)-\theta_1}{\theta_2-\theta(t)}} \begin{bmatrix} \mathscr{C}_4(t) \\ \mathscr{C}_5(t) \\ \mathscr{C}_6(t) \end{bmatrix} \end{bmatrix}^T \odot \begin{bmatrix} \hat{\mathcal{Z}} & \mathcal{X} \\ \mathcal{X}^T & \hat{\mathcal{Z}} \end{bmatrix} \right\rangle \geq \ 0 \text{ which implies}$$

$$-\frac{\theta_2-\theta_1}{\theta(t)-\theta_1} \left\langle \begin{bmatrix} \mathscr{C}_1(t) \\ \mathscr{C}_2(t) \\ \mathscr{C}_3(t) \end{bmatrix}^T \odot \hat{\mathcal{Z}} \right\rangle - \frac{\theta_2-\theta_1}{\theta_2-\theta(t)} \left\langle \begin{bmatrix} \mathscr{C}_4(t) \\ \mathscr{C}_5(t) \\ \mathscr{C}_6(t) \end{bmatrix}^T \odot \hat{\mathcal{Z}} \right\rangle$$

$$\leq - \begin{bmatrix} \mathscr{C}_1(t) \\ \mathscr{C}_2(t) \\ \mathscr{C}_3(t) \end{bmatrix}^T \mathcal{X} \begin{bmatrix} \mathscr{C}_4(t) \\ \mathscr{C}_5(t) \\ \mathscr{C}_6(t) \end{bmatrix} - \begin{bmatrix} \mathscr{C}_4(t) \\ \mathscr{C}_5(t) \\ \mathscr{C}_6(t) \end{bmatrix}^T \mathcal{X}^T \begin{bmatrix} \mathscr{C}_1(t) \\ \mathscr{C}_2(t) \\ \mathscr{C}_3(t) \end{bmatrix}$$

Form the above equations, we can get

$$-(\theta_2-\theta_1)\int_{t-\theta_2}^{t-\theta_1} \langle \dot{x}^T(s) \odot \mathcal{Z} \rangle ds \leq - \left\langle \begin{bmatrix} \mathscr{C}_1(t) \\ \mathscr{C}_2(t) \\ \mathscr{C}_3(t) \end{bmatrix}^T \odot \hat{\mathcal{Z}} \right\rangle - \left\langle \begin{bmatrix} \mathscr{C}_4(t) \\ \mathscr{C}_5(t) \\ \mathscr{C}_6(t) \end{bmatrix}^T \odot \hat{\mathcal{Z}} \right\rangle$$
$$- \begin{bmatrix} \mathscr{C}_1(t) \\ \mathscr{C}_2(t) \\ \mathscr{C}_3(t) \end{bmatrix}^T \mathcal{X} \begin{bmatrix} \mathscr{C}_4(t) \\ \mathscr{C}_5(t) \\ \mathscr{C}_6(t) \end{bmatrix} - \begin{bmatrix} \mathscr{C}_4(t) \\ \mathscr{C}_5(t) \\ \mathscr{C}_6(t) \end{bmatrix}^T \mathcal{X}^T \begin{bmatrix} \mathscr{C}_1(t) \\ \mathscr{C}_2(t) \\ \mathscr{C}_3(t) \end{bmatrix} \quad (9)$$

After a simple rearrangement, from (9), we can get (8).
It should be noted that when $\theta(t) = \theta_1$ or $\theta(t) = \theta_2$, we have $\mathscr{C}_1(t) = \mathscr{C}_2(t) = \mathscr{C}_3(t) = 0$ or $\mathscr{C}_4(t) = \mathscr{C}_5(t) = \mathscr{C}_6(t) = 0$, respectively. Thus (8) still holds, which completes the proof.

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3. Stability Analysis

In this section, we present new delay-dependent asymptotically stability condition for MJSNNs with mode-dependent time-varying delays. For simplicity of presentation, let e_j $(j = 1, 2, ..., 11)$ be $n \times 11n$ matrices with $11n \times n$ blocks, where jth block is an $n \times n$ identity matrix and the others are zero blocks. For example, $y(t) = e_1\xi(t), y(t - d_i) = e_3\xi(t)$, and so on. Here the order of notations are defined as follows:

$$\begin{aligned}\xi(t) = col\{&y(t) \;\; y(t - d_i(t)) \;\; y(t - d_i) \;\; g(\mathcal{M}_i y(t)) \;\; g(\mathcal{M}_i y(t - d_i(t)))\\ &g(\mathcal{M}_i y(t - d_i)) \;\; \frac{1}{(d_i(t))}\int_{t-d_i(t)}^{t} y(s)ds \;\; \frac{1}{(d_i - d_i(t))}\int_{t-d_i}^{t-d_i(t)} y(s)ds\\ &\frac{1}{(d_i(t))^2}\int_{t-d_i(t)}^{t}\int_{\theta}^{t} y(s)dsd\theta \;\; \frac{1}{(d_i - d_i(t))^2}\int_{t-d_i}^{t-d_i(t)}\int_{\theta}^{t} y(s)dsd\theta \;\; \dot{y}(t)\},\end{aligned}$$

$$\mathcal{K}_1 = diag\{k_1^- k_1^+, k_2^- k_2^+, \ldots, k_n^- k_n^+\},$$

$$\mathcal{K}_2 = diag\left\{\frac{k_1^- + k_1^+}{2}, \frac{k_2^- + k_2^+}{2}, \ldots, \frac{k_n^- + k_n^+}{2}\right\},$$

Theorem 3.1 For given scalars d_i, $d = max\{d_i, i \in \mathcal{S}\}$ and μ, the MJSNNs (7) is asymptotically stable, if there exist matrices $\mathcal{P}_i > 0, \mathcal{Q} > 0, \mathcal{R} > 0, \mathcal{S} > 0, \mathcal{U} > 0, \mathcal{V} > 0$, positive diagonal matrix Λ_1, Λ_2, and any matrices $\mathcal{Y}_i$ $(i = 1, 2, ..., 9), \mathcal{W}$ with appropriately dimensions such that the following LMIs holds for all $i \in \mathcal{S}$;

$$\Theta_1 = \Xi_1 - \Upsilon^T\bar{\mathcal{V}}\Upsilon + \Xi_2 + \Xi_3 < 0 \tag{10}$$

$$\Theta_2 = \begin{bmatrix}\hat{\mathcal{V}} & \mathcal{Y}\\ \mathcal{Y}^T & \hat{\mathcal{V}}\end{bmatrix} \geq 0 \tag{11}$$

where

$$\begin{aligned}\Xi_1 = \; & 2e_1^T\mathcal{P}_i e_{11} + \langle e_1^T \odot \textstyle\sum_{j=1}^{s}\pi_{ij}(\mathcal{P}_j)\rangle + \langle e_1^T \odot \mathcal{Q}\rangle - (1 - \dot{d}_i(t))\langle e_2^T \odot \mathcal{Q}\rangle + \langle e_4^T \odot \mathcal{R}\rangle\\ &-(1 - \dot{d}_i(t))\langle e_5^T \odot \mathcal{R}\rangle + \langle e_1^T \odot \mathcal{S}\rangle - \langle e_3^T \odot \mathcal{S}\rangle + \langle e_4^T \odot \mathcal{U}\rangle - \langle e_6^T \odot \mathcal{U}\rangle + d_i^2\langle e_{11}^T \odot \mathcal{V}\rangle,\end{aligned}$$

$$\Xi_2 = \; 2e_{11}^T\mathcal{W}[-e_{11} - \mathcal{A}_i e_1 + e_5],$$

$$\Xi_3 = \left\langle \begin{bmatrix}e_1\\ e_4\end{bmatrix}^T \odot \begin{bmatrix}-\mathcal{K}_1\mathcal{M}_i\Lambda_1 & \mathcal{K}_2\mathcal{M}_i\Lambda_1\\ \maltese & -\Lambda_1\end{bmatrix}\right\rangle + \left\langle \begin{bmatrix}e_2\\ e_5\end{bmatrix}^T \odot \begin{bmatrix}-\mathcal{K}_1\mathcal{M}_i\Lambda_2\mathcal{K}_2\mathcal{M}_i\Lambda_2\\ \maltese - \Lambda_2\end{bmatrix}\right\rangle,$$

$$\begin{aligned}\Upsilon = col[&(e_1 - e_2) \; (e_1 + e_2 - 2e_7) \; (e_1 - e_2 + 6e_7 - 12e_9)\\ &(e_3 - e_2)(e_3 + e_2 - 2e_8)(e_3 - e_2 + 6e_8 - 12e_{10}],\end{aligned}$$

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$$\bar{\mathcal{V}} = \begin{bmatrix} \hat{\mathcal{V}}\mathcal{Y} & \\ \mathcal{Y}^T & \hat{\mathcal{V}} \end{bmatrix}, \hat{\mathcal{V}} = \begin{bmatrix} \mathcal{V} & 0 & 0 \\ 0 & 3\mathcal{V} & 0 \\ 0 & 0 & 5\mathcal{V} \end{bmatrix}, \mathcal{Y} = \begin{bmatrix} \mathcal{Y}_1 & \mathcal{Y}_2 & \mathcal{Y}_3 \\ \mathcal{Y}_4 & \mathcal{Y}_5 & \mathcal{Y}_6 \\ \mathcal{Y}_7 & \mathcal{Y}_8 & \mathcal{Y}_9 \end{bmatrix}.$$

Proof: We choose the following Lyapunov-Krasovskii functional candidate:

$$\begin{aligned} \mathbb{V}(t) = \langle y^T(t) \odot \mathcal{P}_i\rangle + \int_{t-d_i(t)}^{t} \langle y^T(s) \odot \mathcal{Q}\rangle ds + \int_{t-d_i(t)}^{t} \langle g^T(\mathcal{M}_i y(s)) \odot \mathcal{R}\rangle ds \\ + \int_{t-d_i}^{t} \langle y^T(s) \odot \mathcal{S}\rangle ds + \int_{t-d_i}^{t} \langle g^T(\mathcal{M}_i y(s)) \odot \mathcal{U}\rangle ds + d_i \int_{-d_i}^{0}\int_{\theta}^{t} \langle \dot{y}^T(s) \odot \mathcal{V}\rangle ds d\theta. \end{aligned} \tag{12}$$

$\mathcal{L}$ be the weak infinitesimal generator of the random process $\{y(t), r_t, t \geq 0\}$, along the system (7) defined by

$$\begin{aligned} \mathcal{L}\mathbb{V}(t) = & 2y^T(t)\mathcal{P}_i\dot{y}(t) + \langle y^T(t) \odot \sum_{j=1}^{s} \pi_{ij}(\mathcal{P}_j)\rangle + \langle y^T(t) \odot \mathcal{Q}\rangle \\ & - (1 - \dot{d}_i(t))\langle y^T(t - d_i(t)) \odot \mathcal{Q}\rangle + \langle g^T(\mathcal{M}_i y(t)) \odot \mathcal{R}\rangle \\ & - (1 - \dot{d}_i(t))\langle g^T(\mathcal{M}_i y(t - d_i(t))) \odot \mathcal{R}\rangle + \langle y^T(t) \odot \mathcal{S}\rangle \\ & - \langle y^T(t - d_i) \odot \mathcal{S}\rangle + \langle g^T(\mathcal{M}_i y(t)) \odot \mathcal{U}\rangle - \langle g^T(\mathcal{M}_i y(t - d_i)) \odot \mathcal{U}\rangle \\ & + d_i^2\langle \dot{x}^T(t) \odot \mathcal{V}\rangle - d_i \int_{t-d_i}^{t} \langle \dot{x}^T(s) \odot \mathcal{V}\rangle ds, \\ \mathcal{L}\mathbb{V}(t) = & \langle \xi^T(t) \odot \Xi_1\rangle. \end{aligned} \tag{13}$$

Applying Lemma (2.1), we get

$$-d_i \int_{t-d_i}^{t} \langle \dot{x}^T(s) \odot \mathcal{V}\rangle ds \leq -\langle \mathscr{G}^T(t) \odot \Upsilon^T \bar{\mathcal{V}} \Upsilon\rangle, \tag{14}$$

where $\mathscr{G}(t) = col[\mathscr{G}_1(t)\ \mathscr{G}_2(t)\ \mathscr{G}_3(t)\ \mathscr{G}_4(t)\ \mathscr{G}_5(t)\ \mathscr{G}_6(t)]$,
$\mathscr{G}_1(t) = [y(t) - y(t - d_i(t))]$,
$\mathscr{G}_2(t) = [y(t) + y(t - d_i(t)) - \frac{2}{(d_i(t))} \int_{t-d_i(t)}^{t} y(s)ds]$,
$\mathscr{G}_3(t) = [y(t) - y(t - d_i(t)) + \frac{6}{(d_i(t))} \int_{t-d_i(t)}^{t} y(s)ds - \frac{12}{(d_i(t))^2} \int_{t-d_i(t)}^{t}\int_{\theta}^{t} y(s)dsd\theta]$,
$\mathscr{G}_4(t) = [y(t - d_i(t)) - y(t - d_i)]$,
$\mathscr{G}_5(t) = [y(t - d_i(t)) + y(t - d_i) - \frac{2}{(d_i - d_i(t))} \int_{t-d_i}^{t-d_i(t)} y(s)ds]$,

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$\mathscr{G}_6(t) = [y(t-d_i(t)) - y(t-d_i) + \frac{6}{(d_i-d_i(t))}\int_{t-d_i}^{t-d_i(t)} y(s)ds - \frac{12}{(d_i-d_i(t))^2}\int_{t-d_i}^{t-d_i(t)}\int_{\theta}^{t} y(s)dsd\theta]$,

$$\bar{\mathcal{V}} = \begin{bmatrix} \hat{\mathcal{V}} & \mathcal{Y} \\ \mathcal{Y}^T & \hat{\mathcal{V}} \end{bmatrix}, \quad \hat{\mathcal{V}} = \begin{bmatrix} \mathcal{V} & 0 & 0 \\ 0 & 3\mathcal{V} & 0 \\ 0 & 0 & 5\mathcal{V} \end{bmatrix}, \quad \mathcal{Y} = \begin{bmatrix} \mathcal{Y}_1 & \mathcal{Y}_2 & \mathcal{Y}_3 \\ \mathcal{Y}_4 & \mathcal{Y}_5 & \mathcal{Y}_6 \\ \mathcal{Y}_7 & \mathcal{Y}_8 & \mathcal{Y}_9 \end{bmatrix}.$$

where $\Upsilon = col[(e_1 - e_2)\ (e_1 + e_2 - 2e_7)\ (e_1 - e_2 + 6e_7 - 12e_9)\ (e_3 - e_2)\ (e_3 + e_2 - 2e_8)\ (e_3 - e_2 + 6e_8 - 12e_{10}]$,
From the above equations (13) and (14), we get

$$\mathcal{L}\mathbb{V}(t) = \langle \xi^T(t) \odot \Xi_1 - \Upsilon^T \bar{\mathcal{V}} \Upsilon \rangle. \tag{15}$$

For any arbitrary matrices $\mathcal{W}$ with appropriate dimension, the following inequality holds

$$\begin{aligned} 0 &= 2\dot{y}^T(t)\mathcal{W}[-\dot{y}(t) - \mathcal{A}_i y(t) + g(\mathcal{M}_i y(t - d_i(t)))] \\ 0 &= \langle \xi^T(t) \odot \Xi_2 \rangle. \end{aligned} \tag{16}$$

For any $\lambda_{1i} > 0, \lambda_{2i} > 0,\ i = 0, 1, 2, ..., n$, it follows from (6) that

$$\begin{aligned} &[g_i(\mathcal{M}_{ii}y_i(t)) - \mathcal{K}_i^- \mathcal{M}_{ii}y_i(t)]\lambda_{1i} \\ &\times [\mathcal{K}_i^+ \mathcal{M}_{ii}y_i(t) - g_i(\mathcal{M}_{ii}y_i(t))] \geq 0, \end{aligned} \tag{17}$$

$$\begin{aligned} &[g_i(\mathcal{M}_{ii}y_i(t - d_i(t))) - \mathcal{K}_i^- \mathcal{M}_{ii}y_i(t - d_i(t))]\lambda_{2i} \\ &\times [\mathcal{K}_i^+ \mathcal{M}_{ii}y_i(t - d_i(t)) - g_i(\mathcal{M}_{ii}y_i(t - d_i(t)))] \geq 0, \end{aligned} \tag{18}$$

which yields

$$\left\langle \begin{bmatrix} y(t) \\ g(\mathcal{M}_i y(t)) \end{bmatrix}^T \odot \begin{bmatrix} -\mathcal{K}_1\mathcal{M}_i\Lambda_1 & -\mathcal{K}_2\mathcal{M}_i\Lambda_1 \\ \maltese & -\Lambda_1 \end{bmatrix} \right\rangle \geq 0, \tag{19}$$

$$\left\langle \begin{bmatrix} y(t - d_i(t)) \\ g(\mathcal{M}_i y(t - d_i(t))) \end{bmatrix}^T \odot \begin{bmatrix} -\mathcal{K}_1\mathcal{M}_i\Lambda_2 & -\mathcal{K}_2\mathcal{M}_i\Lambda_2 \\ \maltese & -\Lambda_2 \end{bmatrix} \right\rangle \geq 0. \tag{20}$$

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From (19) to (20), we get

$$0 \leq \ \langle \xi^T(t) \odot \Xi_3 \rangle. \tag{21}$$

Adding (15), (16) and (21) yields

$$\mathcal{L}\mathbb{V}(t) = \Big\langle \xi^T(t) \odot \{\Xi_1 - \Upsilon^T \bar{\mathcal{V}} \Upsilon + \Xi_2 + \Xi_3\} \Big\rangle. \tag{22}$$

If $\Xi_1 - \Upsilon^T \bar{\mathcal{V}} \Upsilon + \Xi_2 + \Xi_3 < \ 0$, then $\mathcal{L}\mathbb{V}(t) < \ 0$. Thus, MJSNNs (7) is globally asymptotically stable. This complete the proof.

Remark 3.2 The main objective of the delay-dependent stability criteria is to achieve less conservative results of the developed stability conditions. The main contribution of this work s to develop a novel single integral inequality, which includes Wirtinger-based inequality and Jensens inequality as a special case. Hence, the results derived in this paper are probable to be less conservative which will be established further through standard numerical packages.

Remark 3.3 Generally, the computational complexity mostly depend on the maximum number of decision variables existing in the LMIs. As is known to all, when using delay augmented LKF, and free matrix method, the number of decision variables becomes large, and when the delay subintervals number becomes more, which prompt the complexity and the computational burden of the main results. To overcome this issues, in this work, we have introduced a suitable LKF with more system information and employing new tighter bound inequality for the stability of MJSNNs it can be normal that less conservative outcomes compared than [12], [15], [8]. However, the criteria proposed in this paper not only lead to less conservative stability conditions based on the new integral inequality but also have smaller computational burden, since our theoretical proof is not concerned with any delay-decomposing method or free-weighting matrix method. Hence, the proposed stability conditions provides less conservative results with smaller computational burden simultaneously.

4. Illustrative Examples

In this illustrative section, we show the potency of the proposed new single integral inequality Lemma (2.1) how to reduce the conservatism in the derived stability conditions.

Example 4.1 Consider MJSNNs (7) with $i = 1, 2$, and the parameters as follows:
$\mathcal{A}_1 = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}$, $\mathcal{M}_1 = \begin{bmatrix} 1.6 & 2.9 \\ 0.4 & -1.6 \end{bmatrix}$, $\mathcal{A}_2 = \begin{bmatrix} 0.7 & 0 \\ 0 & 0.8 \end{bmatrix}$, $\mathcal{M}_2 = \begin{bmatrix} 1.9 & 1.7 \\ 0.6 & 0.6 \end{bmatrix}$.
In the system (7), the time-varying delays are assumed to be $d_1(t) = 0.4 + 0.2sint$, $d_2(t) = 0.6 + 0.2sint$. It is obvious that $0 \leq d_1(t) \leq 0.6$, $0 \leq d_2(t) \leq 0.8$. and $\dot{d}_i(t) = 0.3cost \leq 0.3$. By setting the transition probability matrix is $\Gamma = \begin{bmatrix} -2 & 2 \\ 3 & -3 \end{bmatrix}$, then we choose $d_1 = 0.6$, $d_2 = 0.8$, $\mu = 0.3$ by using the MATLAB LMI control Toolbox to solve the LMI (10) in Theorem (3.1) is feasible.

Example 4.2 Consider MJSNNs (7) with $i = 1$ and the parameters as follows:

$$\mathcal{A}_1 = \begin{bmatrix} 7.0214 & 0 \\ 0 & 7.4367 \end{bmatrix}, \quad \mathcal{M}_1 = \begin{bmatrix} -6.4993 & -12.0275 \\ -0.6867 & 5.6614 \end{bmatrix},$$

We consider this example for comparison with the stability criteria developed in [8], [12], [15]. By using the Matlab LMI toolbox to Theorem (3.1), it is found that LMI (10) is feasible. One can obtain the maximum allowable upper bound d_1, under different μ. The obtained maximum admissible upper bound d_1 values is listed in Table 1. From this Table 1, we conclude that the proposed results in this paper is less conservative than those results discussed in [12], [15], [8].

Table 1: The maximum allowable upper bound d_1 for different μ.

Method	μ	0.3	0.5	0.7	0.9
[12]	d_1	0.8402	0.5493	0.4264	0.3214
[15]	d_1	3.8532	3.2011	2.1837	1.7912
[8]	d_1	3.8532	3.2011	2.1837	1.7912
Theorem (3.1)	d_1	3.8932	3.2211	2.9137	1.9012

5. Conclusion

In this paper, the problem of delay-dependent stability analysis of Markovian jump static neural networks with mode-dependent time-varying delays have been investigated. By fully using the available information about time-delays

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and activation functions, a suitable LKF with delay-dependent terms have been constructed. By employing integral inequalities, some sufficient conditions are established that ensure the asymptotically stability for the considered system model. The resulting stability criterion is derived and entrenched in terms of LMI, which can be straightforwardly tested by applying Matlab LMI toolbox. Finally, two numerical examples are provided to demonstrate the less conservatism and effectiveness of the proposed methods.

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